



1  
2  
3  
4

THE EVOLUTION OF TWO-DIMENSIONAL UNDERDEVELOPED  
TURBULENT BOUNDARY LAYER IN A TURBULENT  
FREE STREAM .

Dr. M. Salah E. ABDEL-KAREEM \*

&

Dr. Ibrahim M. M. A. SHABAKA \*\*

ABSTRACT

It is proved that the energy exchange processes during the laminar boundary layer transition into turbulence in the presence of free stream turbulence, take place in accordance with the relationships that govern the same energy exchange processes of the phenomenon in the case of a quiescent free stream .

Bradshaw's method ( Bradshaw et al, 1974 ) for calculating the turbulent boundary layer, succeeded in predicting with a reasonable accuracy, the development of the two-dimensional boundary layer during the final stages of transition and before the turbulent boundary layer is fully developed, without introducing any changes in the original program to account either for the turbulence in free stream or for the fact that the boundary layer is not fully developed. These changes will be dealt with in future work .

This implies that, the development of the turbulent quantities within the transitional-turbulent boundary layer follows the universal turbulent boundary layer mechanisms regardless of the level of the free stream turbulence . It is also an indication of the validity of Abdel-Kareem (1978) conclusion that the mechanism of the laminar boundary layer transition is universal regardless of the free stream turbulence level .

\* Lecturer , department of aeronautics , faculty of engineering , Cairo university .

\*\* Assistance professor , department of aeronautics , faculty of engineering , Cairo university .

INTRODUCTION .

One of the major factors that affect transition over a flat plate is the free stream turbulence . We already know that such a turbulence introduce disturbances into the boundary layer growing over the surface . If such a disturbance is small, it will grow inside the boundary layer according to the stability theory and will tend to excite the boundary layer natural modes ( i.e. Tollmien-Schlichting waves ) . If it is large enough, Reshotko (1969) claims that it can grow by forcing mechanisms to nonlinear levels and lead directly to turbulence inside the boundary layer. Schubauer (1957), Morkovin(1958) and Tani(1969) found that the laminar boundary layer may be highly randomly agitated and yet retains the laminar behaviour .

Chen et al (1971) found that the free stream disturbance spectrum affected not only the location of transition, but also the frequency of the turbulent spot formation within the transitional boundary layer and hence, the extent of the transition zone . Added to all this, is the possible effect of the inactive motion (Townsend, 1961 and Bradshaw, 1967a&b) induced by the free stream turbulence and/or the turbulence in the outer region of the boundary layer on the nature of the energy processes in the transitional-turbulent boundary layer .

The important question here is, whether the resulting turbulent boundary layer beneath a turbulent free stream has the same nature as that growing in a quiescent free stream? That is, are the energy quantities within the former related to each other by the same relationships as the latter ? It is also important to find out whether the energy processes such as production and dissipation

in the fully developed turbulent boundary layer is the same in the two cases or not .

If this turns out to be the case , we may conclude that all what the free stream turbulence is interfering to do is to trigger and speed up a universal process of laminar boundary layer transition without really either changing the universal relationships between the turbulent quantities in the resulting turbulent boundary layer even before it is fully developed .

The idea behind the present work is to predict the development of the mean velocity profile at the end of the transition zone and the beginning of the turbulent zone just before it is fully developed , beneath free stream turbulence level of varying intensities , by means of applying Bradshaw's (1974) method on the data of Abdel-Kareem(1978) for a two dimensional flow over a flat plate in a turbulent free stream .

In doing so, the experimental boundary conditions are to be fed to the computer program as if they were boundary conditions for a non-turbulent free stream . Then the output of the computation is then to be compared with the experimental profiles .

The underlying assumption in the present work is that , the characteristic assumption of Bradshaw's method that there is a functional relationship between  $uv$  &  $q$  , is holding even in the initial stages of the development of the turbulent boundary layer in the turbulent free stream , and also that the shear stress (  $uv$  ) and the turbulent diffusion of the turbulent kinetic energy are related with the same relationships in the two flows . This is a crude assumption that will be investigated in future work ( to be published ) .

