1 # An Auto-Pilot System Design Using Reduced Order

2 # Model Reference Adaptive Control

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4 ABSTRACT

In this paper, the singularly-perturbed design of an auto-pilot system of a high speed fighter aircraft, for the same authors (1985), is modified. The previous suboptimal design based on singular perturbation technique was proper only for aircraft operation near its nominal flight conditions. The new proposed adaptive design would be adequate for all possible changes of aircraft dynamics. The plant with varying parameters, representing aircraft, is controlled to behave nearly similar to a chosen first order reference model.

The proposed discrete model reference adaptive control design uses an adaptive algorithm of Suzuki and Takashima (1978), based on augmented error signal concept and Popov's hyperstability theorem. Computer simulation results are presented to demonstrate the usefulness of the proposed design.

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1. INTRODUCTION

Since a supersonic fighter aircraft does operate in a very wide range of flight parameters; it represents a drastically changing dynamical plant. The model reference adaptive control technique is a useful approach to the control of the dynamic system with unknown parameters (Landau 1974). Such a scheme generally requires complicated computations, so that the adaptive controller is best implemented with a digital computer. From this point of view, the adaptive control system has to be designed in discrete form.

The exact mathematical representation of the global control loop, of longitudinal flight of aircraft system, will lead to a high order plant model. Neglecting the relatively fast dynamics, and taking into consideration the conditions of the considered levelling control mode, the system can be described by a second order model with a reasonable degree of approximation. Using lower order approximate models, suboptimal responses are achieved but they are sufficieutly adequate.

The rest of the paper is organized as follows : the general equations of the adaptive control problem are stated in section 2. In section 3, the used adaptive algorithm is explained. The proposed adaptive controller is developed in section 4. Computer Simulation results are given in section 5, and conclusions appear in section 6.

2. STATEMENT OF THE PROBLEM

Consider the continuous linear dynamic system (plant) interconnected by a set of sampler and zero-order hold device, which is shown in fig.1 Let the continuous transfer function of the plant be described by

$$H(s) = \frac{d_{n-m} s^{m} + \cdots + d_{n-1} s + d_n}{s^n + C_1 s^{n-1} + \cdots + C_{n-1} s + C_n} \quad (m \leq n-1) \qquad (1)$$

where the coefficients c_i and d_i are unknown , or varying according to plant operating conditions.

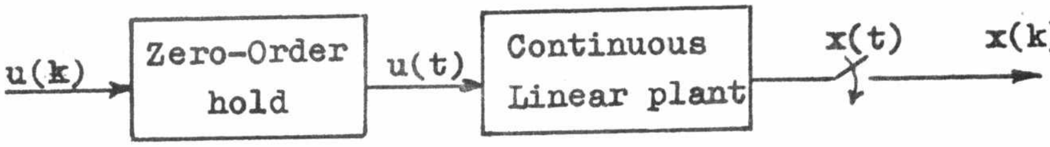

Fig. 1. Plant to be controlled

The sampled output of the plant can be represented by [4],

$$x(k) = \sum_{i=1}^{n} a_i \, x(k-i) + \sum_{i=1}^{n} b_i \, u(k-i) \qquad (2)$$

Where x(k) and u(k) are the plant output and input at the k th sampling instant, respectively, and a_i and b_i are constants related to the coefficients c_i and d_i . Due to variation of the coefficients c_i and d_i, the need for adaptive control for such a plant is obvious.

The design objective is to have the plant output follow the output of a reference model, regardless of the plant parameters.

The reference model is described by

$$x_m(k) = \sum_{i=1}^{n} a_{mi} \, x_{mi}(k-i) + \sum_{i=1}^{n} b_{mi} \, r(k-i) \qquad (3)$$

where x_m (k) is the model output, and r(k) is the reference input. The coefficients a_{mi} and b_{mi} are pre-specified so that the model may yield a stable and desired response to the reference input.

3. USED ADAPTIVE ALGORITHM

It is a discrete model reference adaptive scheme designed about Popov's hyperstability criterion, and it is easily implemented with a small number of on-Line computations. The proof of its hyperstability properties are available in [5], [4] . The flow chart of the adaptive algorithm [8] is shown in Fig.2. An overall view of the function of this controller is given in the block diagram depicted in Fig.3.

The following equations represent the adaptive algorithm itself. The required inputs for the controller (as shown in the block diagram of Fig.3)are the past plant outputs, past control signals, and the next reference model output.

