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MODELLING OF AN AIRCRAFT LONGITUDINAL CONTROL SYSTEM.

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ABSTRACT

This paper deals with the modelling and simulation of an aircraft longitudinal control system including a hydromechanical booster.

A bond graph model of the booster is developed. The equations describing the booster are deduced from the bond graph taking into consideration the basic nonlinearities. The motion of aircraft is represented by a transfer function describing its behaviour during the short period longitudinal oscillations. The validity of the deduced model of the booster is studied by the comparison of calculated transient response of booster with corresponding experimental results. The effect of some constructional and operational parameters on the dynamic behaviour of the hydraulic booster and aircraft is studied by the exploitation of the deduced model.

INTRODUCTION

The study of aircraft stability during flight and the analysis of the effect of different constructional and operational parameters of aircraft and control system has been the subject of several studies. In the case of aircraft with control systems equiped by hydromechanical servo-units; this unit is usually represented by a simple linear model,[1] & [2].

Such simple models may be obtained by linearization of the equations describing the unit or on the basis of experimental evaluation of behaviour of the real units.

In this paper, the control system of an aircraft is the subject of study. The system includes a hydromechanical servo unit; booster, supplied by the hydraulic power from two different systems simultaniously (fig.1) The pilot actuates the booster through a linear transmission system T1. The booster; consequently displaces the elevator by means of another

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transmission system T2.

In order to study the effect of the booster constructional and operational parameters a bond graph model is developed. After assignment of the causality the bond graph carries at the same time both the physical and mathematical structure of the studied booster [3] & [4]. The equations describing the booster could then be obtained systematically from the bond graph. The simulation is realised on a digital computer by the exploitation of the deduced model. The validity of the model is studied by comparing simulation and experimental results.

The behaviour of aircraft longitudinal control is studied during the short period longitudinal oscillation mode of flight. The developed model allows to study the effect of the different constructional and operational parameters of the booster.

MODEL OF AIRCRAFT

The equations describing the motion of aircraft during the short period longitudinal oscillations are deduced assuming that:

- The aircraft is a rigid body subjected to aerodynamic forces and moments in addition to its weight.
- The gyroscopic effects, introduced by the presence of spinning rotors are negligible.
- The center of gravity of aircraft is the origin of coordinate system.
- The horizontal steady flight is the reference flight regime.
- The aircraft velocity is not much affected during the short period longitudinal oscillation.

During this mode of flight, the behaviour of aircraft is described by the following transfer function; [5]:

$$\frac{\Delta \alpha}{\Delta \delta}(s) = G/[s^2/\omega_n^2 + 2\xi s/\omega_n + 1] \qquad (1)$$

The natural frequency and damping coefficient are functions of the aerodynamic parameters of aircraft. In order to evaluate these parameters; some numerical values were found in the aircraft technical description books, others were measured by limited experimental work of the aircraft in addition to the exploitation of the data sheets.

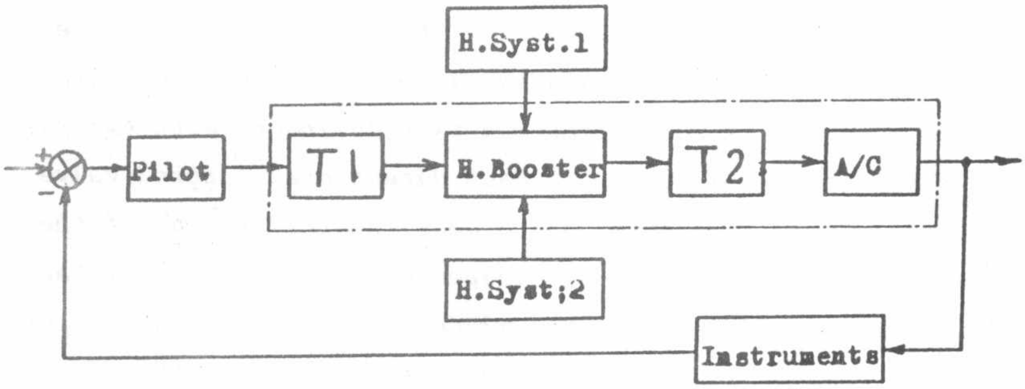

Fig.1 Block scheme of aircraft longitudinal control system.

