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AERODYNAMICS OF AIRFOIL IN SHEAR FLOWS-A NUMERICAL STUDY

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ABSTRACT

A Numerical Study has been carried out to determine the influence of shear flow parameters (shear spread (d), and the vertical orientation of the shear stream (y_s)) on the aerodynamic coefficients C_L and C_d. The results are obtained for a Joukowski Airfoil immersed in Gaussian exponential shear stream at Reynolds number of 20,000. In order to exclude the effect of the incoming stagnation pressure and study only the effect of the vorticity carried by the Gaussian shear stream, the aerodynamic coefficients are calculated with respect to the dynamic pressure of the separating streamline at the Gaussian profile.

The integro-differential formulation is used to solve the governing Navier-stokes equations. The results show that the Gaussian shear parameters (y_s) and (d) have small influence on the aerodynamic coefficients. This conclusion recommend that the strong interaction between the vorticity carried by the shear stream and the circulation around the airfoil (predicted by the inviscid analysis) is considerably weakend by viscous dissipation.

1. INTRODUCTION

A great number of researchs concerning uniform flows over airfoil exists in the literature. There are, however, many important applications in which shear flows should be included. One such example is presented by a wing in natural wind as occurs near the ground, where the predominant feature of the wind stream is a considerable vertical gradient in velocity. Other applications include flows behind propellers and wings.

In 1942, Tsien [1] used the earlier studies of Sanden (1912) to develop the theory of symmetrical Joukowsky airfoils in a stream

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of uniform shear. Tsien generalized the well-known Blasius theorem for calculating aerodynamic forces acting on an airfoil immersed in a shear flow. Tsien's results indicate that the effect of a velocity gradient on the lift coefficient is equivalent to a shift in the angle of zero lift. The amount of the shift depends on the magnitude of the velocity gradient and the thickness-to-Chord ratio. For realistic values of velocity gradient, the shift in the angle of zero lift is less than 0.2 deg.

In 1958, Sowyrda [2] investigated the magnitude of the shear effect on cambered profiles. Sowyrda extended the classical (inviscid) theory of Tsien for cambered airfoils. He showed that the shear strength increases the lift coefficient at low angle of attacks. The combined effect of both the shear strength and the camber was predominate at zero angle of attack in Sowyrda's work.

In 1973, Frost and Hutto [3] analytically studied the influence of wind shear on lift, drag, roll and yaw moments of a wing in a horizontal wind gradient at various elevations and all angles. Frost and Hutto employed a general series solution for the distribution of lift along the wing span to compute loads and moments on the airfoil. The system of governing equations for the lift, induced drag, roll and yaw moments for a variety of wind conditions was solved numerically. The results indicated that wind shear can have a significant effect on the rolling and Yawing moments of an aircraft. The influence of the wind shear on lift and induced drag was not thoroughly investigated by Frost [3] . Chow etc...all [4] solved numerically the inviscid, incompressible rotational flow over airfoils. The free stream is represented by the Gaussian exponential velocity profile. The numerical results demonstrated strong interaction between the vorticity carried by the free stream and the circulation around the airfoil. The changes in pressure distribution due to the vorticity and stagnation pressure effects are much more pronounced on the upper surface than those on the lower surface. The numerical results showed that there is an optimum vertical location of the airfoil relative to the upstream profile (y_s) for maximum lift. A correlation between the maximum lift and a nonuniformity parameter of the upstream profile (d) was obtained also in reference [4] .

Experimental research related to the influence of wind shear on the aerodynamic coefficients is very rare. Recently Payne etc. ... all [5] conduced an experimental wind tunnel investigation to determine the influence of a nonuniform wind profile on the static aerodynamic coefficients of an airfoil. Force balance and surface pressure measurements were obtained from NACA 0018 airfoil in a linear velocity profile. The airfoil was tested in a Reynolds number range from 7.5×10^4 to 2×10^5 with end plates to simulates an infinite wing. It should be noted that Payne 's results are restricted only for linear shear flows. The influence of the velocity gradient on the aerodynamic characteristics was found to be small in Payne's work.

