

GENERALIZED CORRELATIONS FOR THE PROCESS OF
ENERGY SEPARATION IN VORTEX TUBES

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ABSTRACT:

The Ranque-Hilsch vortex tube is a simple device which splits a compressed gas stream into a cold stream and a hot stream. The process of energy separation taking place in the vortex tube has been investigated theoretically and experimentally. The analysis of the dimensional groups that govern the tube performance as a cooling device have shown that; for geometrically similar tubes, the inside diameter of the tube is a parameter. This conclusion was verified experimentally. The experimental results have also indicated that there is an optimum tube diameter for best cooling performance. It was also shown that the ratio between the temperature drop of the cold stream achieved at a certain pressure ratio to its maximum value $(T_o - T_c)/(T_o - T_c)_{max}$ can be expressed by a sole function of the cold mass fraction U (U: kg. cold air per kg. supply air to the tube), and does not depend on other conditions of operation.

On the other hand, the correlation between the analytical and experimental results have shown that the overall performance of the vortex tube can be expressed by a generalized formula which is valid for all conditions of operation. Such formula can predict the cooling performance of the vortex tube from knowledge of its inside diameter, type of the working medium (expressed by the specific heat ratio C_p/C_v), its inlet condition (temperature T_o and pressure P_o), the required cold stream pressure (P_c) and cold mass fraction (U).

INTRODUCTION:

The process of energy separation taking place inside the vortex tube is very complex. It consists of three dimensional flow of two rotating gas streams moving in opposite directions and interacting with each other. Such a process, in which energy is transferred radially outwards from the inner layers of the fluid to its outer layers, is highly irreversible. Therefore, experimental data about the distribution of pressure, velocity and temperature inside the tube have often been used as a basis for the theoretical analysis of the vortex tube phenomenon.

The experimental data of many investigations [eg. 1,2,3] have shown that the static pressure inside the tube is maximum at the wall and minimum along the axis. The ratio between the wall and axial pressures is maximum near the inlet nozzle and is of the order of 1.5 to 2.

The study of the velocity distribution inside the tube [eg. 3,4,5] have shown that the air as it leaves the nozzle forms a free vortex ($V \propto 1/r$) which is then transformed by friction into a forced one ($V \propto r$). Now taking two fluid particles existing in two successive stream tubes and laying at the same axial plane, the outer particle lies in a higher pressure region than the inner particle. Since the two particles have started expansion from the same condition as they emerged from the side of the vortex chamber; then the inner particle will have a higher velocity than the outer one. Now, introducing the viscous effect; then the outer particle will try to force the inner particle to move with a uniform angular velocity. On the other hand, the inner particle tries to accelerate the outer particle and therefore work is done by the inner fluid particle on the outer particle. This viscous work is taken at the expense of the kinetic energy of the inner particle causing its velocity to decrease till a uniform angular velocity distribution (solid body rotation) becomes dominant in the core region. The overall result of this process is that the fluid in the core region does shear work on the fluid in the outer region as the former expands while travelling towards the tube axis. Thus energy is transferred from the core region to the outer region with a resultant temperature drop.

On the other hand, many trials have been made to present a general mathematical formulation for the process of energy separation taking place in the vortex tube [eg. 4, 8-10]. However, the mathematical treatment of such a problem was found to be extremely difficult and led to nowhere.

So far, all attempts to characterize the vortex tube performance are based on experimental trial and error procedures and do not provide a basis for design. Not only that but also no attempts have been reported in which these experimental results are correlated to establish mathematical relationships that predict the cooling performance of the vortex tube. Therefore, the main objective of the present work is to establish generalized relationships which can describe the vortex tube performance as a function of the tube size, type of inlet gas, its temperature, pressure and the cold stream fraction required. Thus, the present work is divided into two sections one leading to another. In the first section, the effect of tube diameter on its performance was established and in the second, a generalized correlation expressing the tube performance was obtained.