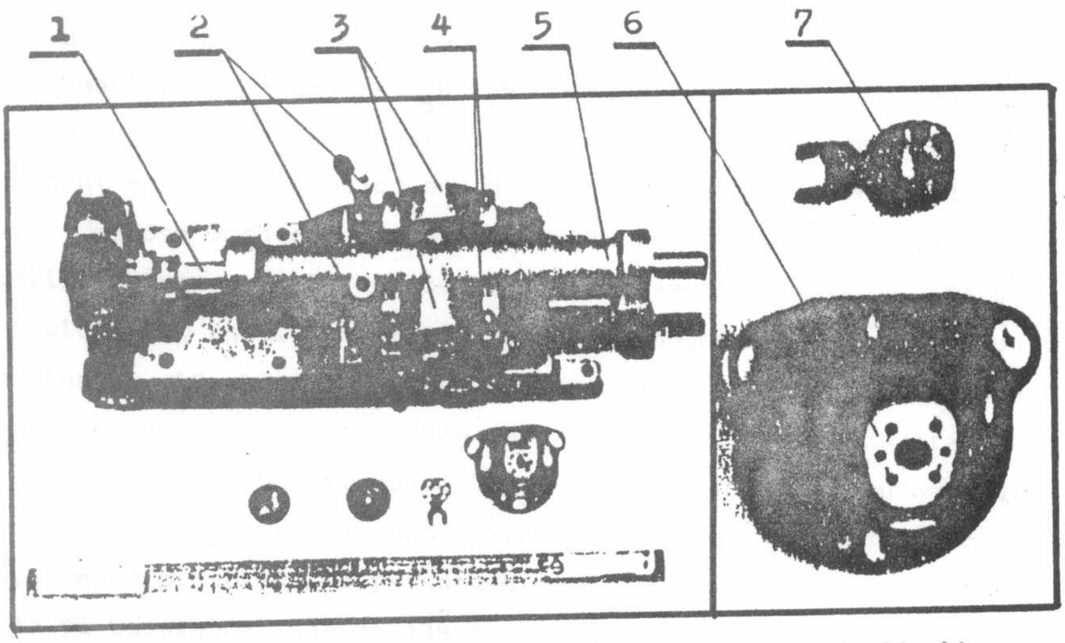

Fig.2 Hydraulic booster with one of the infinite-position D.C.V.disassembled.
1-Piston rod. 2-Connections for hyd.supply.
3-Two infinite-position directional control valves.
4-Return connections.5-Twin piston cylinder.
6-Fixed disc.7-Flat rotary slide valve.

DESCRIPTION OF THE HYDRAULIC BOOSTER

The hydraulic booster used is a hydromechanical unit with a follow up control system. The booster has two infinit-position directional control valves with flat rotary-type slide valves. The two slide valves are interconnected within a common distributing assembly and operate in concert.

The actuating assembly is made up from a twin cylinder with two pistons and one common piston rod (Fig.2 &3). Each directional control valve is fed from a seperate hydraulic system to increase the reliability of the aircraft control system. The booster is equiped with a feed back mechanism (not shown in figures) which brings the slide valve to the neutral position when the piston rod reaches a final position corresponding to that of the control rod.

BOND GRAPH OF THE HYDRAULIC BOOSTER

The hydraulic booster is supplied by the hydraulic power from two seperate constant pressure systems, represented by two sources of effort SE1 and SE41 (fig.4). The four restrictions of each of the two directional control valves are represented by the resistances: R_a, R_b, R_c and R_d, modulated by the slider rotation angle θ, Each of the four resistances in the first valve is commonly manufactured to be identical with the corresponding one in the second valve. The elements SE9 and SE49 impose the constant return pressures of both the power systems. The effect of fluid compressibility in actuator cavities is represented by the capacitances C1 and C2 modulated by the piston displacement; y. The resistances R_i of bonds 21, 32 and 71 represent the resistance to internal leakage, while the resistance to external leakage is represented by by the element R_e of bonds 17 and 57. The Transformation of hydraulic power to mechanical one and vice versa is insured by four TF elements. The external load, depending on the deflection angle of control surface is imposed by the source of effort SE27 modulated by the piston displacement y. The resistance R_{25} represents the effect of friction forces opposing the piston motion. The inertia of the moving parts is taken into consideration by the inertia element I_{26}.