THE CALCULATION METHOD

Bradshaw's method considers the equations of motion for the two-dimensional turbulent boundary layer in the following form :

\* The continuity equation :

$$1- \quad \frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0$$

\* The momentum equation :

$$2- \quad U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = -\frac{1}{\rho} \frac{\partial P}{\partial X} + \frac{1}{\rho} \frac{\partial}{\partial Y} \left( \frac{\partial U}{\partial Y} - \int \overline{u'v'} \right) \\ = U_{\infty} \frac{\partial U_{\infty}}{\partial X} + \frac{1}{\rho} \frac{\partial \tau}{\partial Y}$$

\* The turbulent energy equation : ( outside viscous sublayer )

$$\left( U \frac{\partial}{\partial X} + V \frac{\partial}{\partial Y} \right) \frac{1}{2} \overline{q^2} = - \frac{\partial U}{\partial Y} - \frac{1}{\rho} \frac{\partial}{\partial Y} \left( \overline{p v} - \frac{1}{2} \rho \overline{q^2 v} \right) - \epsilon$$

where  $\overline{q^2} = \overline{u^2} + \overline{v^2} + \overline{w^2}$  ,  $\tau = -\int \overline{u'v'}$  &  $\epsilon = \nu \left( \frac{\partial u_i}{\partial x_j} \right)^2$

The last equation can be regarded as an equation for the advection or the rate of change of the turbulent kinetic energy  $\left( \frac{1}{2} \overline{q^2} \right)$  ( the L.H.S. of the equation ) through a point .

The method then defines :

$$a- \quad a_1 = \frac{\tau}{\rho \overline{q^2}}$$

$$b- \quad L = \frac{(\tau/\rho)^{3/2}}{\epsilon}$$

$$c- \quad G = \left( \frac{\overline{p v}}{\rho} + \frac{1}{2} \overline{q^2 v} \right) / \left( \left( \frac{\tau_{\max}}{\rho} \right)^{1/2} \frac{\tau}{\rho} \right)$$

Which transform the energy equation into :

$$3- \quad U \frac{\partial}{\partial X} \left( \frac{\tau}{2a_1 \rho} \right) + V \frac{\partial}{\partial Y} \left( \frac{\tau}{2a_1 \rho} \right) = \frac{\tau}{\rho} \frac{\partial U}{\partial X} - \left( \frac{\tau_{\max}}{\rho} \right)^{1/2} \frac{\partial}{\partial Y} \left( G \frac{\tau}{\rho} \right) - \frac{(\tau/\rho)^{3/2}}{L}$$

Where  $a_1$  ,  $\frac{L}{\delta}$  & G are dimensionless functions of  $y/\delta$  and they relate the turbulent kinetic energy , the dissipation of turbulent energy and the turbulent energy diffusion to the local value of the shear stress and can all be estimated from experiments .

Equations 1 , 2 & 3 form a set of equations for the three unknowns U , V & T . This set of equations is hyperbolic, as it contains the diffusion term that takes into account the upstream effects . This means that there exist as many characteristic lines as there are equations along which the partial differential equations reduce to ordinary differential equations containing the gradients along the characteristics only . A full listing of the FORTRAN computer program (also available on a floppy disc ) together with the report explaining the method is available from the author . Mr. Elbeheery in his M.Sc. report has explained the different routines in the program in greater details .

The method is proved to be a great success , largely in my opinion due to its sound physical bases compared with other available methods .

The calculation method is used in the present work as a measure of the deviation of the boundary layer in question from the fully developed turbulent boundary layer case , based on the assumption that the relations between the turbulent quantities are always governed by the proposed relations ( a , b & c ) and any deviation from this assumption has a simple effect on the rest of the calculation that diminishes as this deviation decreases .

## RESULTS

The computations were performed by means of an IBM XT computer .

Figures 1 to 5 show the results of the computations for two cases, the first case using the Coles' family of profiles to generate the starting profile for the calculations , and in the second case, the first experimental profile is used as a starting profile .

## Comments

- 1 - The collapse of the calculated profiles on the experimental ones is not bad , and it is improving as the calculations proceeds .
- 2 - The calculations recovered rather quickly to the experimental values in the case of using the first experimental profile compared with the case of using the synthetic starting profiles .
- 3 - There is a pronounced waviness in the calculated profiles which we think needs further investigations ( to be studied in future work, to be published ) .
- 4 - The rather fast recovery of the calculated profiles to the free stream value is perhaps due to the fact that the diffusion in this case in the outer layer of the boundary layer is larger due to the assumed quiescent free stream .
- 5 - There is the expected discrepancy between the calculated and the experimental profiles in all the cases in the inner layer in the early stages of development of the turbulent boundary layer where the mean velocity profile is still not an accurate representation of the high turbulent activities of the flow there, due to the artificially speeded up boundary layer instability .
- 6 - When the diffusion is given enough chance, the mean velocity profiles are brought back to the normal shape

and the calculations predict the flow with an increasing accuracy .