EXPERIMENTAL SET-UP:

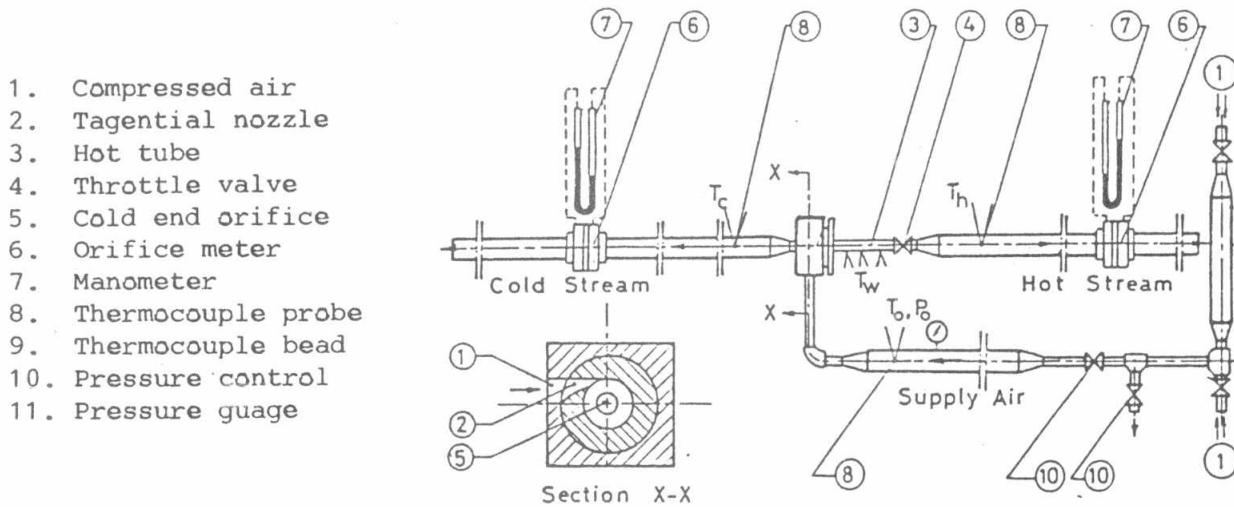


Fig. 1 Experimental Set Up

Figure 1 shows a schematic diagram for the experimental rig used. Compressed air supply (1) is introduced to the vortex chamber through a tangential nozzle (2). The nozzle is arranged peripherally at one end of the hot tube (3) to produce a high velocity annular flow swirling towards its far end. At this end, a throttle valve (4) is fitted to generate a reverse flow at the core of the tube. This flows back and out through the cold end orifice (5) fitted adjacent to the inlet nozzle. At the same time, the annular flow swirling along the tube passes out through the throttle (hot) valve. The throttle valve thus controls the mass fractions of the (cold) stream leaving through the cold orifice and the hot stream leaving through the valve. The mass rates of flow of these two streams are measured by standard orifice meters (6) and manometers (7). The temperatures of the supply (compressed) air, cold and hot streams are measured by thermocouple probes (8). The wall temperature T_w was measured by thermocouple beads (9) imbedded in special grooves at the outer surface of the tube. Temperature measurements were taken at three axial locations along the tube at $4D$, $12D$ and $20D$ from the inlet nozzle. The supply air pressure was controlled via the pressure control circuit (10) and measured by pressure gauge (11). The vortex tube and all piping were perfectly insulated.

A set of four geometrically similar vortex tubes of different sizes are used in the present work. Design details of these tubes are as follows: Tube diameter, $D = 11.5, 14, 16$ and 20 mm. Tube length, $L = 24D$. Cold orifice diameter, $D_c = 0.5D$. Nozzle: tip area, $A_n = 0.12$ Tube cross section area, with simple profile and rectangular tip having width/height ratio = 2.

RESULTS AND DISCUSSIONS

I. Effect of Tube Diameter:

In order to establish the effect of the vortex tube diameter on its performance; the four geometrically similar, vortex tubes were tested under identical conditions of operation. Sample of results of these tests are

given in Figs. 2 through 5. These curves show the concerned performance parameters: ΔT_c , RE, ΔT_h , M_o and COP versus the cold fraction U for the 16 mm diameter tube at different inlet pressures ($P_o = 2, 3, 4$ and 5 bar). Qualitatively similar results are obtained for the other three tubes however the values are different.