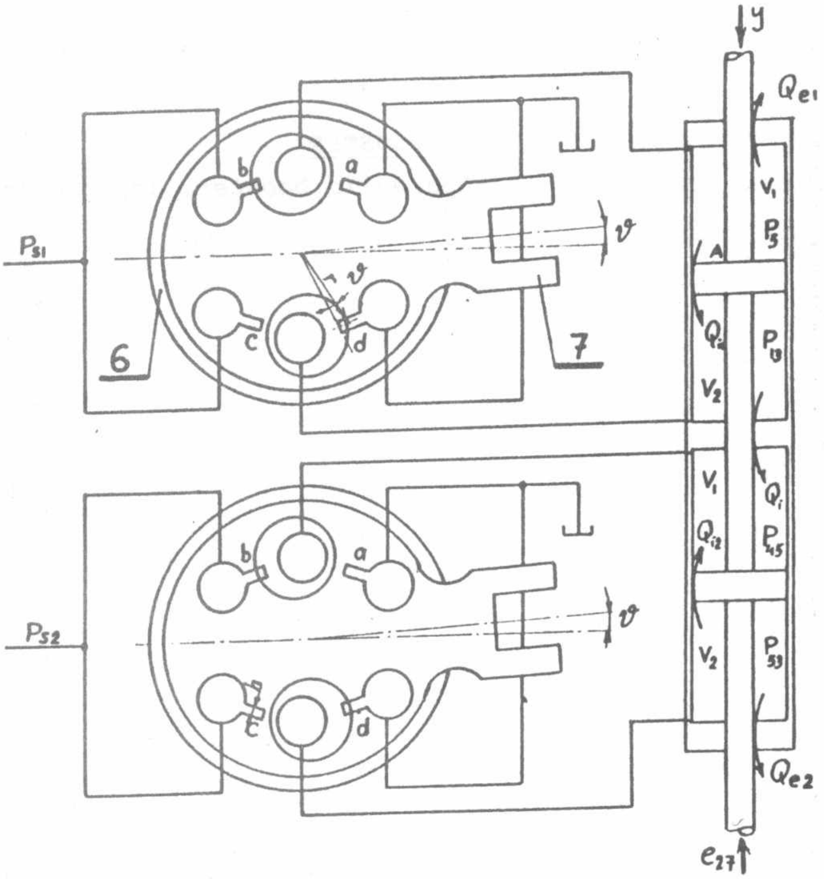

Fig.3 Functional scheme of the hyd. booster.

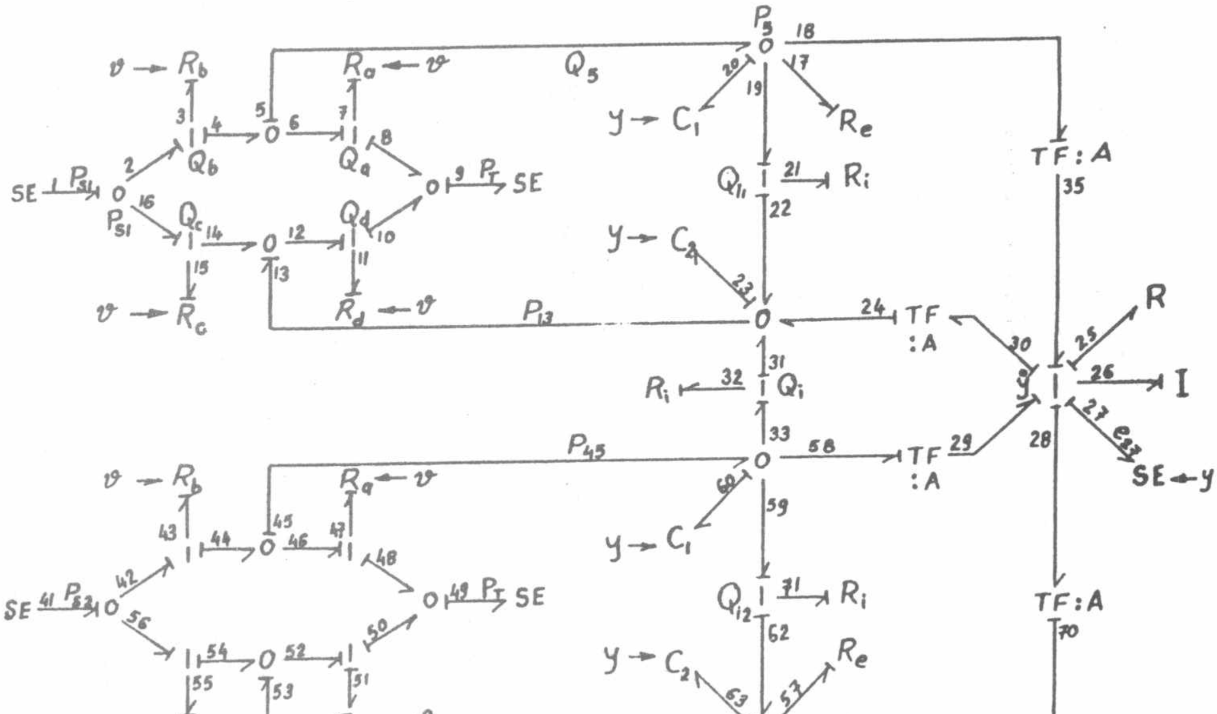

Fig.4 Augmented bond graph of the hyd.booster

MATHEMATICAL MODEL OF THE HYDRAULIC BOOSTER

The equations describing the hydraulic booster, deduced from the bond graph are as follows:

