MILITARY TECHNICAL COLLEGE

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THE CONTRIBUTION OF AUTOMATIC CONTROL TO ENERGY SAVING

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ABSTRACT:

The present work shows the contribution of optimal control to energy saving for two different dynamical systems. An optimal control consists in determining the manipulable inputs variables which transfer the system from an initial to a final state satisfying the system dynamics, and optimizing a certain quality criterion related to the system performance. These optimal control problems are solved by a systematic, general usage and easy Computer Aided Optimization procedure, with which the originally dynamic optimization problem is transformed to an easier static parametric optimization task. The evolution of the control inputs with respect to a suitable independent variable is expressed by adequate system of analytic functions. The function coefficients are then iterated by a search algorithm to values that optimize the objective function. The technique is applied to two energy systems. The first is a Domestic Hot Water for which it is required to maximize the collected energy. The second is a space heating system for which it is required to minimize the auxiliary energy cost and a discomfort index. For the two systems substantial energy gain is realized over the ON-OFF classical controller. The gain increases as the constant value of the ON-OFF controller is far from the optimum. A sensitivity analysis is done to calculate the variations of the optimal control to variations of the system parameter with respect to nominal conditions. This can be used to simplify the implementations of the optimal control.

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1. INTRODUCTION:

Solar energy systems optimally controlled to minimize auxiliary energy consumption may represent one of the most significant ways to achieve energy conservation through automatic control in the area of domestic and industerial heating and cooling. Another application of energy savings through automatic control is in the determination of the optimal trajectory of a vehicle moving between two fixed points providing an optimal reconciliations between two conflicting objectives - the total energy consumed and the travel time required by each trajectory. Several control strategies such as differential proportional, derivative integral controllers or combination of them can be devised. However such systems has been designed for a constant worst case ambient conditions with no direct concern for minimization of energy consumption.

More complex control strategies can be designed to satisfy the basic requirements and minimize some means of cost and/or discomfort. These more complex control strategies are referred to as optimal control strategies.

An optimal control problem is characterized by the following three items:
i) A dynamic model of the system relating the state and control variables.
ii) A quality criterion expressed in terms of state and control variables.
iii) Constraints on the state and control variables.

2. SYSTEM DYNAMICS AND CONSTRAINTS:

The necessity of considering the system notion is now well known. We remined only the theory of Bellman which states that the assembly of optimal subsystems does not necessarily lead to an optimal one. we are always confronted with the immortal problem of modelling, considering detailed complex acurate system, yet keeps the model simple enough to permit feasable computational tools.

For the solar systems under study accurate results are obtained by adopting distributed parameters model in which the temeprature variable satisfy partial differential equations referred to as the heat equations. However sufficient accuracy can be obtained for most problems by aggregating the spatial temperature and using lumped model with a single independant variable; time. This is based on the assumption of uniform temperature distribution.

For the vehicle model the equations are written based on the assumption of a point mass system rather than rigid body. The dynamic model is obtained by applying the fundamental law of mechanics.

3. PROBLEMS FORMULATION:

a) Water Heating System:

This system consists of a flat plate collector and well mixed storage tank at uniform temperature connected to a load. Basic lumped models for the system components are developed in /1/ and used in the present study. For this system the heat balance equation for the storage tank yields to the following dynamical equation;

$$C_s \frac{dT_s}{dt} = \dot{Q}_u - (AU)_s (T_s - T_a) - (\dot{m}\, C_p)_L (T_s - T_m) \qquad (3.1)$$

with $T_s(t_0) = T_0$

$$\dot{Q}_u = A_c F_R [H_T \tau\alpha - U_L (T_s - T_a)]$$

The parameter definitions and their simulated values are listed with the results.

Our objective is to calculate the optimal collector mass flow which maximizes the integral of the difference between the energy collected and the cost of pumping fluid through collector;

$$J_a = \int_{t_o}^{t_f} [\dot{Q}_u (T_s, \dot{m}, .., t) - C \dot{m}^3]\, dt \tag{3.2}$$

Where C is the proportionality coefficient for pumping which includes the efficiency of conversion of heat into mechanical energy. The power three for $\dot{m}$ is based on the assumption of fully turbulent flow. The time period of optimization is taken from 6 AM to 6 PM. The entire load was 80 liters/day per square meter of collector area. The storage tank volume was 80 liters per square meter of collector area. Both the solar radiation H and the ambient temperature were assumed to be sinusoidal functions of time between 6AM and 6PM.