The graphs of Figs. 2 and 3 show that an increase in the inlet pressure P_o would increase the values of ΔT_c and M_o over the whole range of change of cold fraction U . As a consequence, the refrigerating effect produced $U \cdot (T_o - T_c)$ also increases over the whole range of U . It can also be noted

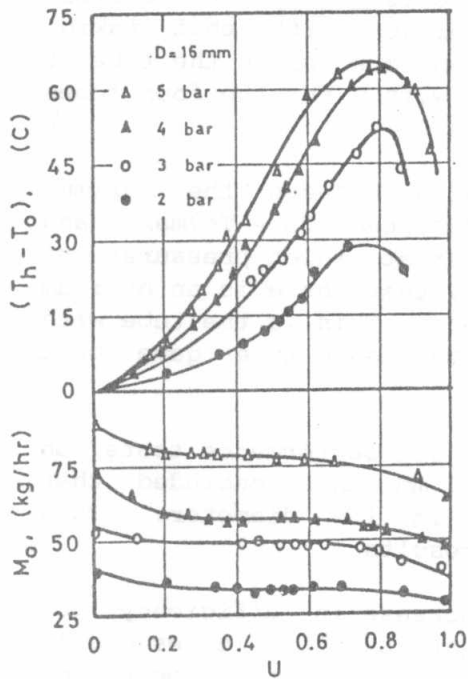


Fig. 2 $(T_h - T_o)$ and M_o Vs. U
(Tube: $D = 16$ mm)

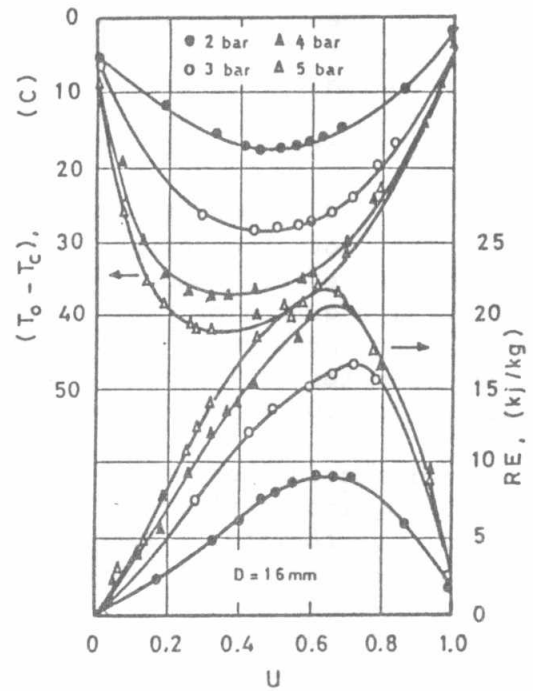
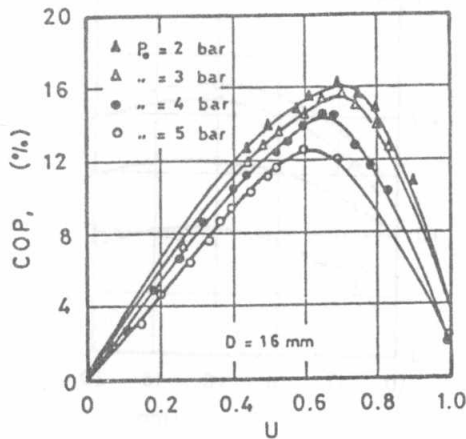
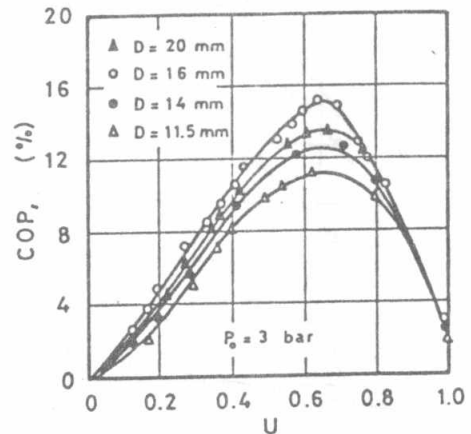


Fig. 3 $(T_o - T_c)$ and RE Vs. U
(Tube: $D = 16$ mm)



a. Tube : $D = 16$ mm



b. Different tubes

Fig. 4 Coefficient of Performance of Various Tubes

from Fig. 3 that the value of the cold air fraction U corresponding to the maximum temperature drop $(T_o - T_c)_{max}$ decreases as the inlet pressure increases ($U = 0.5$ at pressure $P_o = 2$ bar and $U = 0.37$ at $P_o = 5$ bar).