$$Q_{a1} = C_d . A_a \sqrt{\frac{2(P_5 - P_T)}{\rho}} \qquad (2)$$

$$Q_{b1} = C_d . A_b \sqrt{\frac{2(P_{s1} - P_5)}{\rho}} \qquad (3)$$

$$Q_{c1} = C_d . A_c \sqrt{\frac{2(P_{s1} - P_{13})}{\rho}} \qquad (4)$$

$$Q_{d1} = C_d . A_d \sqrt{\frac{2(P_{13} - P_T)}{\rho}} \qquad (5)$$

Where the areas of valve orifices A_a, A_b, A_c & A_d are given, for the small angular displacement of the slider, by;

$$\left.\begin{aligned} A_b &= A_d = rh\theta \\ A_a &= A_c = 0 \end{aligned}\right\} \quad \text{for } \theta \geq 0 \qquad (6)$$

$$\left.\begin{aligned} A_b &= A_d = 0 \\ A_a &= A_c = rh|\theta| \end{aligned}\right\} \quad \text{for } \theta < 0 \qquad (7)$$

$$Q_5 = Q_{b1} - Q_{a1} \qquad (8)$$

$$Q_{13} = Q_{d1} - Q_{c1} \qquad (9)$$

$$Q_{17} = P_5 / R_e \qquad (10)$$

$$Q_{18} = A.\dot{y} \qquad (11)$$

$$Q_{i1} = (P_5 - P_{13})/R_i \qquad (12)$$

$$Q_{20} = Q_5 - Q_{17} - Q_{18} - Q_{i1} \qquad (13)$$

$$P_5 = \frac{1}{C_1} \int Q_{20}.dt \qquad (14)$$

$$C_1 = V_1/B \quad , \qquad V_1 = V + A.y \qquad (15)$$

$$Q_{24} = A.\dot{y} \tag{16}$$

$$Q_i = (P_{45}-P_{13})/R_i \tag{17}$$

$$Q_{23} = Q_{24} + Q_{il} + Q_i - Q_{13} \tag{18}$$

$$P_{13} = \frac{1}{C_2}\int Q_{23}.dt \tag{19}$$

$$C_2 = V_2/B \quad , \quad V_2 = V - A.y \tag{20}$$

$$e_{35} = A.P_5 \tag{21}$$

$$e_{30} = A.P_{13} \tag{22}$$

$$e_{29} = A.P_{45} \tag{23}$$

$$e_{28} = A.P_{53} \tag{24}$$

$$e_{27} = k_L.y \tag{25}$$

$$e_{25} = f_v.\dot{y} + F_c \ \ sign\ (\dot{y}) \tag{26}$$

$$e_{26} = e_{35} - e_{20} - e_{29} - e_{28} - e_{27} - e_{25} \tag{27}$$

$$\dot{y} = \frac{1}{I}\int e_{26}.dt \tag{28}$$

The equations describing the lower part of the booster are similar to these equations. They are also deduced in a similar way from the lower half of the bond graph.

The hydraulic booster is equiped with a solid negative feed back mechanism, for the small input displacement, characterizing the studied unit, the feed back mchanism is described by;

$$\theta = a_1.z - a_2.y \tag{29}$$

MEASUREMENT OF THE HYDRAULIC BOOSTER TRANSIENT RESPONSE

The transient response of the hydraulic booster is evaluated experimentally by the simultaneous measurement and plotting of the control rod and piston rod displacements. The control rod is rapidly displaced by means of a double acting electromagnetic solenoid of adjustable stroke, 2, (Fig.5 & 6).

The measurement is realised by means of two inductive displacement pick-ups (3) & (4). The recording of the measured variables is carried out by means of a Hottinger system with multi channel hot pin plotter.

Both of the two directional control valves are fed from a hydraulic generator of constant pressure. The hydraulic generator includes a variable displacement piston pump driven by means of a high power electric servomotor of adjustable speed. The pump input pressure is boosted by means of a centrifugal pump driven electrically. Both of the main pump input and output pressures can be measured and controlled.

A hydraulic accumulator is installed to smooth out the pressure pulsation resulting from the pump flow pulsation. The experimental work is realized in the laboratories of chair of aircrafts of the M.T.C. The measured response of the booster, y(t), for several magnitudes of input displacement Z(t) is given in figure 7.