### Conclusion .

The presented results clearly show that the process of transition in the presence of turbulence in the free stream produces turbulent boundary layers that are predictable to an acceptable accuracy by the ordinary methods of predicting the turbulent boundary layer in a non-turbulent free stream .

This implies that the turbulent quantities within the transitional-turbulent boundary layer follow the universal turbulent boundary layer mechanisms regardless of the level of the free stream turbulence .

This conclusion supports Abdel-Kareem's (1978) conclusion that the mechanism of the laminar boundary layer transition is universal regardless of the level of the free stream turbulence .

### REFERENCES

- 1 - ABDEL-KAREEM, M.S.E. (1978)  
Effect of free stream turbulence on boundary layer transition .  
Ph.D. thesis , University of London .
- 2 - BRADSHAW, P. , FERRISS, D.H. & ATWELL, N.P. (1967)  
JFM , vol. 28, part 3 , pp 593 - 616 .
- 3 - BRADSHAW, P. (1967) a  
"Irrotational fluctuations near a turbulent boundary layer "  
JFM , vol. 27 , pp 209 - 230 .
- 4 - BRADSHAW, P. (1967) b  
"Inactive motion and pressure fluctuations in turbulent boundary layers".  
JFM , vol. 30 , pp241 .
- 5 - CHEN, K.K. & THSON, N.A. (1971)  
"Extension of Emmons' spot theory to flows on blunt bodies".  
AIAA J. , vol. 9 , no, 5 , pp 821 .

- 6 - MORKOVIN, M.V. (1958)  
"Transition from laminar to turbulent shear flow . A review of some recent advances in its understanding".  
Trans. ASME , vol. 80 , pp 1121 .
- 7 - RESHOTKO, E. (1976)  
"Boundary layer stability and transition".  
Annual review of fluid mechanics, vol. 8, pp 311.
- 8 - SCHUBAUER, G.B. (1957)  
"Mechanism of transition of subsonic speeds".  
Boundary layer research symposium, Fresburg (Ed. H. Gortler) .  
Berlin Springer Verlag , pp 85 .
- 9 - TANI, I. (1969)  
"Boundary layer transition ".  
Ann Rev. fluid mech., vol. 1 , pp 169 .
- 10- FL BEHEERY, Z.I.A. (1984)  
M.Sc. thesis  
Department of Aero. , Cairo University .