Thorough investigation of the graphs of Fig. 3 would show that the rate of change of the temperature drop produced decreases as the inlet pressure P_o increases. For this reason; the coefficient of performance COP plotted in Fig. 4-a decreases as P_o increases.

Note also that the values of COP are relatively small. This is due to the necessity of compressing more air than that available for refrigeration. The hot stream absorbs energy in compression but makes no direct contribution towards the cooling capacity of the system. Moreover, the process of energy separation inside the vortex tube is highly irreversible causing the temperature drop to be small in comparison with that taking place in an isentropic expansion. Also the expansion work inside the tube is not utilized. Fig. 4-b shows the change of COP with U for the four tubes under consideration at $P_o = 3$ bar.

Figures 5 and 6 show respectively the relationships between the maximum values for both the temperature drop of the cold stream $(T_o - T_c)_{max}$ and refrigerating effect RE_{max} versus the tube diameter at inlet pressures of 2-5 bar. The graphs of these figures show clearly that there is an optimum size for the vortex tube that gives best performance. This is the tube with 16 mm diameter. It should be noted here that this same tube gave best values of the COP as shown in Fig. 4-b.

It is worth noting here that, Hilsch [11] carried out performance tests on tubes having inside diameters of 4.6, 9.6 and 17.6 mm and concluded that the tube performance improves with an increase in its diameter. This conclusion is in good agreement with the present results.

However, it was further stated [11] that the increase in efficiency for larger tube diameters is attributed to the fact that the heat losses due to heat conduction from the axis to the circumference of the vortex tube become

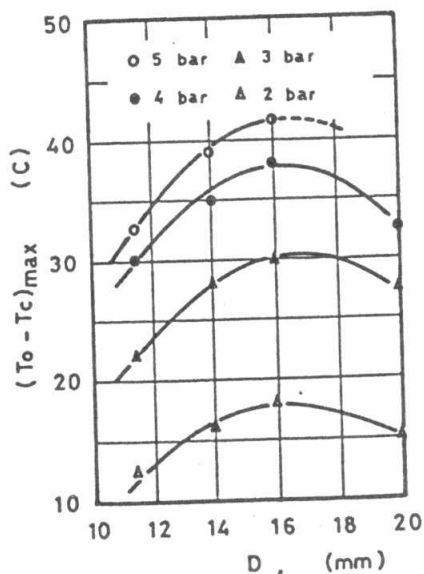


Fig. 5 Maximum Temperature Drop Vs Tube Diameter

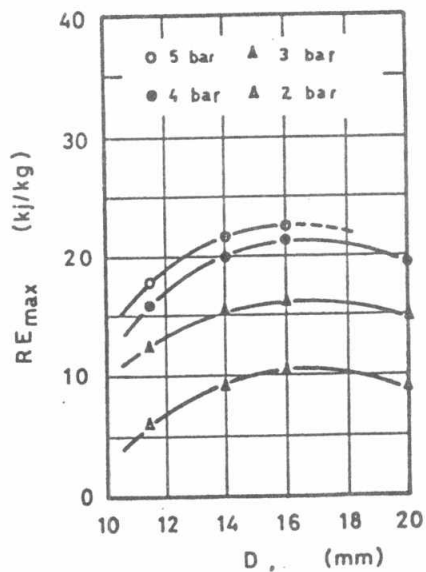


Fig. 6 Maximum Refrigerating Effect Vs Tube Diameter

less important for larger tube diameters. Also the larger the tube diameter D , the smaller is the ratio D_c/D , where D_c is the most favorable diameter of the cold end orifice. It should be noted here that, this comment of Hilsch is not correct because temperature measurements inside the tube have shown that the static temperature is minimum at the axis of the tube and maximum at the circumference. Thus the heat conduction must be in the opposite direction to that from Hilsch's point of view. Also, for geometrically similar tubes the value of D_c/D is the same, therefore should not be considered as an independent variable.