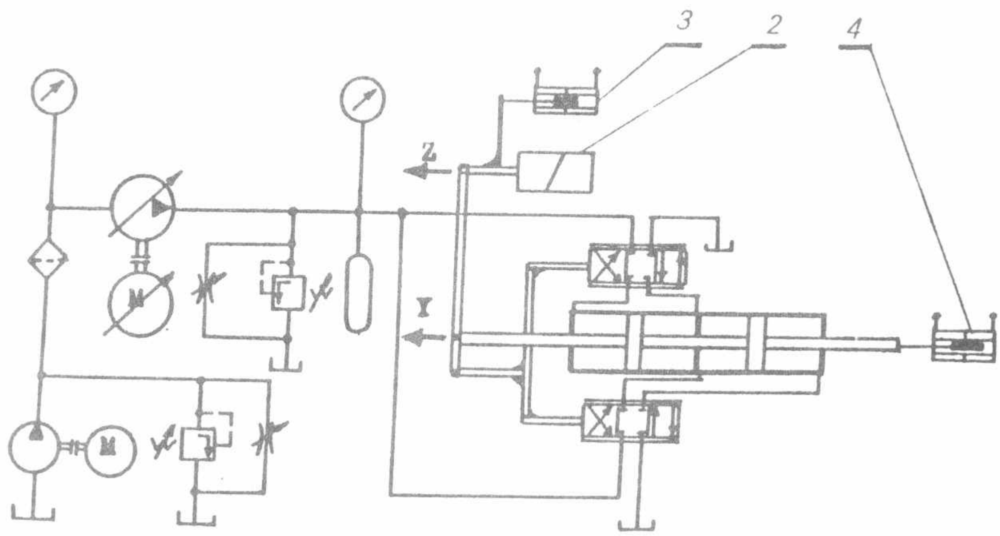

Fig. 5 Model scheme of the experimental arrangement for the measurement of booster trasient response.

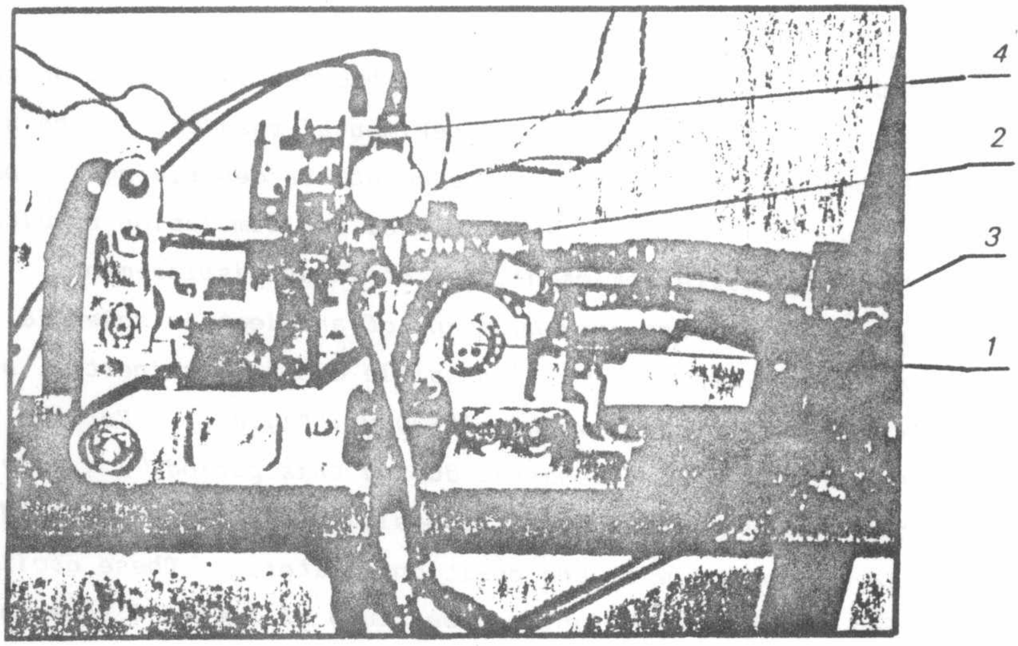

Fig. 6. Test Stand

1....... Hydraulic booster.

2....... Electromagnetic Solenoid.

3 & 4..: Displacement inductive pick-ups.

