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DIGITAL FLIGHT CONTROL SYSTEM DESIGN USING OUTPUT FEEDBACK & MODEL FOLLOWING TECHNIQUES

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ABSTRACT

Recent advances in control technology provide the means for designing multimode control laws which achieve improved weapon delivery capability and reduced pilot workload.

The control objectives for these advanced modes include the ability to command a chosen variable without experiencing significant motion in other, specified varables.

Direct Lift Control (DLC) is characterized by flight path command without change in angle of attack.

Two design methods are used to generate a direct lift control law for an advanced aircraft, fast-sampling error-actuated digital controllers method and model following technique, [2] and [8].

Simulation illustrates that fast-sampling error-actuated digital controllers offer several advantages over model following technique.

INTRODUCTION

It has been recognised in recent years, that it is possible to make direct digital control of flight modes which provide ; the aircraft to maneuvre in ways which are impossible with conventional control systems ; the capability for implementing specialized modes for bombing, strafing, and air-to-air combat for advanced fighter aircraft.

The control objectives for these advanced modes include the ability to command a chosen variable without experiencing significant motion in other, specified variables.

For longitudinal dynamics of an aircraft, such decoupled modes include direct lift control fuse lage pitch pointing, and vertical translation.

Direct lift Control (DLC) is characterized by flight path command (or normal acceleration command) without change in angle of attack.

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DLC provides an aircraft with enhanced capabilities in both offensive and defensive situations besides the terrain following missions. Defensively, the change in flight path is essential, and the fact that there are no pitching cues aids deception. Offensive improvements are in improved flight path control resulting in more precise bomb runs, quicker dive recovery and lessened altitude loss.

In this paper, two design methodologies are used to generate a direct lift control law for an advanced aircraft model.

Porter and Bradshaw design method [2], [4], indicates that non-interacting control is achievable by the implementation of fast-sampling error-actuated digital controllers only if extra plant output measurements are generated by the introduction of appropriate transducers and processed by inner-loop compensators.

Model following technique, achieve the required maneuvre, or the specialized mode formulated as desired mode, by minimizing the integral square error between the desired model and aircraft model (performance index), generates constant feedback gains which match the aircraft with desired model [7], [8].

Simulations for normal acceleration command, illustrates that Porter method offer several advantages over model following technique such as, the avoidance of a complex cost critiria and its associated mathematical anyalysis, moreover, the fast-sampling digital controllers are extremely robust in the sense that these maneuvres can be effected throughout a wide range of different flight conditions by the same controller.

I. Fast-Sampling error-actuated digital Controllers Method

i. System Configuration

Linear multivariable time invariante plant may be represented (by transformation of states, if necessary) by the state and output equations of the respective forms [1,2,3]

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} x_1(t) \\ X_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ B_2 \end{bmatrix} U(t) \qquad (1)$$

and

$$Y(t) = \begin{bmatrix} C_1 & C_2 \end{bmatrix} \begin{bmatrix} X_1(t) \\ X_2(t) \end{bmatrix} \qquad (2)$$

where n = no. of states, ℓ = no. of outputs = no. of inputs, X_1 state vector partition of length $(n - \ell)$, X_2 state vector partition of lenght (ℓ), and B_2 is a square, nonsingular matrix.

In order to achieve tracking, new output measurements W(t) are selected, given by

$$W(t) = Y(t) + M\,\dot{x}_1(t)$$
$$= \begin{bmatrix} C_1 + MA_{11} & C_2 + MA_{12} \end{bmatrix} \begin{bmatrix} X_1(t) \\ X_2(t) \end{bmatrix}$$
$$W(t) = \begin{bmatrix} F_1 & F_2 \end{bmatrix} \begin{bmatrix} X_1(t) \\ X_2(t) \end{bmatrix} \qquad (3)$$

Where M is an extra plant measurement matrix which must be designed so that

$$\text{rank } F_2 = \text{rank } \left| C_2 + MA_{12} \right| = \ell \qquad (4)$$

A fast-sampling error-actuated digital controller governed on the discrete-time set $I_t = (0_1, T_1, 2T, ..)$ by the proportional plus integral control - law Fig.1 . of the form.