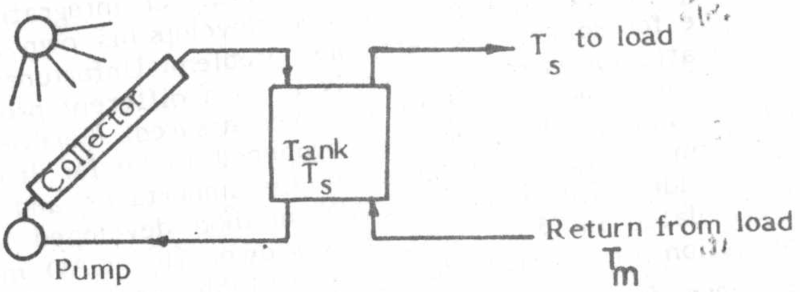

Fig. (1): Domestic Hot Water System

b) <u>**Space Heating System,**</u> (See Figure 2).

It consists of the same components as the previous system but the load is a space at a uniform temperature T_R. Performing an energy balance for the storage tank and the enclosureyields to the following dynamical equations;

$$C_s \frac{dT_s}{dt} = \dot{Q}_u - [(UA)_s + (\dot{m} C_p)_R] (T_s - T_R) \tag{3.3}$$

$$C_R \frac{dT_R}{dt} = (\dot{m} C_p)_R (T_s - T_R) + \dot{Q}_{aux} - (UA)_R (T_R - T_a) \tag{3.4}$$

Where $\dot{Q}_{aux}$ is auxiliary heat energy. A performance measure of the form;

$$J_b = \int_{t_0}^{t_f} [f\dot{Q}_{aux} + C (T_R - T_{set})^2]\, dt \tag{3.5}$$

is selected for minimization. The first term in J_b represents a measure of the energy cost and the second is a measure of discomfort. Ideally, the objective function should be optimized over the entire heating season. This however is not practical because of the difficulty of the prediction of the weather conditions. As an approximation the minimization is taken on a daily basis, and T_R is forced to equal T_{set} at midnight. Midnight was chosen because optimization performed over one day resulted in a temprature near T_{set} at midnight /2/.

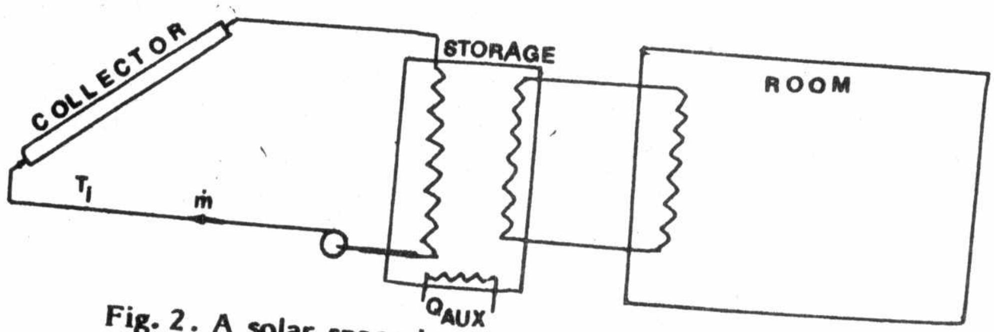

Fig. 2. A solar space heating system.

4. METHODOLOGY:

The application of Pontryagin maximum principle to the previousely formulated optimal control problems leads to well known problems in automatic control called Two Point Boundary Value Problems TPBVP. The solution of which is a very difficult task. Numerous methods are available in the literature to solve such problem numerically such as finite difference, weighted residuals quasi-linearization and shooting methods. So far no general satisfactory method has been developed. Quite often these methods are so sensitive that small changes, even in one single parameter such as time interval for integration causes divergence. When facing such problems, each one develops his own algorithm which is perfectly suitable for solving his particular problem. Unfortunately the method usually fails if an attempt is made to apply it to a different problem. Generally the convergence of non-linear TPBVP's is never assured. Morever, if the initial guesses appearing in any of the procedure hyppened to be far from the required values, the algorithm diverges. In this study the importance and the application of the Computer Aided Optimization (CAO) method developed in /4/ to solve the problems formulated in (3-a- , and c) is shown. This CAO method is based on the representation of control inputs by suitable analytical functions. The coefficients of these functions are determined iteratively by a static search algorithm to values optimizing the quality criterion allocated to each system. With this method the originally complicated dynamic optimization problem is transformed to an easily solvable static parameter optimization task.