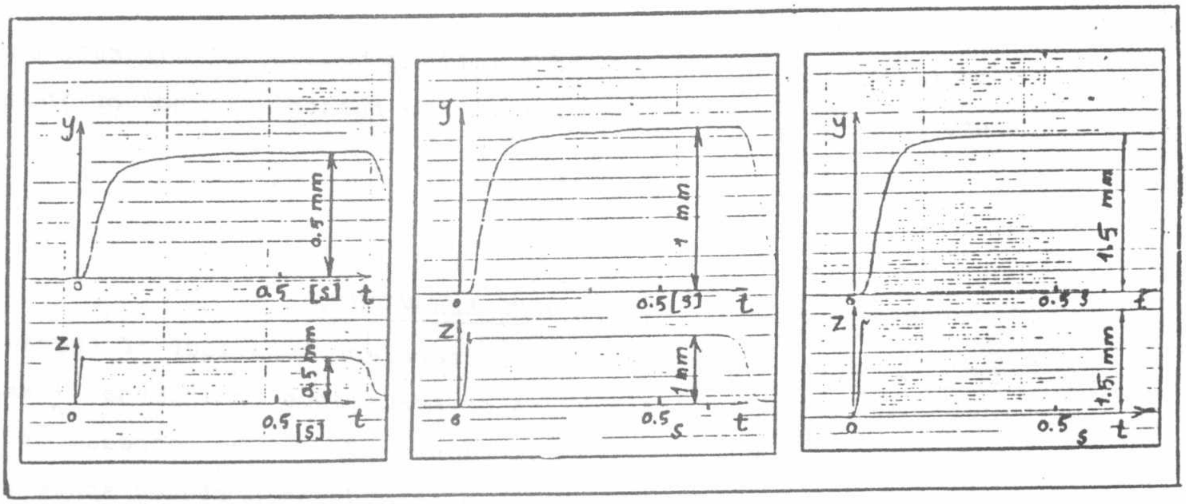

Fig.7 Records of measured transient response of booster.

VALIDITY OF THE HYDRAULIC BOOSTER MODEL.

The validity of the model of hydraulic booster is studied on the basis of the comparison of simulation and experimental results. The simulation is realized, by the exploitation of the equations describing the booster, on a digital computer. Both the experimental and simulation results are given in figure 8 for different magnitudes of input displacement.

The control rod is displaced by means of an electric solenoid. Sudden step displacement of this rod was impossible due to its inertia, but it reaches the required position within 15-20 ms,(Fig.8). For the simulation the input displacement is represented during this period by a ramp.

A small damped oscillations in the position of the control rod is observed mostly due to the impact with the position limiter. These oscillations were neglected during simulation. The study of figure 8 shows acceptable agreement between the simulation and experimental results. The linearity of booster model is checked by calculating the response of booster to step inputs of different magnitudes, the results are given in figure 9, from this figure, one can conclude that the model presents -practically- a linear behaviour. A linear empirical model representing the booster in the design conditions is deduced according to the method given by Schwarzenback [6].

The obtained model is a simple first order transfer function.

$$Y/Z = 1/ [1+0.0569s] \tag{30}$$

Such a simple model can be applied for the modelling of aircraft control systems when the problems concerning the booster are not of interest [1] & [2].

EFFECT OF INTERNAL LEAKAGE ON THE BOOSTER BEHAVIOUR.

The effect of internal leakage on the transient behaviour of hydraulic booster is calculated for several values of resistance to internal leakage.

The calculation results are given in figure 10. The decrease of the resistance to internal leakage introduces a considerable steady state error regardless to its stabilizing effect, therefore a special attention should be payed to check the state of inner seals. The permissible values of internal leakage can be found taking into consideration both the permissible