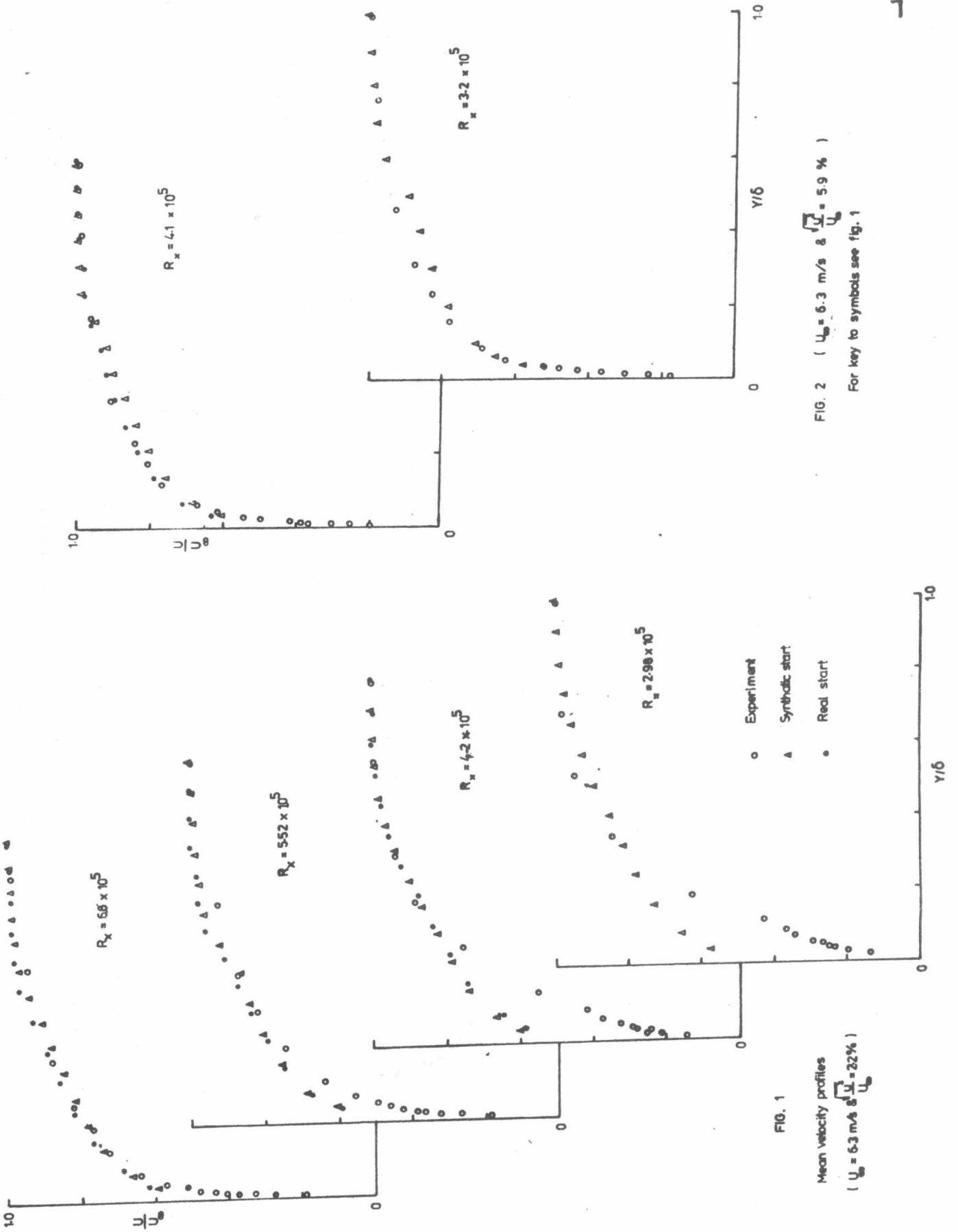


FIG. 2 (  $U_0 = 6.3$  m/s &  $\sqrt{\frac{U}{U_0}} = 5.9\%$  )  
For key to symbols see fig. 1

FIG. 1  
Mean velocity profiles  
(  $U_0 = 6.3$  m/s &  $\sqrt{\frac{U}{U_0}} = 22\%$  )