$$U(kT) = f\left[\mathcal{K}_o e(KT) + \mathcal{K}_1 Z(KT)\right] \qquad (5)$$

Where f = gain equal to sampling frequency, $\mathcal{K}_o$ = proportional error controller matrix, $\mathcal{K}_1$ = integral of error controller matrix, T sampling time, e(kT) represents the error betweem the command input vector V (kT) and the measurement vector W(t), and Z(kT) is the integral of the error vector

$$Z\left[(k+1)T\right] = Z(kT) + T\,e(kT) \qquad (6)$$

$$e(kT) = V(kT) - W(kT) \qquad (7)$$

It is required to generate the control input vector

$$U(t) = U(kt),\ t \in \left(kT, (k+1)T\right),\ kT \in I_t \qquad (8)$$

So as to cause the output vector Y(T) to track any constant command input vector V(t) on I_t in the sense that

$$\lim_{k \to \infty} \left[V(kT) - Y(kT)\right] = 0 \qquad (9)$$

as a consequence of (7) at steady state

$$\lim_{k \to \infty} e(kT) = \lim_{k \to \infty} \left[V(kT) - W(kT)\right] = 0 \qquad (10)$$

for arbitrary initial conditions.

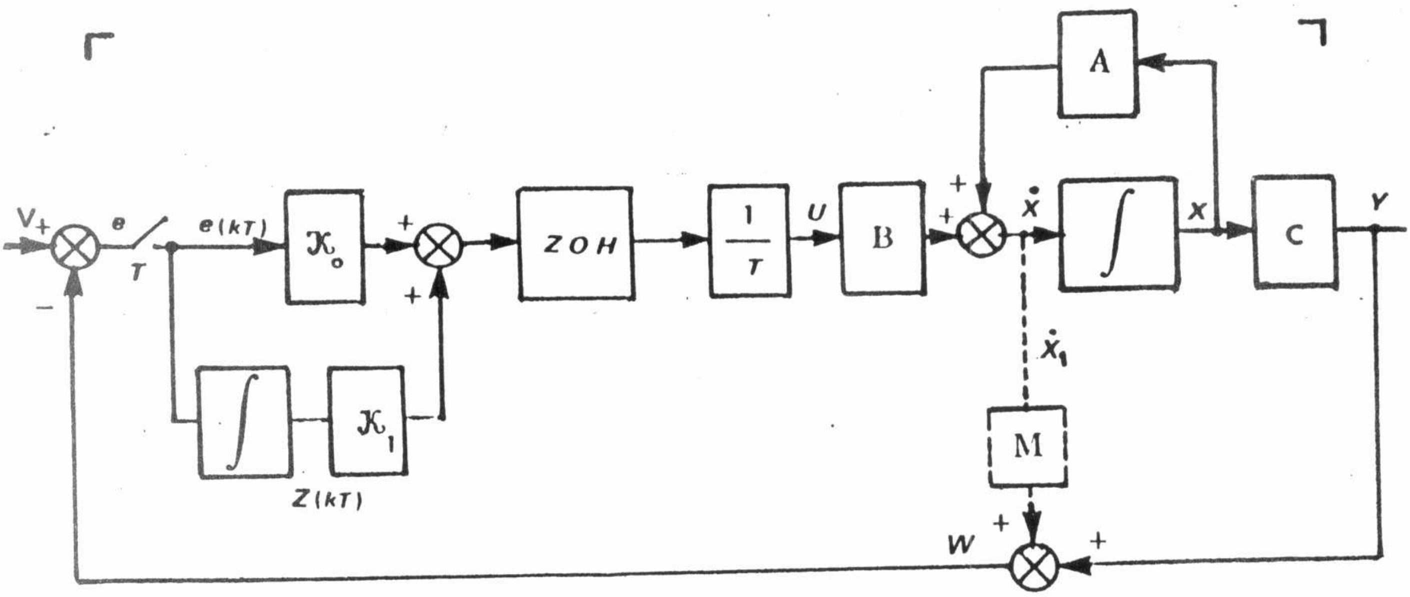

Fig 1. Block Diagram Of Controllers Design

In equations (1) to (7), $X_1(t) \in R^{n-\ell}$, $X_2(t) \in R^{\ell}$, $U(t) \in R^{i}$, $W(t) \in R^{\ell}$, $A_{11} \in R^{(n-\ell)\times(n-\ell)}$, $A_{12} \in R^{(n-\ell \times \ell)}$, $A_{21} \in R^{(\ell \times n-\ell)}$, $A_{22} \in R^{\ell\times\ell}$, $B_2 \in R^{\ell\times\ell}$, $C_1 \in R^{\ell\cdot(n-\ell)}$, $C_2 \in R^{\ell\times\ell}$, $F_1 \in R^{\ell\times(n-\ell)}$, $F_2 \in R^{\ell\times\ell}$, $e(t) \in R^{\ell}$, $V(t) \in R^{\ell}$, $\mathcal{K}_0 \in R^{\ell\times\ell}$, $\mathcal{K}_1 \in R^{\ell\times\ell}$, $M \in R^{\ell\times(n-\ell)}$

Equations (2), (3) clearly indicate that the vector of extra measurement is

$$W(t) - Y(t) = \begin{bmatrix} MA_{11} & MA_{12} \end{bmatrix} \begin{bmatrix} X_1(t) \\ X_2(t) \end{bmatrix} \qquad (11)$$

Such that V(kT) and Y(kT) satisfy the tracking condition (9) for any matrix M if e(kT) satisfy the steady-state condition (10), since equation implies that