In most of the foregoing literature review, the analytical and numerical investigations were carried out for only inviscid shear flows. However, nowadays it is widely accepted that viscous effects do control and regulate the basic mechanisms in the flow field. It is expected that there will be a considerable interaction between the vorticity carried in the nonuniform stream and the viscous stress near the airfoil. For this reason, it is the purpose of the present investigation to determine, numerically, the influence of the shear parameters of the Gaussian shear profile (shear spread(d) and the vertical orientation of the shear stream (y_s) on the aerodynamic coefficients C_L and C_d (Figure 1). The present results are compared with the inviscid results of Payne [5].

The exact Navier-Stokes equations rescast in an integro-differential formulation [6] is used to study the viscous flow over Joukowski airfoil. This formulation has many advantages over the prevailing pure finite difference approach [6] . The Joukowski profile is obtained from mapping of radius 0.5 with the center located at x = - 0.05 and y = 0.05. The chord length is 1.81. The study was done for a constant Reynold number of 20,000. The Reynolds number is based on the circle diameter the basic uniform evelocity U_o(Figure 1).

2- MATHEMATICAL FORMULATION

2.1 Governing Equations

The vorticity transport equation for an incompressible unsteady flow is written as [6] :

$$\frac{\partial\omega}{\partial t} = \frac{1}{H^2} [-\vec{\nabla} \cdot (\vec{V}_a\omega) + \frac{1}{Re} \nabla^2 \omega]^2 \qquad (1)$$

where ω is the magnitude of the vorticity vector and $\vec{V}_a$ is the apparent velocity vector in the transformed plane. H^a represents the scale factor of transformation.

The kinematic aspect of the flow is presented as [7] :

$$\left.\begin{aligned} \vec{V}_a(r_o,t) = &- \frac{1}{2\pi} \int_R \frac{\vec{\omega}(r,t) \times (\vec{r} - \vec{r}_o) dR}{|\vec{r} - \vec{r}_o|^2} \\ &- \frac{1}{2\pi} \int_{R_o} \frac{\vec{\omega}_u (r,t) \times (\vec{r}-\vec{r}_o) dR}{|\vec{r} - \vec{r}_o|^2} \end{aligned}\right\}$$

$$- \frac{1}{2\pi} \int_B \frac{(\vec{U} \times \vec{n}) \times (\vec{r}_b - \vec{r}_o) r_b \, d\theta}{|\vec{r}_B - \vec{r}_o|^2}$$

$$+ \frac{1}{2\pi} \int_B \frac{(\vec{U} \cdot \vec{n}) (\vec{r}_b - \vec{r}_o) r_b \, d\theta}{|\vec{r}_b - \vec{r}_o|^2} \qquad (2)$$

where; R is the computational region, R_o is the outer region and it extends to infinity, B represents the outer boundary (Figure 1). $\vec{n}$ is the unit vector normal to the outer boundary, $\vec{r}_o$ is the position vector at the required point r_o. $\vec{U}$ and $\vec{\omega}_u$ are the velocity vector and the wake vorticity vector of the free stream Gaussian profile (Figure 1):

$$\bar{U} = 1 + a \exp - \{\frac{y-y_s}{d}\}^2 \text{, for basic velocity } U_o = 1 \ldots \qquad (3)$$

The fluid domain is divided into two regions R and R_o. The inner region (Figure 1) is the region where the computations are carried out. This inner region contains the grid nodes that have vorticity different from the undisturbed wake vorticity $\vec{\omega}_u$. The kinematic boundary condition for the external flow problem requires that the velocity has to reach the free stream velocity at an infinite distance away from the solid surfaces. In the present work this requirement is satisfied by equation (2).

Equations (1) and (2) describe the Integro-differential formulation [6] . This formulation is derived from the Navier-stokes equations. However, it posses many advantages over the prevailing primitive variable Navier-Stokes system. Details can be seen in reference [7].

2.2 Initial and Boundary Conditions

The Gaussian profile is used as an initial condition in the present work. Along the body surface, the no-slip condition is used to describe the velocity boundary conditions. The surface vorticity at the successive times is calculated by satisfying the no-slip condition as well as the conservation of total vorticity [7] . At the outer computational boundary (Figure 1), the Gaussian profile is used as the outer boundary conditions.