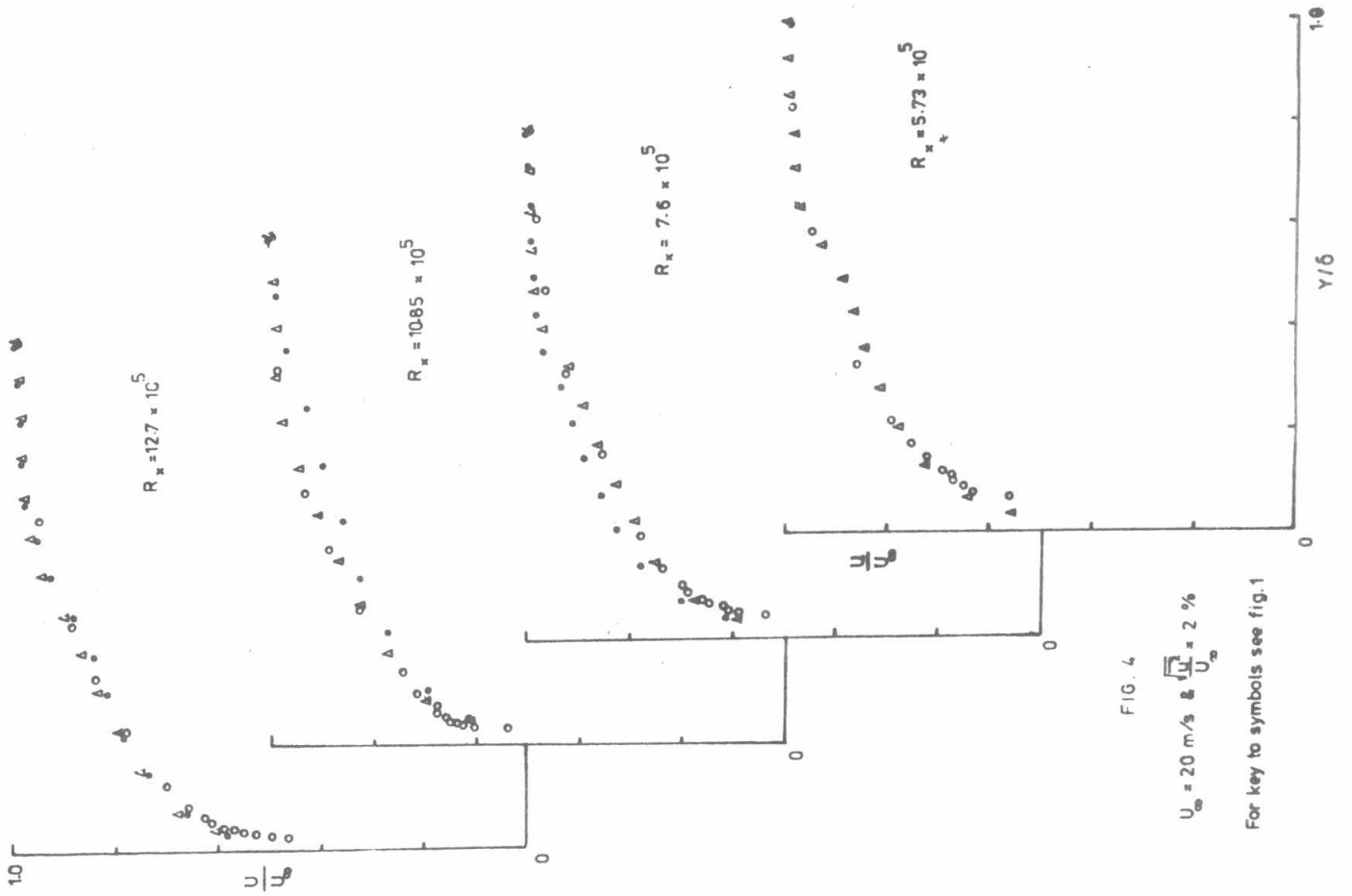


FIG. 4

$U_0 = 20 \text{ m/s}$  &  $\frac{\sqrt{U}}{U_0} = 2\%$

For key to symbols see fig.1

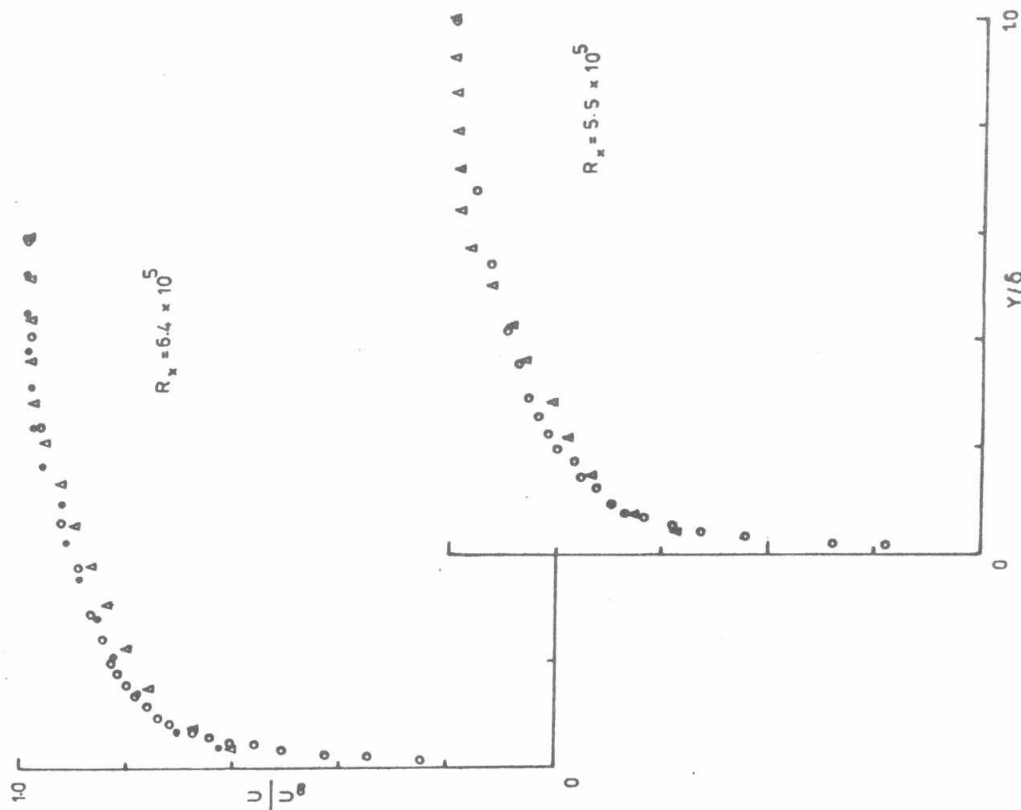


FIG. 3 (  $U_0 = 8.5 \text{ m/s}$  &  $\frac{\sqrt{U}}{U_0} = 1.85\%$  )