On the other hand, the dimensional analysis made by Stephan [12] has come to the conclusion that the ratio: $(T_o - T_c)/T_o$ can be expressed as:

$$(T_o - T_c)/T_o = f_1(Re).f_2(Pr).f_3(E).f_4(R).f_5(B).f_6(Po).f_7(Pa).f_8(U) \quad (1)$$

For the same condition of operation (Po , T_o and U) and same working medium; the properties and all parameters in equation 1 are constants thus:

$$(T_o - T_c)/T_o = f(Re) \quad (2)$$

Equation 2 indicates that, for geometrically similar tubes and for the same inlet conditions, the tube performance (temperature drop) depends only on Reynolds number (Re), defined as the ratio of the inertia forces to the viscous forces. For the same inlet conditions we can express Reynolds number as a value proportional to the tube diameter:

$$Re = \rho.V.D/\mu = K.D \quad (3)$$

Hence;

$$(T_o - T_c) / T_o = f(D) \quad (4)$$

Equation 4 shows that; for geometrically similar vortex tubes, the tube diameter is a parameter that affects the temperature drop of the cold stream.

For a small diameter of vortex tubes, Reynolds number will also be small. This means that the inertia forces (the driving force), representing the strength of the vortical motion inside the tube is so small that the vortex effect is weak and the performance deteriorates. As the tube diameter increases, Reynolds number increases as well. Thus the vortex motion becomes stronger and the inertia forces and viscous forces become of the same order of magnitude causing the tube performance to improve (as in case of $D = 14$ and 16 mm). For larger diameters ($D > 16$ mm) the performance deteriorates again. This is because, the inertia forces become very large compared with the viscous forces (friction forces) responsible for the energy separation process.

It should thus be emphasized here that the Reynolds number is an important factor which affects the vortex tube performance. That is for containing the inertia force term responsible for the vortex effect and viscous force term responsible for the energy separation process. The latter term does shear work on the gas layers causing energy separation. It can therefore be concluded that, there is an optimal value of Reynolds number for any pressure ratio which gives the maximum energy separation in the vortex tube.

In order to correlate the experimental results of the four tubes under consideration together with those of previous investigations, the values of $(T_o - T_c)_{max}$ produced by the tubes of different diameters were divided by a reference value taken as $(T_o - T_c)_{max}$ for the 16 mm diameter tube. Table I

shows the values of $[(T_o - T_c)_{\max}]D / [(T_o - T_c)_{\max}]16$ and the diameter ratio $[D/16]$ for the present results together with those of other investigations. The empirical correlation that found to fit these results is:

$$\frac{[(T_o - T_c)_{\max}]D}{[(T_o - T_c)_{\max}]16} = K_r$$

$$= 0.56 + 0.44 (D/16) - 0.11 (D/16)^2 \quad (5)$$

where D is the tube diameter in mm.

		$[(T_o - T_c)_{\max}]D / [(T_o - T_c)_{\max}]16$						
Po bar	D mm D/16	Scheper [6]	Hilsch [11]	Present work				Dubinviski[13]
		3.8 0.24	9.6 0.66	11.5 0.72	14 0.88	16 1	20 1.25	54 3.38
2		0.63	0.72	0.73	0.89	1	0.83	-
3		-	0.78	0.80	0.88	1	0.87	-
4		-	0.82	0.82	0.89	1	-	0.76
5		-	-	0.79	0.88	1	-	0.78

Table I Values Of : $[(T_o - T_c)_{\max}]D / [(T_o - T_c)_{\max}]16$
And Dimensionless Diameter $[D/16]$ For Different Tubes

On the other hand, the effect of tube diameter on the rate of heat loss from uninsulated vortex tubes was also studied. This was achieved by determining the percentage heat loss (Q_l) $Q_l = \text{Convective heat loss} / Mo.Cp.T_o$ from each tube at different conditions of operation (P_o and U). The results have shown that the values of Q_l are approximately equal for all geometrically similar tubes. Hence under the same conditions of operation; the convective heat loss from geometrically similar tubes is not a parameter that affects the performance. Thus Eq. 5 is valid for both insulated and uninsulated vortex tubes.

II. Generalized Correlation for Temperature Drop:

The temperature of the cold stream is the most important variable to be determined as far as energy separation is concerned in the vortex tube. For the purpose of obtaining a similarity relation for predicting the cold stream temperature; Stephan [12] made a dimensional analysis of the parameters governing the vortex tube performance thus arrived at :

$$(T_o - T_c) / (T_o - T_c)_{\max} = f(U) \quad (6)$$

Equation 6 indicates that the ratio $(T_o - T_c) / (T_o - T_c)_{\max}$ is independent of the operating conditions and the working medium of the vortex tube. Therefore, this equation is designated as the similarity relation for energy separation in a vortex tube.