3. NUMERICAL FORMULATION

3.1 Airfoil Transformation and Grid System

The transformation function:

$$z = \xi - \xi_o + \frac{k^2}{\xi - \xi_o} \qquad \text{where } \xi_o = 0.05 - 0.05i$$

$$\text{and } K = \cdot 0.4174$$

produces a cambered Joukowski airfoil with a rounded trailing edge. The Chord length is 1.81 relative to the diameter of the circle. The above transformation function also transforms the region outside the airfoil to the outside of a unit circle in the transformed plane. In the transformed plane, the grid net is chosen such that a stretching r-coordinates is used as follows:

$$r = \text{Exp}\,(s_o + (J-1)\,\Delta s) + C\,, \quad J = 1\,, \ldots 50$$

$$\text{with } s_o = -1.44\,, \quad \Delta s = 0.082$$

The grid spacing in θ-direction was $\Delta\theta = \frac{\pi}{24}$.

3.2 Numerical Model

The finite-differencing approximation is used to express the vorticity transport equation (1) in difference form. Central differences are used to approximate the derivatives terms in equation (1).

The velocity expression (equation 2) includes the evaluation of the integrals in equation (2). In the present work the finite-Fourier series approach is used to calculate the velocities and the surface vorticities [6].

3.3 Numerical Procedure

(i) With all the dependent variables known at the previous time level, the vorticity finite-difference equation is solved at all the grid points (using successive under relaxation technique) to obtain the vorticity values at the new time level.

(ii) New values of the boundary vorticity are obtained using the integral representation. The steps (i) and (ii) are repeated until the surface vorticity satisfies a prescribed convergence criterion.

(iii) Using the new values of vorticities, the velocities are calculated at this new time level.

(iv) The procedure is advanced for successive time levels.

4. NUMERICAL RESULTS AND DISCUSSION

4.1 Numerical Experiment

The present Integro-differential Formulation is coded into a computer program. In order to check the validity of the method as well as the accuracy of the computer code, a numerical experiment was conducted on a circular cylinder. Figure (2) shows the comparison between the present results and the results of References [8, 9] . The comparison is done for uniform flow at Reynold number of 40. It is evident in Figure (2) that the present method compares very good with experiments.

4.2 Results of Studied Cases

The computer code was used to obtain results for different combinations of the Gaussian profile parameters; (y_s), (d) and (a). Table (1) presents the cases that are studied.

Table (1)

Test case number	a	y_s	d	Test case number	a	y_s	d
1	1.0	-0.3	1.0	8	1.0	0	0.5
2	1.0	-0.2	1.0	9	1.0	0	$\frac{1}{\sqrt{2}}$
3	1.0	-0.1	1.0	10	1.0	0	1
4	1.0	0	1.0	11	1.0	0	1
5	1.0	0.1	1.0	12	1.0	0	1.5
6	1.0	0.2	1.0	13	1.0	0	2.0
7	1.0	0.3	1.0				

Reference (4) indicated that both the upstream stagnation pressure and the vorticity carried by the Gaussian stream control the values of the lift and drag coefficients. The effect of the stagnation dynamic pressure is excluded in the present study since the objective of this study is to study the influence of the vorticity carried by the Gaussian profile on the aerodynamic coefficients C_L and C_d. Indeed the vorticity of the profile depends mainly on the Gaussian parameters (d) and (y_s).

It should be noted that the exclusion of the dynamic pressure is achieved by normalizing the lift and drag with respect to the profile dynamic pressure of the separation stream line ($1/2\rho U_\infty^2$) (Figure 1). Figure 3 shows the variation of the present lift coefficient with the vertical orientation length (y_s), for a = 1 and d = 1. It is evident in the figure that(y_s)has a negligible effect on the lift and drag coefficients. However, for inviscid calculations (Reference 4)(y_s)has a small (but not negligible) effect on C_l (about(8 %).