Two primary intermediate variables are used in the computations because they greatly simplify the notation. V (k) is a 2n x1 vector comprised of the last n plant outputs, the last (n-1) inputs, and the next model output.

$$V(k) = \begin{bmatrix} Y_p(k) \\ Y_p(k-1) \\ \vdots \\ Y_p(k-n+1) \\ \hline U(k-1) \\ U(k-2) \\ \vdots \\ U(k-n+1) \\ \hline Y_m(k+1) \end{bmatrix} \qquad (4)$$

Zeta (ξ) is the sum of the squares of the components of V(k) multiplied by corresponding adaptive gains

$$\xi(k-1) = \sum_{i=1}^{2n} \gamma_i \, V_i^2(k-1) \tag{5}$$

With these variables, the coefficients of the cubic equation for the augmented error Eta (η) can be computed

$$C_0 = -\phi(k)$$
$$C_1 = 1 - G_0(k-1)\,\xi(k-1) \tag{6}$$
$$C_3 = \gamma_0 \, \xi^2(k-1)$$

$$C_3 \eta^3 + C_1 \eta + C_0 = 0 \tag{7}$$

After η has been determined from Eqn. (7), it is used to alter the adaptive variables in nonlinear difference equations

$$G_0(k) = G_0(k-1) - \gamma_0 \eta^2 \sum_{i=1}^{2n} \gamma_i \, V_i^2(k-1) \tag{8}$$
$$G_i(k) = G_i(k-1) + \gamma_i V_i(k-1)\,\eta \quad (i=1,\ldots,2n) \tag{9}$$

These adaptive variables are then used to compute the control signal for the plant input.

$$U(k) = \sum_{i=1}^{2n} g_i(k)\, V_i(k) \tag{10}$$

From (10) and (4), control input can be described by

$$U(k) = \sum_{i=1}^{n} G_i(k)\, Y_p(k+1-i) + G_{2n}(k)\, Y_m(k+1) + \sum_{i=1}^{n-1} G_{n+i}(k).U(k-i) \tag{11}$$

For simulation studies the sampled plant output is computed, while in application, the actual plant output is used instead.

4. APPLICATION ON AIRCRAFT SYSTEM

4.1 Mathematical Model of Aircraft System

The open loop transfer function between aircraft pitch angle (θ) and elevator surface deflection (δ_e), for the considered levelling control mode of a typical aircraft [7], can be approximated by the simple form :

$$\frac{\theta(s)}{\delta(s)} = \frac{K_p}{s(1+\tau_p s)} \tag{12}$$

Where k_p, τ_p are the gain and time constant of pitch motion of aircraft respectively. Numerical values, estimated at asumed nominal flight conditions [9] are given by