Figure 7 shows the relationship between $(T_o - T_c) / (T_o - T_c)_{\max}$ versus the cold fraction U for the 16 mm diameter tube at inlet pressures of 2, 3, 4

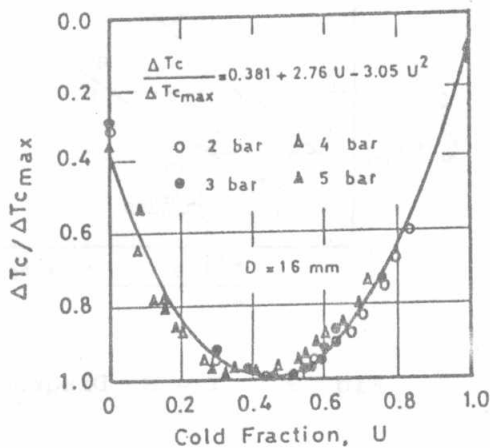


Fig. 7 $(T_o - T_c)/(T_o - T_c)_{max}$
Vs. Cold Fraction U
(Tube : $D = 16$ mm)

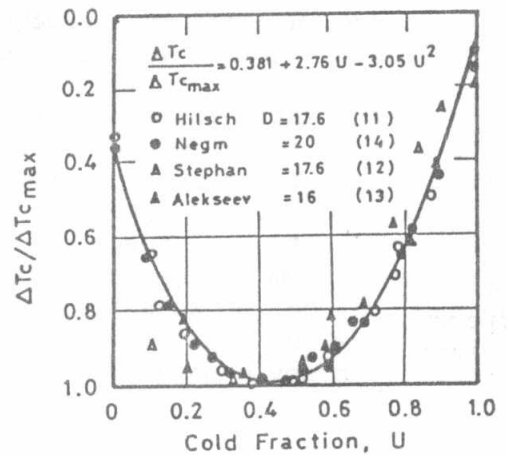


Fig. 8 $(T_o - T_c)/(T_o - T_c)_{max}$
Vs. Cold Fraction U
(Other Investigations)

and 5 bar. Figure 8 shows the corresponding relationship for data from other investigations. Thus by correlating the experimental results of the present work and those of previous works, a unified empirical formula that expresses the ratio between the temperature drop produced and its maximum value at the same inlet pressure P_o is obtained as follows:

$$\frac{(T_o - T_c)}{(T_o - T_c)_{max}} = 0.381 + 2.76U - 3.05 U^2 \quad (7)$$

Now, if we introduce another relation between the maximum temperature drop $(T_o - T_c)_{max}$ and the pressure ratio $n (P_o/P_c)$, a general equation can be obtained for the vortex tube performance.

The process of energy separation taking place in the vortex tube is a constant energy one. Thus, for a perfect gas; the entropy change can be calculated from:

$$S_2 - S_1 = [C_p \cdot \ln(T_2 / T_1) - R \cdot \ln(P_2 / P_1)] \quad (8)$$

where 2 and 1 are the final and initial states.

In case of vortex tube, the process taking place is of constant total enthalpy, thus: $T_2 = T_1$. Hence, for a given pressure ratio $n (P_1/P_2)$, and for a certain working medium; the value of Equation 8 is equal to constant;

$$S_2 - S_1 = R \cdot \ln(n) = B \quad (9)$$

From the law of entropy conservation, the change of specific entropy is:

$$S_2 - S_1 = U \cdot \Delta S_c + (1-U) \cdot \Delta S_h \quad (10)$$

In the present work, the most important term is the change of specific entropy of the cold stream (ΔS_c).

As the process taking place inside the vortex tube is adiabatic and irreversible; then the entropy must increase. As illustrated in Fig. 9; such an increase (ΔS_c) is ranging from a certain minimum value ($\Delta S_c)_{min}$ to a value equals to the constant B; as U varies from 0 to 1.