Figure (4) compares the influence ofthe shear spread (d) on the lift coefficient between the present viscous calculations and the inviscid calculations of Reference[4] for y_s=0.0.The inviscid results predict a strong influence of the shear spread (d) on the lift coefficient while the present results show a weak influence on C_l. The discrepancy between the the present viscous results and the inviscid results is a consequence of the dissipation action of the viscous flow. As indicated by reference [4] the decrease of shear spread (d) increases the vorticities at and around the separating streamline (Figure 1). These vorticities have the same positive since as the circulation around the airfoil. Thus the lift coefficient increases since it depends on both the airfoil circulation and the vorticities carried in the incoming stream. This is true in the inviscid calculations however, for viscous flow the viscous shear stress strongly interact with these vorticies and greatly discripate them (Figure 5).

Figure (4) shows also the drag coefficient variation with the shear spread (d). The coefficient is almost constant and compares very good with the uniform drag coefficient (0.0043). Payne [5] showed experimentally that the effect of shear gradient on the aerodynamic coefficients is negligibly small for 75,000<Re<200,000. This conclusion agrees with the present results and assures that the viscous effects strongly dissipate the shear gradients for low angle of attacks cases.

5. CONCLUSIONS

The influence of the Gaussian shear parameters (y_s) and (d) on the aerodynamic coefficients of airfoil was found to be negligibly small for viscous computations. The strong interaction between the vorticity carried by the Gaussian profile and the airfoil circulation is greatly dissipated by viscous effects.

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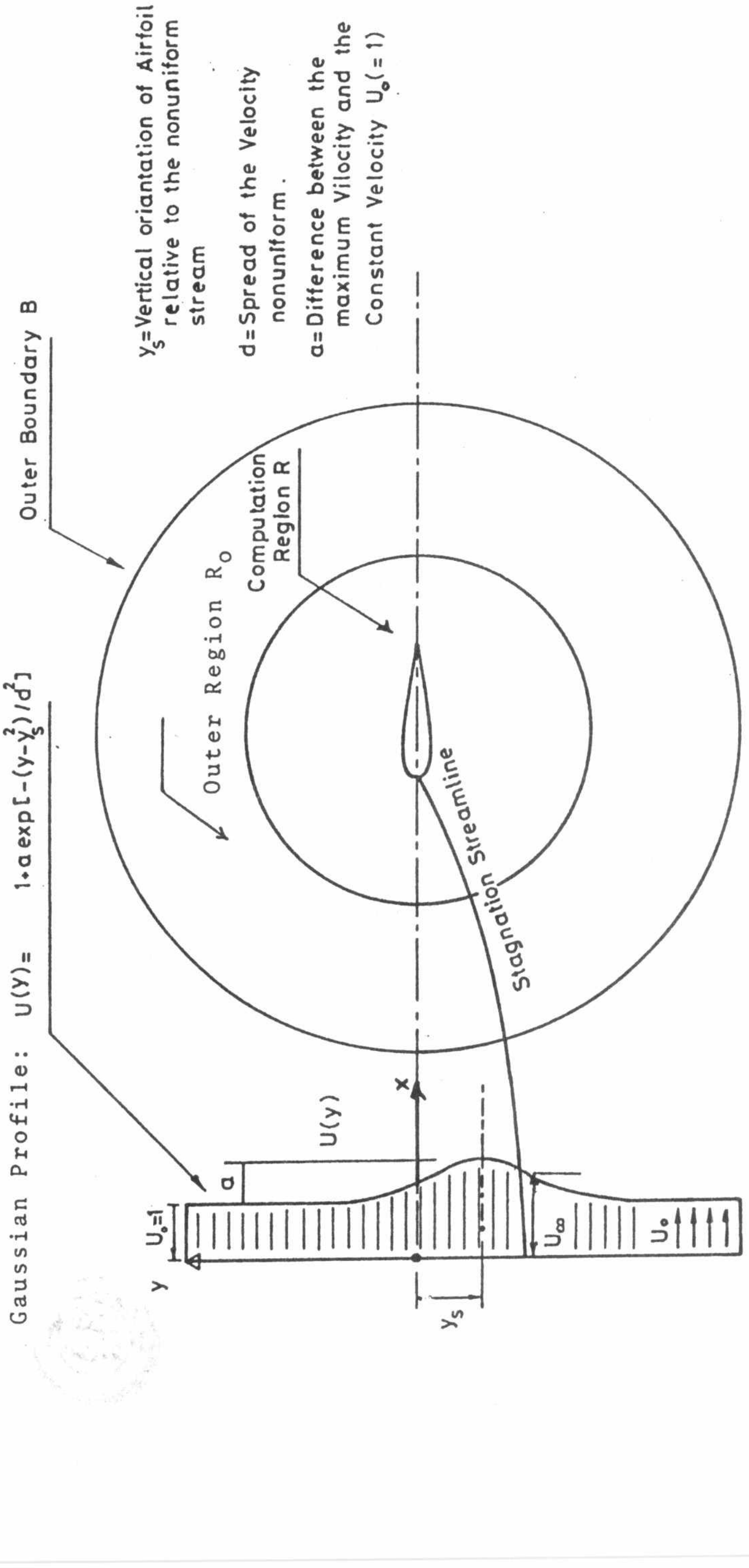