The developed program consists mainly of four subroutines:

) The dynamic model subroutine allows the integration of the process under consideration system a, or c. The integration is performed by a fourth order Rung-Kutta method.

) The structure of the control input is approximated by an efficient spline function.

i) The quality criterion subroutines, in which the criterion J_a, or J_c is calculated at each step and added over the whole integration length.

) The optimization subroutine permits to determine the coefficient of the spline functions iteratively so that the optimal of the quality criterion is achieved. This is a very efficient subroutine which can determine up to 30 parameters with constraints on these parameters by adopting the search step. Also it does not need to calculate the gradient of the quality criterion to determine the search direction.

The basic steps of the developed algorithm are:

1. Assume a certain number of initial values for each control inputs at discrete values of the independent variable. The number of these points is assumed appropriately to permit adequate representation of the control variables.
2. Obtain an analytical function for each control variable by fitting the assumed points with cubic spline functions with coefficients C_i to enable the calculation of the control variables at any instant by interpolation.
3. Knowing the values of the control variables and the initial conditions on the state variables, the dynamic equations of the system are integrated forward either in a closed form or numberically by using a fourth order Rung-Kutta method.
4. Knowing the values of the control and state variables on the whole interval $(t_o - t_f)$, the value of the criterion J is calculated by summing up its instantaneous value on each integration step.
5. The values of the criterion J are transferred to the subroutine EXTREME that adjusts the coefficients C_i of the spline function to values that optimize (maximize or minimize) the criterion J. These coefficients define the shape of the required optimal control. The optimization procedure is shown shematically in Fig. 3.

The subroutine EXTREME allows to determine the extremum of a function with N variables where the analytical properties of this function are not knowns. The algorithm consists of the following steps:

1. Choice of search direction
   The first search direction is given by the user in the form of a vector DX (N). This vector contains the N initial steps, appropriately chosen to define the initial step around the initial vector X_i. The extremum along this first direction is determined, say X_{i+1}. A second search direction passing by X_{i+1} and perpendicular to the first is constructed, see Fig. 4. The extremum along this new direction is denoted by X_{i+2}. A third search direction passing by X_{i+2} and perpendicular to the previous directions is constructed. This procedure is repeated until we get the X_{i+N} point. The first stage (N iterations) is thus completed. The new principal direction for the next stage is now given by the line joigning the initial point X_i and the extremum X_{i+N}.
2. Determination of the extremum along a line.
   Three values of the function, F_1, F_2 and F_3 are calculated at the three points X_1 - DX, X_1, X_1 + DX respectively. The extremum of a fictitous parabola that passes by the previous three points is determined. The locus of the extremum is given by the following form.

$$X_{i+1} = X_i - DX \frac{F_3 - F_1}{2|F_1 - 2F_2 + F_3|K}$$

   Where K = +1 for the search of maximum and K = -1 for the search of minimum.
3. Definition of the search step.
   In the first stage the initial values of the step along each direction are given by the user in the table DX (N). This step is divided by four if the distance between the new and the previous points is smaller than one quarter of the current step along this direction. The step is multiplied by two if the new point is situated at twenty times the current step.
4. Consideration of the constraints.
   Before the calculation of the function, a test is introduced in the subroutine describing the function to be optimized to examine whether the current point is situated in the admissible domain or not. If not a RETURN to subroutine EXTREME is done to deliver another point that satisfy the constraints.