$$\lim_{t \to \infty} \begin{bmatrix} A_{11} & A_{12} \end{bmatrix} \begin{bmatrix} X_1(t) \\ X_2(t) \end{bmatrix} = 0 \qquad (12)$$

in any steady state.

It is evident from equations (1), (2), (3), and (5) that the discrete form of the tracking system are :

$$\begin{bmatrix} Z\{(k+1)T\} \\ X_1\{(k+1)T\} \\ X_2\{(k+1)T\}\end{bmatrix} = \begin{bmatrix} I_\ell & -TF_1 & -TF_2 \\ \Gamma\Psi_1\mathcal{K}_1 & \Phi_{11}-\Gamma\Psi_1\mathcal{K}_0F_1 & \Phi_{12}-\Gamma\Psi_1\mathcal{K}_0F_2 \\ \Gamma\Psi_2\mathcal{K}_1 & \Phi_{21}-\Gamma\Psi_2\mathcal{K}_0F_1 & \Phi_{22}-\Gamma\Psi_2\mathcal{K}_0F_2\end{bmatrix}\begin{bmatrix}Z(kT)\\X_1(kT)\\X_2(kT)\end{bmatrix}$$

$$+ \begin{bmatrix} T I_\ell \\ \Gamma\Psi_1\mathcal{K}_0 \\ \Gamma\Psi_2\mathcal{K}_0\end{bmatrix} V(kT) \qquad (13)$$

and
$$Y(kT) = \begin{bmatrix} 0 & C_1 & C_2 \end{bmatrix} \begin{bmatrix} Z(kT) \\ X_1(kT) \\ X_2(kT)\end{bmatrix} \qquad (14)$$

where
$$\begin{bmatrix} \Phi_{11} & \Phi_{12} \\ \Phi_{21} & \Phi_{22}\end{bmatrix} = \exp.\left\{\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22}\end{bmatrix} T\right\} \qquad (15)$$

and
$$\begin{bmatrix} \Psi_1 \\ \Psi_2 \end{bmatrix} = \int_0^T \exp.\left\{\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22}\end{bmatrix} t\right\}\begin{bmatrix} 0 \\ B_2 \end{bmatrix} dt \qquad (16)$$

The transfer function matrix of the closed - loop discrete system governed by equation (13), (14) is

$$G(\lambda) = \begin{bmatrix} 0 & C_1 & C_2 \end{bmatrix}\begin{bmatrix} \lambda I_\ell - I_\ell, & TF_1, & TF_2 \\ -\Gamma\Psi_1\mathcal{K}_1, & \lambda I_{n-\ell}-\Phi_{11}+\Gamma\Psi_1\mathcal{K}_0F_1, & -\Phi_{12}+\Gamma\Psi_1\mathcal{K}_0F_2 \\ -\Gamma\Psi_2\mathcal{K}_1, & -\Phi_{21}+\Gamma\Psi_2\mathcal{K}_0F_1, & \lambda I_\ell-\Phi_{22}+\Gamma\Psi_2\mathcal{K}_0F_2 \end{bmatrix}\begin{bmatrix} TI \\ \Gamma\Psi_1\mathcal{K}_0 \\ \Gamma\Psi_2\mathcal{K}_0 \end{bmatrix} \qquad (17)$$