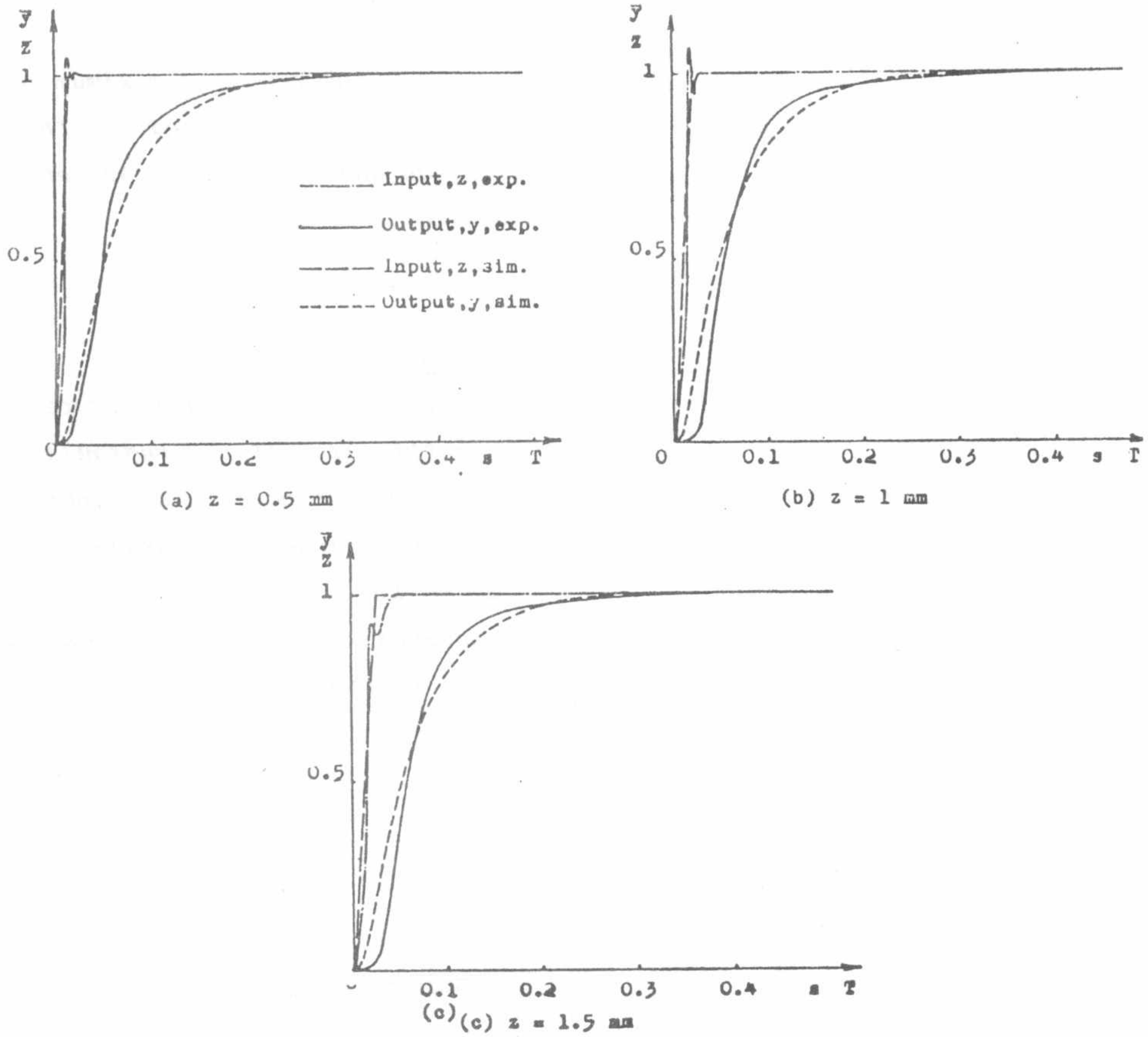

Fig. 8 Simulation and experimental results of the hydraulic booster transient response evaluation,for different magnitudes of input displacement,z.

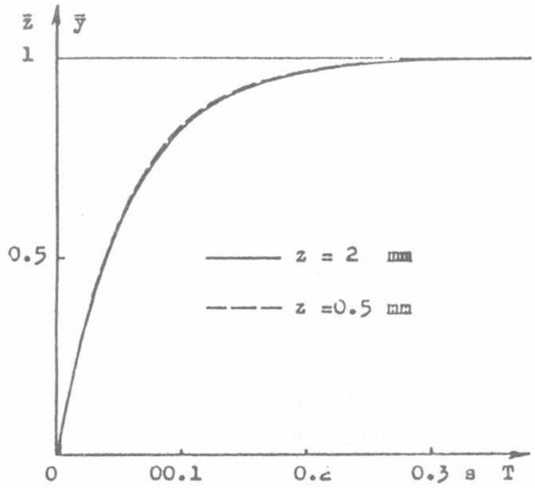

Fig. 9 Step response of hyd.booster for different amplitudes of input displacement.

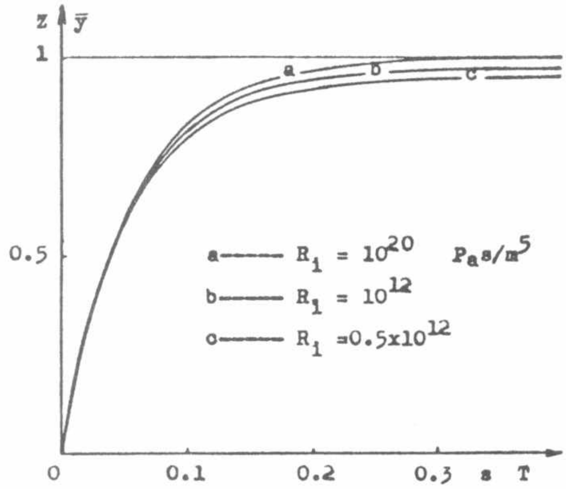

Fig.10 Effect of the resistance to internal leakage on the transient response of hyd.booster.

error and the steady state requirements, mainly the required maximum booster load. The effect of Bulk's modulus of fluid is also studied. The variation of Bulk's modulus within its real range gave no significant effect on the dynamic behaviour of the hydraulic booster.

SIMULATION OF AIRCRAFT LONGITUDINAL CONTROL SYSTEM.

The simulation of aircraft longitudinal control system, (Fig.1) is realized by the exploitation of equations describing the aircraft and hydraulic booster.