Fig. 10 shows the relationship between the change in specific entropy of the cold stream (ΔS_c) versus cold air fraction U for the optimal tube (D=16 mm) at different inlet pressures ($P_0=2-5$ bar). Fig. 11 shows the relationship between the ratio $(\Delta S_c)/B$ versus the cold fraction U for the D16 tube at different inlet pressures. It could be concluded from this figure that the minimum change of specific entropy $(\Delta S_c)_{min}$ is about 0.67 from the maximum specific entropy change B:

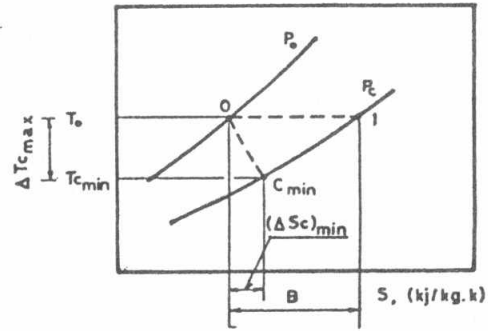


Fig. 9 T - S Diagram

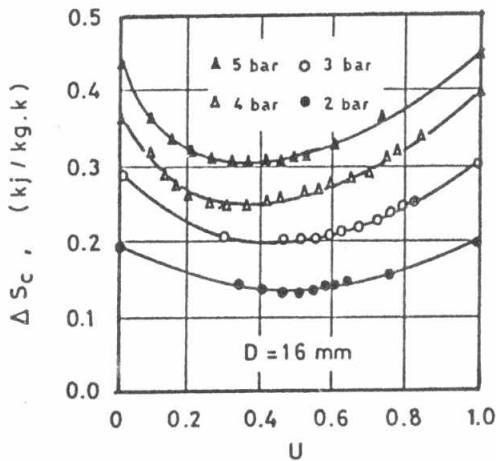


Fig. 10 Entropy Change Vs U
(Tube : D = 16 mm)

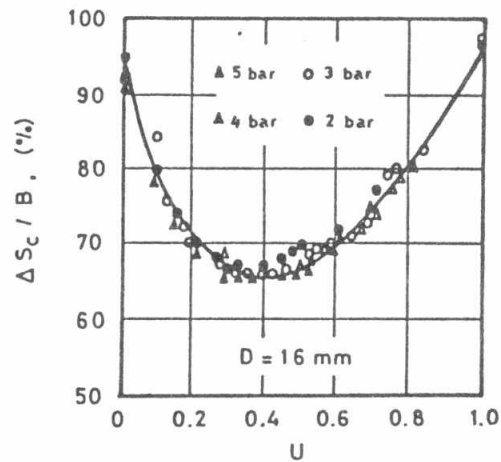


Fig. 11 Percentage Entropy Change
Vs. U (Tube: D = 16 mm)

$$(\Delta S_c)_{min} / B = a \quad (\text{where } a = 0.67) \quad (11)$$

At the same time, from Eq. 8 :

$$(\Delta S_c)_{min} = C_p \ln[(T_c \text{ min}) / (T_0)] + R \cdot \ln(n) \quad (12)$$

Substituting Eq. 12 into Eq. 11 :

$$[C_p \cdot \ln(T_c \text{ min} / T_0) + R \cdot \ln(n)] / [R \cdot \ln(n)] = a \quad (13)$$

Hence;

$$\ln(T_c \text{ min} / T_0) = [(a-1)R/C_p] \cdot \ln(n) \quad (14)$$

$$T_c \text{ min} = T_0 \cdot n^{(a-1)R/C_p} \quad (15)$$

$$(T_0 - T_c)_{max} = T_0 \left[1 - n^{(a-1)R/C_p} \right]$$

Substituting the value: $C_p = k \cdot R / (k-1)$ into Eq. 15 :

$$(T_o - T_c)_{\max} = T_o \left[1 - n \right]^{\frac{(a-1)(k-1)}{k}} \quad (16)$$

Equation 16 predicts the value of the maximum temperature drop produced by the vortex tube at any condition of operation: pressure ratio n , inlet temperature T_o and the working medium (different specific heat ratio k).

The specific heat ratio [$C_p/C_v=k$] has a significant effect on the vortex tube performance. From the Eq. 16 and Fig. 12, it could be noted that monoatomic gasses ($k=1.66$) has better performance than the diatomic ($k=1.4$) and polyatomic ($k=1.33$) gasses. This conclusion agrees well with the experimental results of Stephan [8], who used air, oxygen and helium as working media. Air and oxygen represent diatomic gasses ($k=1.4$) and helium the monoatomic gas with ($k=1.66$). The results showed that the monoatomic gasses have better performance than diatomic gasses.