At fast-sampling, it follows from equation (15) and (16) that

$$\lim_{f \to \infty} f \begin{bmatrix} \Phi_{11}-I_{n-\ell} & \Phi_{12} \\ \Phi_{21} & \Phi_{22}-I_{\ell} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \tag{18}$$

and

$$\lim_{f \to \infty} f \begin{bmatrix} \Psi_1 \\ \Psi_2 \end{bmatrix} = \begin{bmatrix} 0 \\ B_2 \end{bmatrix} \tag{19}$$

From results of Porter and Shenton [2], as $f \to \infty$ the transfer function matrix $G(\lambda)$ assumes the asymptotic form

$$\Gamma(\lambda) = \tilde{\Gamma}(\lambda) + \hat{\Gamma}(\lambda) \tag{20}$$

Where, $\tilde{\Gamma}(\lambda)$ contains "slow" modes and $\hat{\Gamma}(\lambda)$ contains "fast" modes.

$$\tilde{\Gamma}(\lambda) = C_0(\lambda I_n - I_n - I A_0)^{-1} \; I B'_0 \tag{21}$$

$$\hat{\Gamma}(\lambda) = C_2(\lambda I_\ell - I_\ell - A_4)^{-1} \; B_2 \mathcal{K}_0 \tag{22}$$

$$A_0 = \begin{bmatrix} -\mathcal{K}_0^{-1}\mathcal{K}_1 & 0 \\ A_{12}F_2\mathcal{K}_0^{-1}\mathcal{K}_1 & A_{11}-A_{12}F_2^{-1}F_1 \end{bmatrix} \tag{23}$$

$$B_0 = \begin{bmatrix} 0 \\ A_{12} \; F_2^{-1} \end{bmatrix} \tag{24}$$

$$C_0 = \begin{bmatrix} C_2F_2^{-1}\mathcal{K}_0^{-1}\mathcal{K}_1 & C_1-C_2F_2^{-1}F_1 \end{bmatrix} \tag{25}$$

and

$$A_4 = -B_2\mathcal{K}_0F_2 \tag{26}$$

From equations (21) and (23), the slow modes Z_s of the tracking system as $f \to \infty$ are the poles $Z_1 \cup Z_2$ of $\tilde{\Gamma}(\lambda)$ where

$$Z_1 = \left\{ \lambda \in C : \left| \lambda I_\ell - I_\ell + T\mathcal{K}_0^{-1}\mathcal{K}_1 \right| = 0 \right\} \tag{27}$$

and

$$Z_2 = \left\{ \lambda \in C : \left| \lambda I_{n-\ell} - I_{n-\ell} - TA_{11} + TA_{12}F_2^{-1}F_1 \right| = 0 \right\} \tag{28}$$

While the fast modes Z_f as $f \to \infty$ are the poles Z_3 of $\hat{\Gamma}(\lambda)$ where

$$Z_3 = \left\{ \lambda \in C : \left| \lambda I_\ell - I_\ell + F_2 B_2 \mathcal{K}_0 \right| = 0 \right\} \tag{29}$$

Furthermore, it follows from equations (21), (23), (24) and (25) that the slow transfer function matrix reduces to the form

$$\tilde{\Gamma}(\lambda) = (C_1 - C_2 F_2^{-1} F_1)(I_{n-\ell} - I_{n-\ell} - TA_{11} + TA_{12}F_2^{-1} F_1)^{-1} TA_{12}F_2^{-1} \tag{30}$$

because the slow modes corresponding to poles Z_1 are asymptotically unstable as $f \to \infty$

Also from equations (22) and (26), the fast transfer function

$$\hat{\Gamma}(\lambda) = C_2 F_2^{-1} (\lambda I_\ell - I_\ell + F_2 B_2 \mathcal{K}_0)^{-1} F_2' B_2 \mathcal{K}_0 \tag{31}$$

Hence, in view of equations (20), (30) and (31), it is evident that the asymptotic closed-loop transfer function $\Gamma(\lambda)$, as $f \to \infty$ and the state and output equations (1),(2) and (3), are equivalent [1].