Fig.1. Airfoil in a Nonuniform Parallel Stream

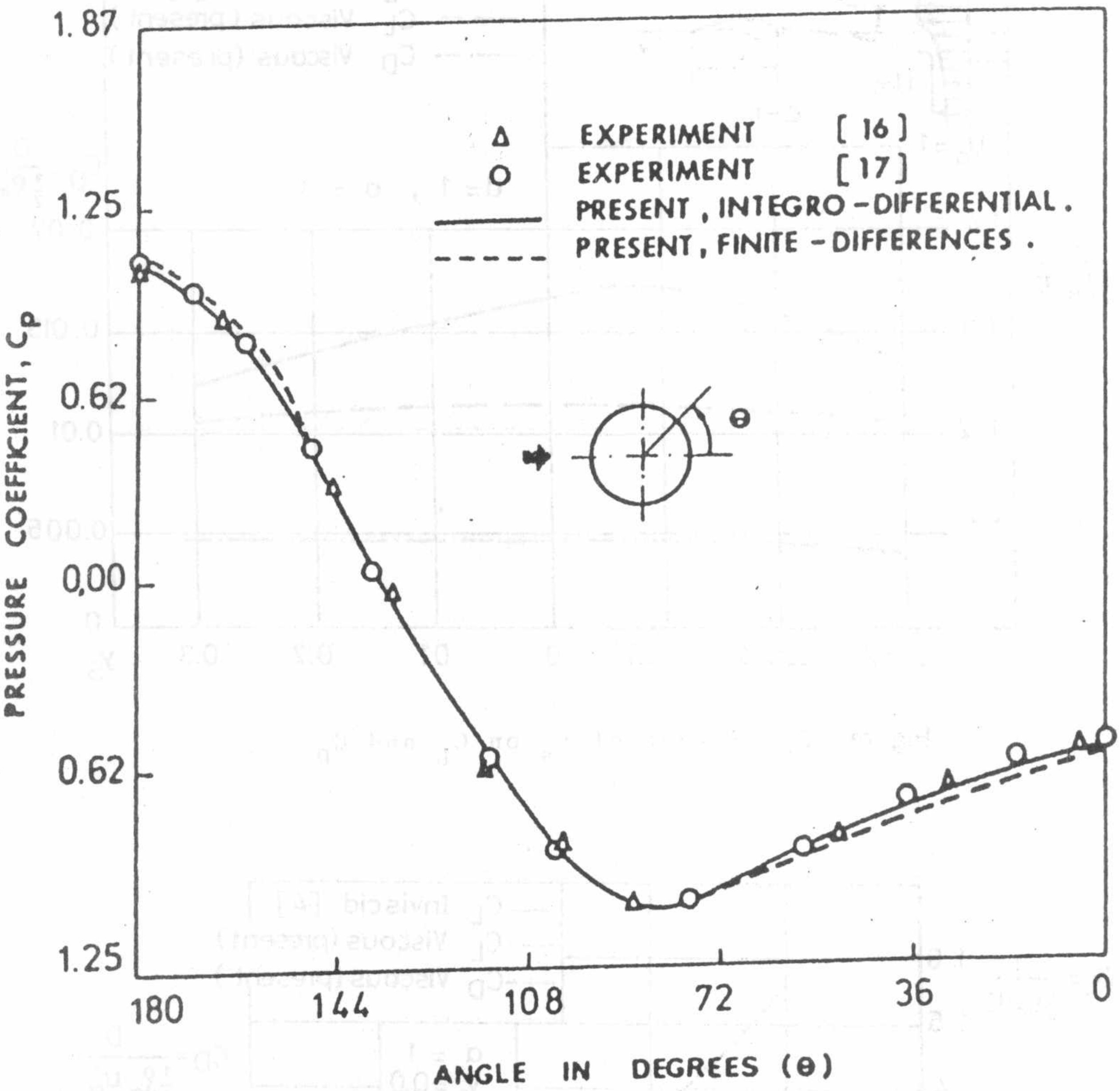

Figure 2 Pressure Distribution on Circular Cylinder in Uniform Flow (RE = 40)