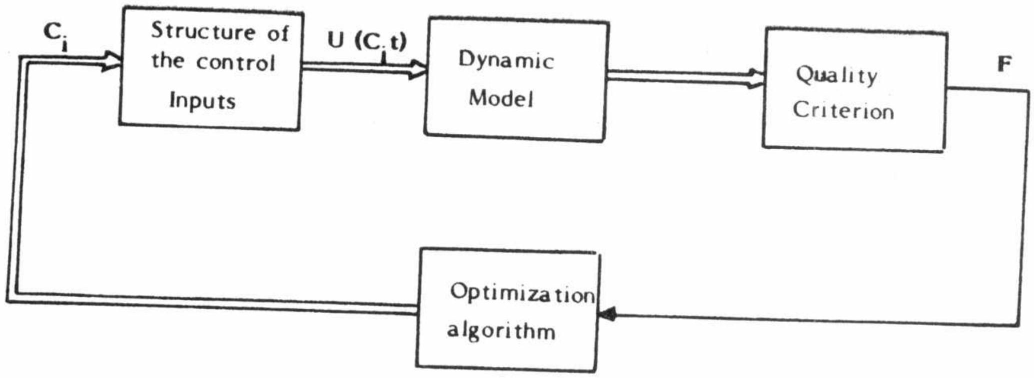

Fig. (3): The Optimization Procedure

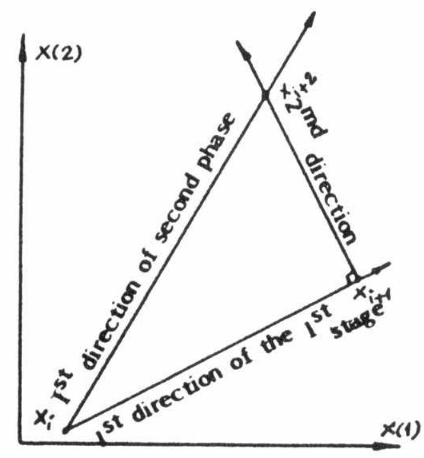

Fig. (4a) : Search direction

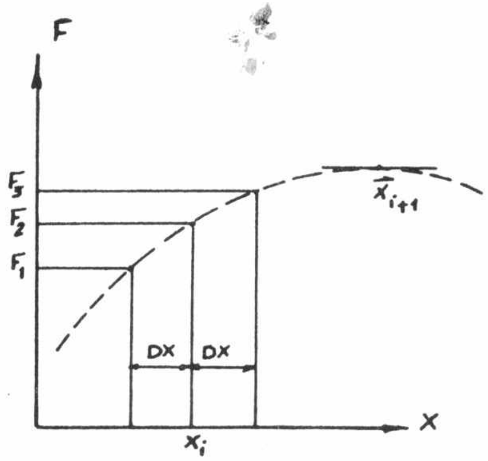

Fig. (4b): The extremum along a line.

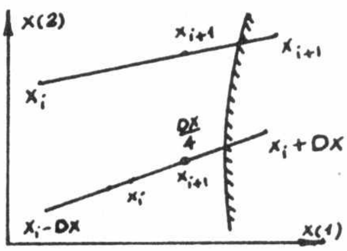

Fig. (4c): Situation near the boundaries in the case of two dimensions.

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5. NUMERICAL RESULTS AND DISCUSSION:

a) Domestic Hot Water DHW-system:

Simulations were carried out with the parameter values given in table (1).

For the purpose of comparison both the optimal flow rate and ON-OFF controller with different setting collector flow rates were considered.

Results given in figure (5) shows the variation of the criterion J_a with ocllector mass flow rate with the ON-OFF controller. Mass flow rate of 0.025 kg/s/m² is shown to give a maximum value of J_a at two different weather conditions as characterised by incident solar radiation and ambient temperature.

Figure (6) shows the collector mass flow rate per unit area/versus the time of the day for the considered uniform load profile. At the begining of the day, the mass flow rate rises, reaches a peak value at about 12 noon and then falls back down to zero at sun set. This behaviour is similar to that of incident solar radiation. Variations of storage, temperature are shown in figure (7) for two different initial storage temperature and the load considered.

The comparison of the net energy J integrated over the day for optimal flow and ON-OFF controller shows that the optimal flow rate produces a value of J that is a little higher than the best value that can be obtained by a constant mass flow rate.

Substantial reduction of J is noticed for any decrease/increase from that value. The comparison is summarized in Figure (8)

CONCLUSION:

The present work has described a computationally viable method to determine the optimal control for three different systems-Domestic Hot Water, space heating systems and train trajectory. This is a modular general usage method. For design purposes the controlling system cost must be considered. However the results obtained in this study show that:

For the DHW system an ON-OFF low price controller operating at a well chosen setting value of mass flow rate, may produce nearly the same performance as the fully optimaly controlled system.