ii <u>System Synthesis</u>

System model defined by equations (13) and (14) will achieve required tracking in the sense of equation (9) provided only that

$$Z_s \cup Z_f \subset \text{Unit Circule} \tag{32}$$

for sufficiently small sampling periods. This will be satisfied if the controller and transducer matrices $\mathcal{K}_0$, $\mathcal{K}_1$ and M are chosen such that the poles Z_1, Z_2 and Z_3 lies inside a unit circule . (equations (27), (28) and (29)). And matrix M must simultaneously satisfy equation (4).
Moreover, for increasing non-interacting tracking behaviour, $\mathcal{K}_0$ and M chosen so that equation (20) be a diagonal transfer function by requiring that

$$F_2 B_2 \mathcal{K}_0 = (C_2 + MA_{12}) B_2 \mathcal{K}_0 = \text{diag}(\sigma_1, \sigma_2, \ldots, \sigma_\ell) \qquad (33)$$

II. Model-Following System Synthesis

At linear optimal control theory, the quadratic performance index does not always express the real design objective, such transient response which may be more meaningful for modern control systems. Model following technique take into consideration such criteria, required dynamics are formulated as desired linear model, the integral square error between the model and system is minimized in the performance index [7]

The linear optimal control theory is applied to the two types of model following systems (implicit and explicit model following) [8], which in both cases generates constant feedback gains which match the sytem with the required model.

Design starts by description of the required response of the aircraft as desired model

$$\dot{Z} = A_m Z \qquad (34)$$

while the main plant (aircraft) model

$$\left.\begin{aligned} \dot{X} &= A_a X + B_a U \\ Y &= C_a X \end{aligned}\right\} \qquad (35)$$

and the performance index becomes

$$J_e = \min \int_0^\infty \left[(Y-Z)^T Q (Y-Z) + U^T R U \right] dt \qquad (36)$$

where Q,R are the wighting matrices.

After the appropriate choice of wighting matrices Q,R and sampling period, the system and criteria in equation (35) and (36) are descretized [8].

$$\left.\begin{aligned} X_{k+1} &= \phi X_k + \psi U_k \\ J_D &= \sum_{k=1}^{N} (X^T_k Q_D X_k + 2X^T_k M_D U_k + U^T_k R_D U_k) \end{aligned}\right\} \qquad (37)$$

so the optimal control law will be

$$U_k = G_k X_k \qquad (38)$$

where

G_k : The optimal gain

$$G_k = -(R_D + \psi^T P_{k+1} \psi)^{-1} . (\psi^T P_{k+1} \phi + M^T_D) \qquad (39)$$

and P_k : The solution of the discrete matrix Riccatti equation

$$P_k = (\phi^T P_{k+1} \phi + Q_D) - (\phi^T P_{k+1} \psi + M_D) \cdot (R_D + \psi^T P_{k+1} \psi)^{-1} \cdot (\psi^T P_{k+1} \phi + M_D^T) \qquad (40)$$

Discrete matrices ϕ, ψ, Q_D, M_D, and R_D were calculated by the program DISCG2, also P_k SOLUTION OF Riccatti equation was obtained by using an efficient program RICAT2 [8].

SIMULATION RESULTS

The linearised longitudinal dynamics of an aircraft flying at a mach number of 0,6 at an altitude of 3000 ft are governed by the state and output equations of the respective forms

$$\begin{bmatrix} \dot{\theta} \\ \dot{q} \\ \dot{\alpha} \end{bmatrix} = \left[\begin{array}{c|cc} 0 & 1 & 0 \\ \hline 0 & -,869 & 43,2 \\ 0 & 0,993 & -1,34 \end{array}\right] \begin{bmatrix} \theta \\ q \\ \alpha \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ \hline -1,58 & -17,3 \\ -,252 & -1,169 \end{bmatrix} \begin{bmatrix} \delta_f \\ \delta_e \end{bmatrix} \qquad (41)$$

$$\begin{bmatrix} n_A \\ \gamma \end{bmatrix} = \left[\begin{array}{c|cc} 0 & -,268 & 47,8 \\ 1 & 0 & -,1 \end{array}\right] \begin{bmatrix} \theta \\ q \\ \alpha \end{bmatrix} \qquad (42)$$

porter's method was used to compute the control law for direct lift control. Number of inputs equal number of output = 2, $\ell = 2$, $n - \ell = 1$. In view of equations (27), (28), (29) and (33), matrices M, $\mathcal{K}_0$, $\mathcal{K}_1$ were calculated such that equations (4), (32) and (33) are satisfied.