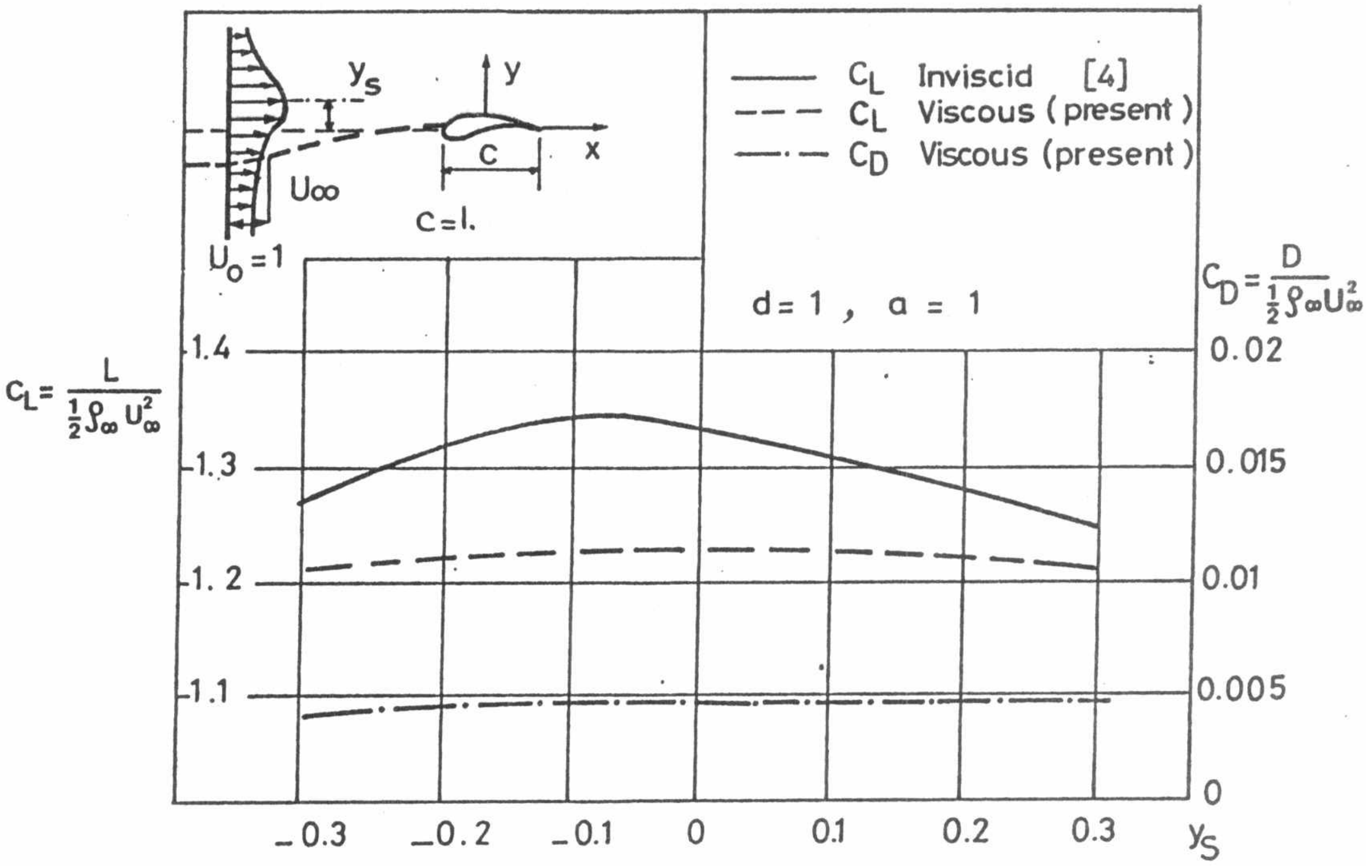

Figure 3 Effect of y_s on C_L and C_D

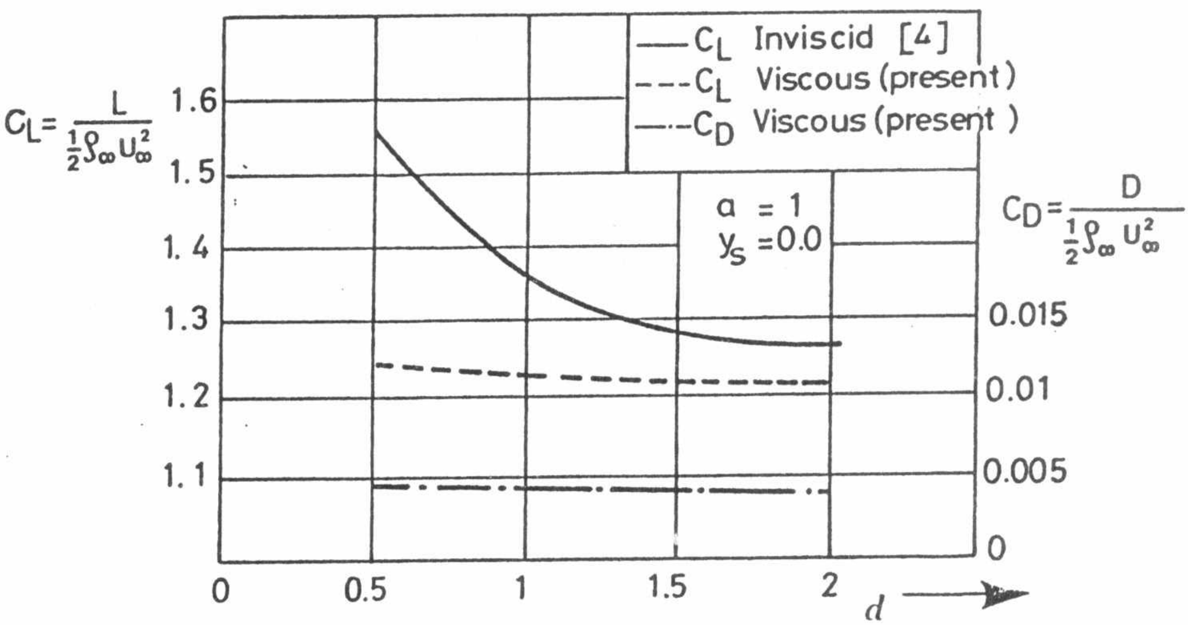

Figure. 4 Effect of d on C_L and C_D.

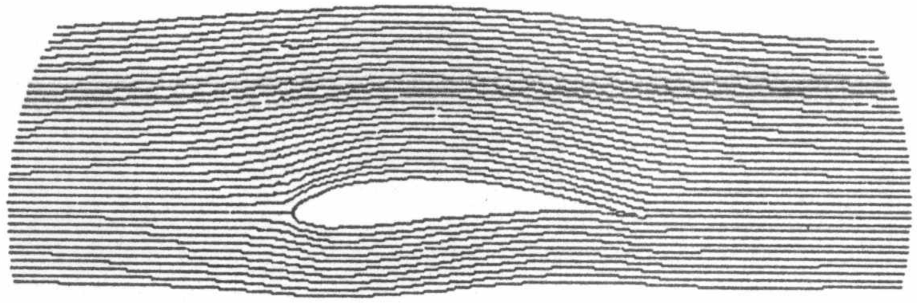

Figure (5) Streamline Contours over Joukowski Airfoil for Re = 20,000 (at early time levels).

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