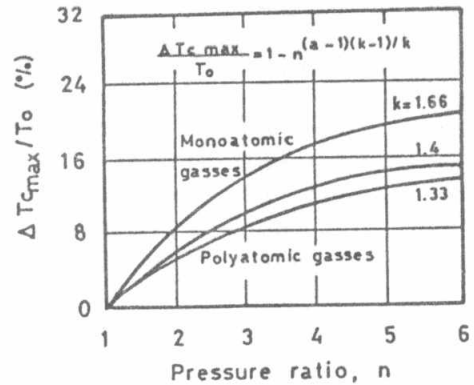


Fig. 12 Effect of Specific Heat Ratio on Max. Temperature Drop

The above conclusion can also be supported by the experimental results of Martynovskii and Alekseev [13], who studied the vortex tube performance under various working media, where air, methane, carbon dioxide, and ammonia were used. The results have not shown any correlation, however air was slightly better than the other gases. This is because the value of k for air (1.4) is slightly higher than that for the other gasses (1.33).

The effect of the temperature level of the inlet air T_o can also be seen from Eq. 16. An increase in inlet temperature T_o , leads to an increase in the temperature drop $(T_o - T_c)_{\max}$.

Now a generalized formula that describes the temperature drop in a vortex tube can be obtained by combining the Eq. 16 for the maximum temperature drop ($T_c \max$), Eq. 7 for the relationship between the ratio of the temperature drop to the maximum temperature drop ($\Delta T_c / \Delta T_c \max$): and Eq. 5 for the tube size correction factor K_r . Thus:

$$(T_o - T_c) = K_r \cdot \left[(0.381 + 2.76U - 3.054U^2) \cdot T_o \left[1 - n \right]^{\frac{(a-1)(k-1)}{k}} \right] \quad (16)$$

This is the generalized formula that predicts the temperature drop for different tube diameters, various conditions of operation, working media, at different inlet temperature levels T_o , and cold gas fraction U .

CONCLUSIONS:

The cooling performance of a set of four geometrically similar vortex tubes, having inside diameters of 11.5, 14, 16 and 20 mm., was investigated experimentally and analytically. The following conclusions may be drawn:

1. The analytical study of the factors affecting the performance of

geometrically similar vortex tubes, have shown that such performance is dependent upon the physical size of the tube (inside diameter) as expressed by Eq. 4. This dependence has been verified experimentally. Moreover, the experimental results have shown that the 16 mm. diameter tube is the optimal size for best performance (maximum temperature drop, maximum refrigerating effect and maximum coefficient of performance).

2. The correlation between the theoretical and experimental results for geometrically similar vortex tubes have shown that the ratio between the produced temperature drop to its maximum drop ($T_c / T_{c \max}$) could be expressed by a sole function of cold fraction U (Equation 7). Also, The ratio between the maximum temperature drop produced by a tube having diameter D, to that produced by the tube of optimal diameter (16 mm.) can be expressed by the tube size correction factor given by Eq. 5 .

3. The generalized correlation that predicts the temperature drop produced by the vortex tube is expressed as (Eq. 16):

$$T_o - T_c = (0.381 + 2.76U - 3.054U^2) \cdot T_o \cdot (1 - n^{(a-1)(k-1)/k}) \cdot Kr \quad (16)$$

This relationship indicates that the temperature level of the inlet gas (T_o) has a direct effect on the tube performance. An increase in the value of T_o increase the temperature drop produced (T_c) and vice versa. Also, an increase in the value of k improves the tube performance. Thus, the above formula could be considered as a generalized relationship which can be used to predict tube performance at various conditions of operation.

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NOMENCLATURE:

Cp	Specific heat at constant pressure	[kJ/kg.K]
D	Vortex tube diameter	[mm]
k	Specific heat ratio = Cp/Cv	
M	Mass rate of Flow	[kg/hr]
n	Inlet pressure ratio = Po/Pc	
P	Pressure	[bar abs.]
Pr	Prandtl number	2
E	Eckert number = Vo / To.Cp	
R	Gas constant	
Re	Reynolds number	
S	Specific entropy	[kJ/kg.K]
U	Mass fraction of cold stream	[kg cold gas stream/ kg supply gas]
V	Velocity of gas	[m/s]
β	Coefficient of thermal expansion	[1/K]
μ	Dynamic Viscosity	[N.s/ m ²]
ρ	Density	[kg/m ³]

Subscripts

c	Cold Stream
h	Hot stream
o	Inlet condition