It was assumed that $\{\sigma_1, \sigma_2\} = \{1, 1\}$, $\mathcal{K}_1 = \mathcal{K}_0$ and matrix

$M = \begin{bmatrix} 0,25 \\ 0 \end{bmatrix}$, it follows from equation (3) that the corresponding transducers measurement equation is

$$\begin{bmatrix} W_1(t) \\ W_2(t) \end{bmatrix} = \begin{bmatrix} 0 & -0,018 & 47,8 \\ 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} \theta \\ q \\ \alpha \end{bmatrix} \qquad (43)$$

and from equations (5) and (33) the control law equations

$$\begin{bmatrix} U_1(kT) \\ U_2(kT) \end{bmatrix} = \frac{1}{T}\left\{ \begin{bmatrix} -2,2869 & -105,1 \\ 3,14 & 162,6 \end{bmatrix} \begin{bmatrix} e_1(kT) \\ e_2(kT) \end{bmatrix} + \right.$$

$$\left. + \begin{bmatrix} -2.2869 & -105,1 \\ 3,14 & 162,6 \end{bmatrix} \begin{bmatrix} Z_1(kT) \\ Z_2(kT) \end{bmatrix} \right\} \qquad (44)$$

For sampling period $T = 0,02$ Sec., the normal acceleration is shown in Fig. 2, when the command input vector $[V_1(t) \quad V_2(t)]$ changed from [1 1] to [8 1] in 0,8 sec.

It is apparent that the pull up maneuvre is satisfactory made with minimum transient interaction between γ and α .

Model following technique was applied to carry out the same maneuvre, i.e., to achieve an 8-g pull up in 0,8 sec.

(For calculation details, referred to reference [8])

Fig. 3 illustrates that the direct lift control mode was effected, but with interaction control associated with overshoot, which does not improve neither offensive nor defensive maneuvres.

CONCLUSION

It is evident that the fast-sampling error-actuated digital control design method is straight-forward and offers several advantages over model following technique for control system design such as (1) the avoidance of a complex cost function and its associated mathematical analysis (discretization-calculating of triple integral of exponential function, solution of matrix Riccati equation,... (2) the avoidance of the requirement for complete state feedback.

Furthermore, Porter's method exhibit high accurancy in the fact of plant-parameter variations provided that the steady state condition expressed by equation (12) correspond to kinematic relationships which hold between the state variables as a consequence of the fundamental dynamical structure of the plant, therefore, the control law operates over a wide range of flight condition successfully.

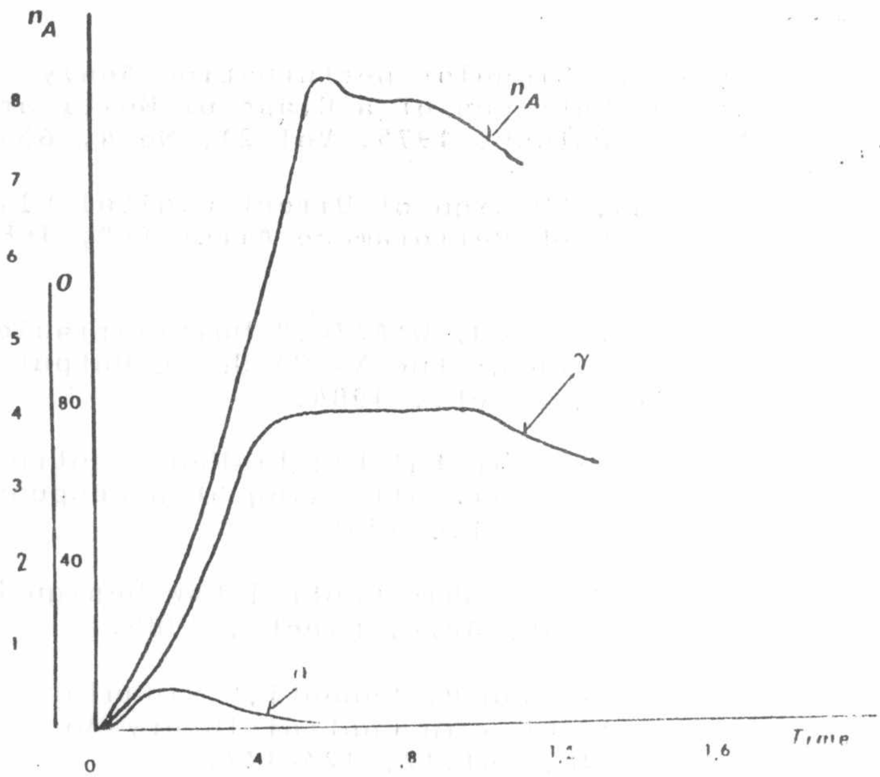
n_A
8
7
6
5
4
3
2
1
80
40
0
n_A
γ
0
.4
.8
1,6
Time