The step response of the aircraft to a step displacement of control stick is calculated for several values of resistance to internal leakage in the booster, The results are plotted in figure 11.

It is well seen that the increase of internal leakage introduces a considerable steady state error to the aircraft response.

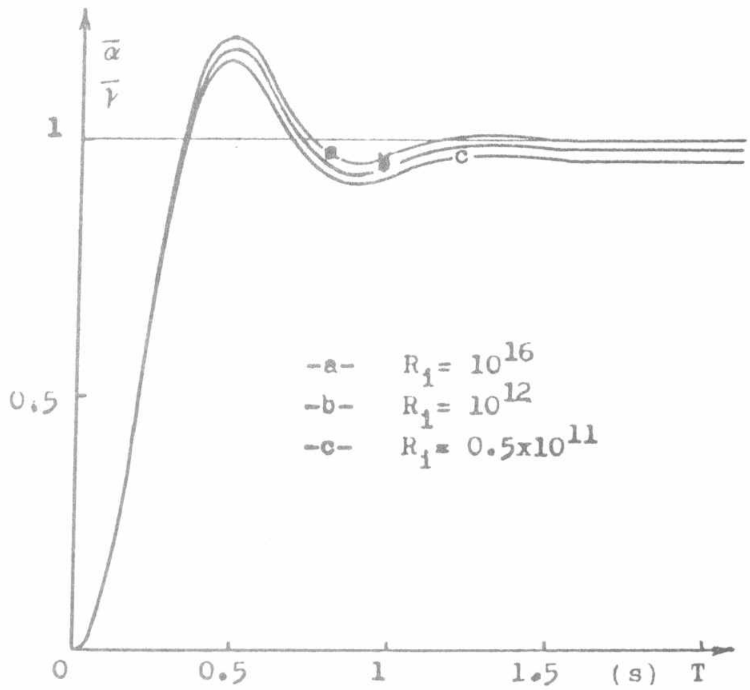

Fig. 11 Effect of internal leakage in the booster on the transient response of aircraft

CONCLUSION

This paper deals with the modelling and simulation of an aircraft longitudinal control system. A bond graph model of the hydromechanical servo-unit; booster; is developed. The equations describing the booster were deduced from the bond graph taking into consideration the basic nonlinearities.

The aircraft is represented by the transfer function describing its behaviour during the short period longitudinal oscillations, The simulation is realised on a digital computer by the exploitation of the equations describing the system. The validity of the model of booster is studied by comparing simulation and experimental results. The comparison shows that the deduced model represents the real booster behaviour with acceptable accuracy.

This model is used to study the effect of internal leakage in the cylinder on the transient behaviour of the system. The simulation results proved that the internal leakage,regardless of its stabilizing effect; introduces a steady state position error to the response of hydraulic booster and aircraft.

LIST OF SYMBOLS

a_1 , a_2	parameters of feedback mechanism. (rad/m)
A	piston area. (m^2)
A_a, A_b, A_c, A_d	areas of restrictionsof slide valve. (m^2)
B	Bulk's modulus of fluid. (N/m^2)
C_1 , C_2	capacitances. (m^5/N)
C_d	discharge coefficient. (-)
e_{27}	loading force. (N)
$e_{25}, e_{26}, e_{28}, e_{30}, e_{35}$	forces. (N)
f_v	viscous friction coefficient. (Ns/m)
F_c	Static friction force. (N)
G	Gain of aircraft transfer function. (-)
h	Width of spool valve slots. (m)
I	inertia; reduced mass of all movable parts. (kg)
K_L	loading coefficient. (N/m)
P_{s1} , P_{s2}	supply pressure. (pa)
P_T	return pressure. (pa)
P_5 , P_{13} , P_{45}	pressures. (pa)

$Q_{a1}, Q_{b1}, Q_{c1}, Q_{d1}$ Flow rates through valve ports. (m^3/s)

Q_i , Q_{i1} , Q_{i2} internal leakage flow rates. (m^3/s)

Q_{e1} , Q_{e2} external leakage flow rates. (m^3/s)

$Q_5, Q_{17}, Q_{18}, Q_{20}, Q_{23}, Q_{24}$ flow rates. (m^3/s)

r radius. (m)

R_e resistance to external leakage. (Ns/m^5)

R_i resistance to ternal leakage. (Ns/m^5)

s Laplace operator.

t, T time. (sec.)

V , V_1 , V_2 volumes. (m^3)

y piston displacement. (m)

z control rod displacement. (m)

α angle of attack of aircraft. (rad.)

γ deflection angle of control stick. (rad.)

δ deflection angle of elevator. (rad.)

θ rotation angle of rotary spool. (rad.)

ρ specific mass of fluid. (kg/m^3)

ω_n natural frequency. (rad/s)

ξ damping coefficient. (-)

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