For key to symbols see fig.1

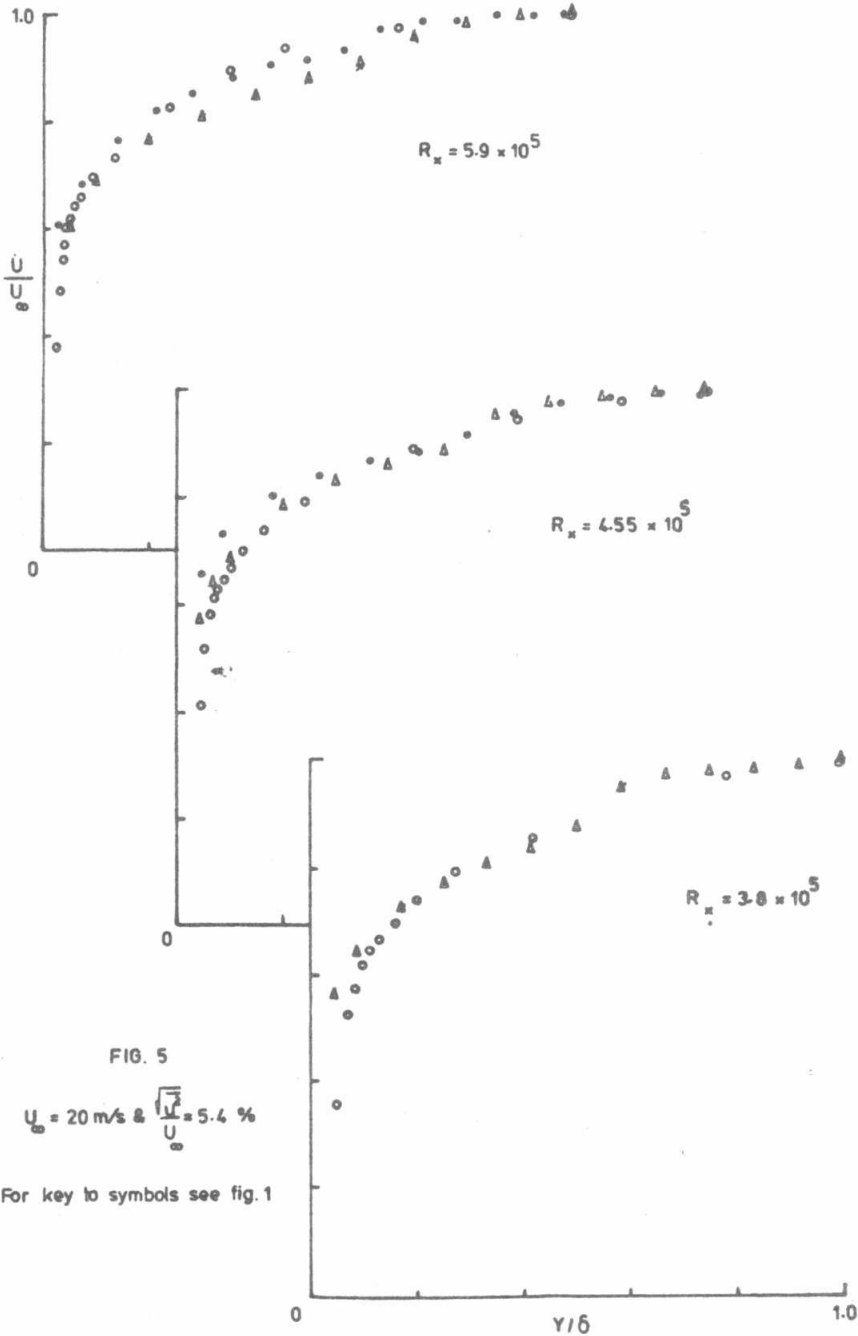


FIG. 5  
 $U_\infty = 20 \text{ m/s}$  &  $\frac{\sqrt{U}}{U_\infty} = 5.4 \%$   
 For key to symbols see fig. 1