Fig. 2

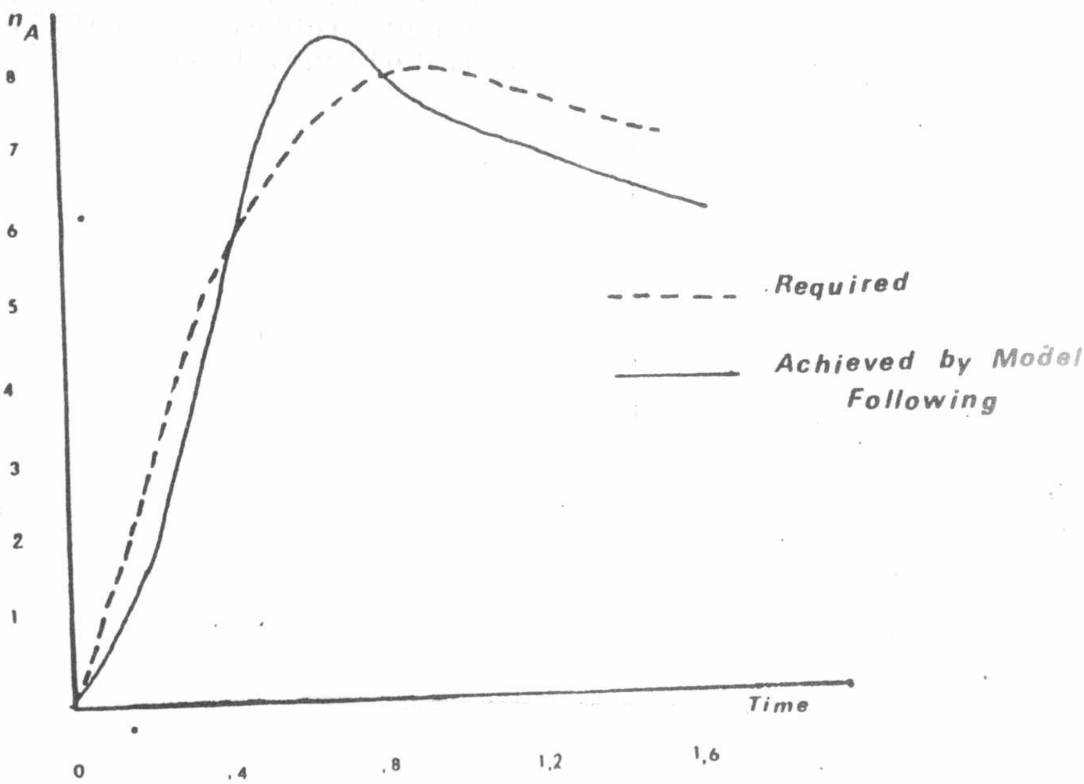
n_A
8
7
6
5
4
3
2
1
Required
Achieved by Model Following
Time
0
.4
.8
1,2
1,6

Fig. 3

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NOMENCLATURE.

θ ..... *Pitch Angle*

q ..... *Pitch Rate*

α ..... *Angle of Attack*

n_A .... *Normal Acceleration*

γ .... *Flight Path Angle*

δ_f .... *Flaperon Deflection*

δ_e .... *Elevator Deflection*