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Table 1: Input data for DHW system.

Model parameter	Symbol	UNits	Value
Collector area	A_c	m^2	8
Collector efficiency factor	F^c	-	0.98
Pumping cost coefficient	C	$W/(Kg/S)^3$	1000
Collector loss coefficient	U_L	$W/m^2 {}^\circ K$	4
Storage loss coefficient	U^L	$W/m^2 {}^\circ K$	0.4
Specific heat	C^s	$J/kg^\circ K$	4187
Transmittance-absorbtence product	τ^p_α	-	0.8
Switch-on temperature differential	T_{on}	°K	1

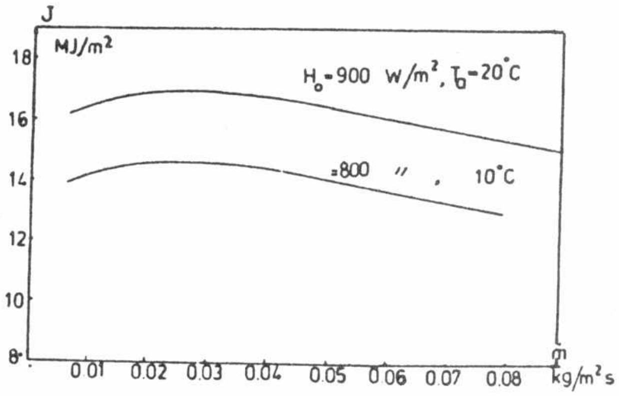

Fig. 5. Variation of the criterion with the mass flow rate for the ON-OFF controller.

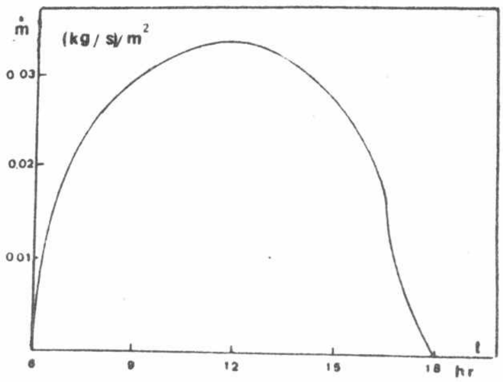

Fig. 6. Optimal collector mass flow rate.

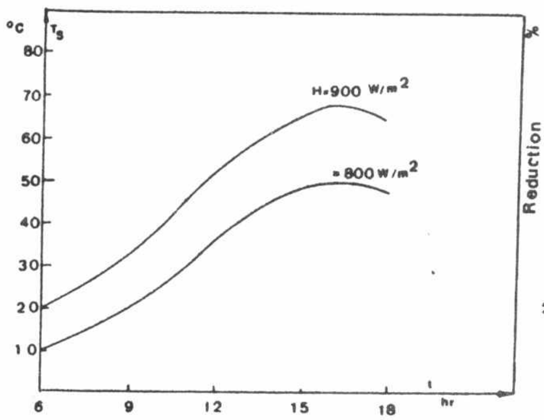

Fig. 7. Variation of storage temperature for the optimal mass flow rate.

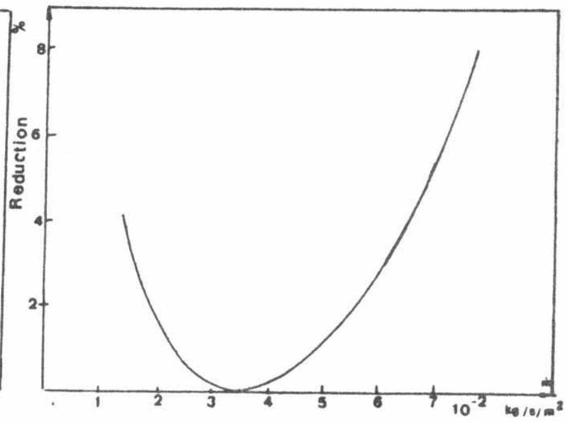

Fig. 8. Reduction of the criterion at different values of collector mass flow rate.