$k_p = 15$, $\tau_p = 7.5$ seconds

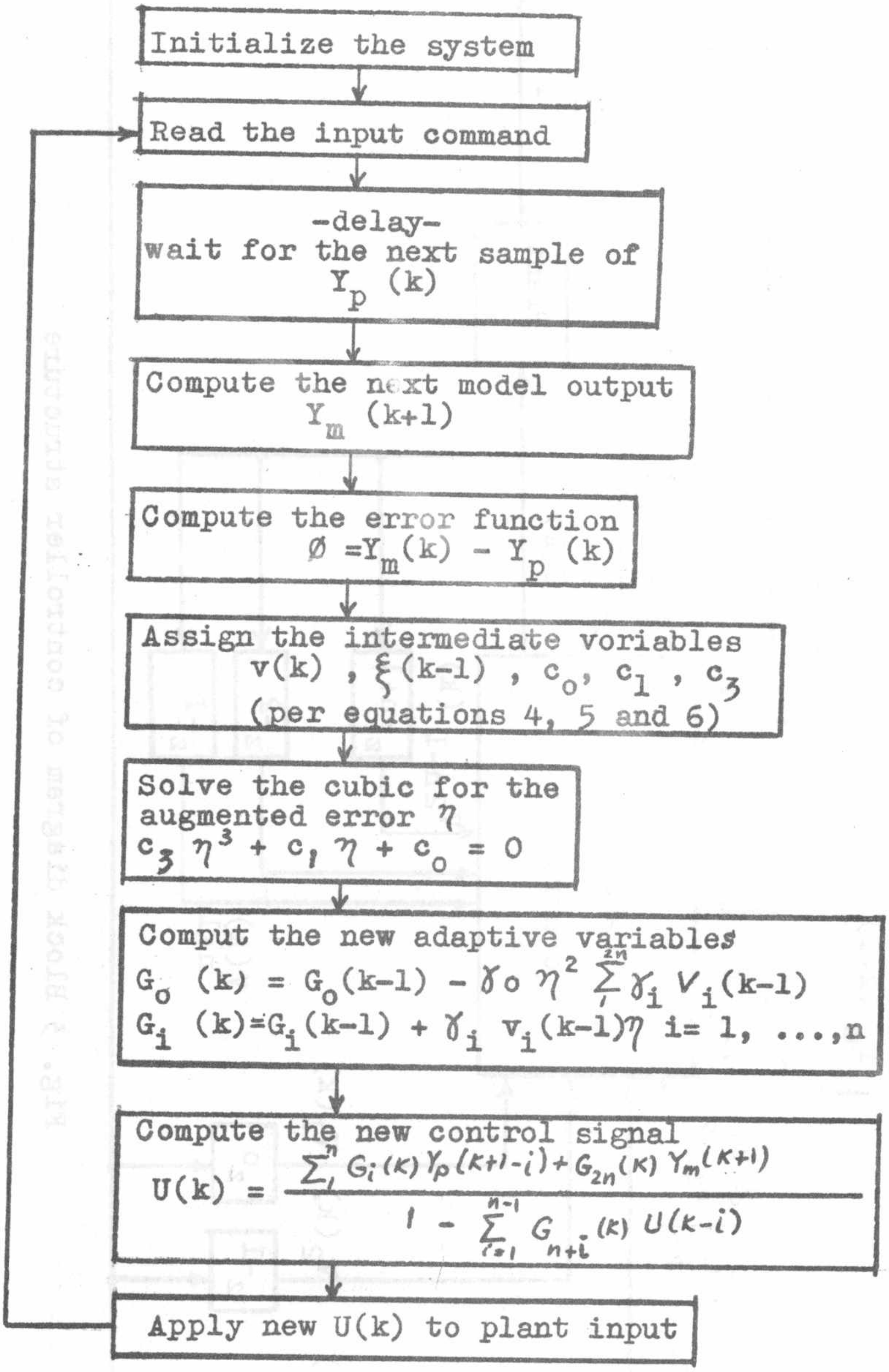

Fig. 2 Flowchart of the adaptive algorithm

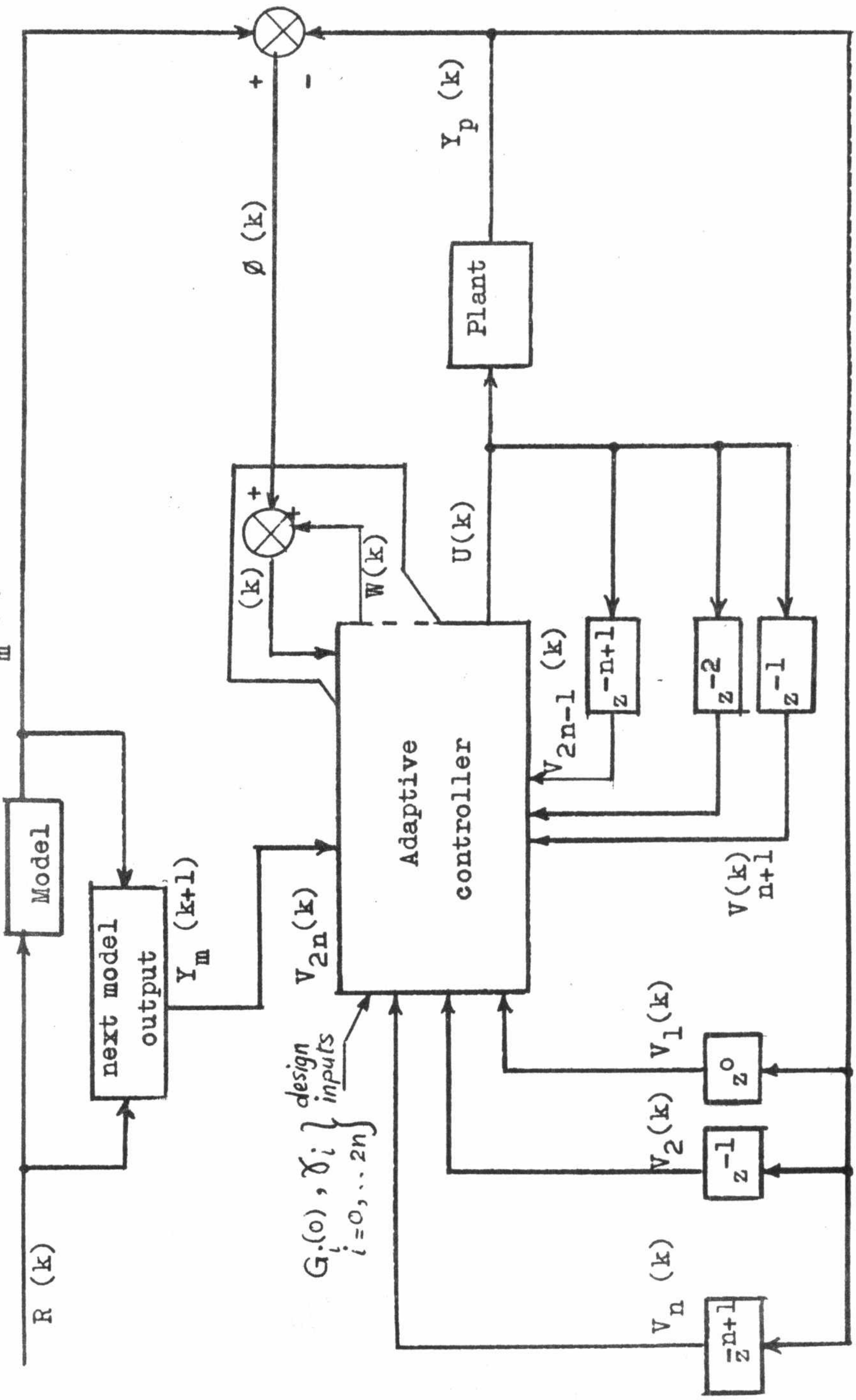

Fig. 3 Block diagram of controller structure

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4.2 Proposed Adaptive Controller

The adaptive controller used is shown in the block diagram of Fig.3. The plant in our case, is the closed loop controlled aircraft with constant feed back gains (k_θ , $k_{\dot\theta}$) as illustrated in Fig.4.

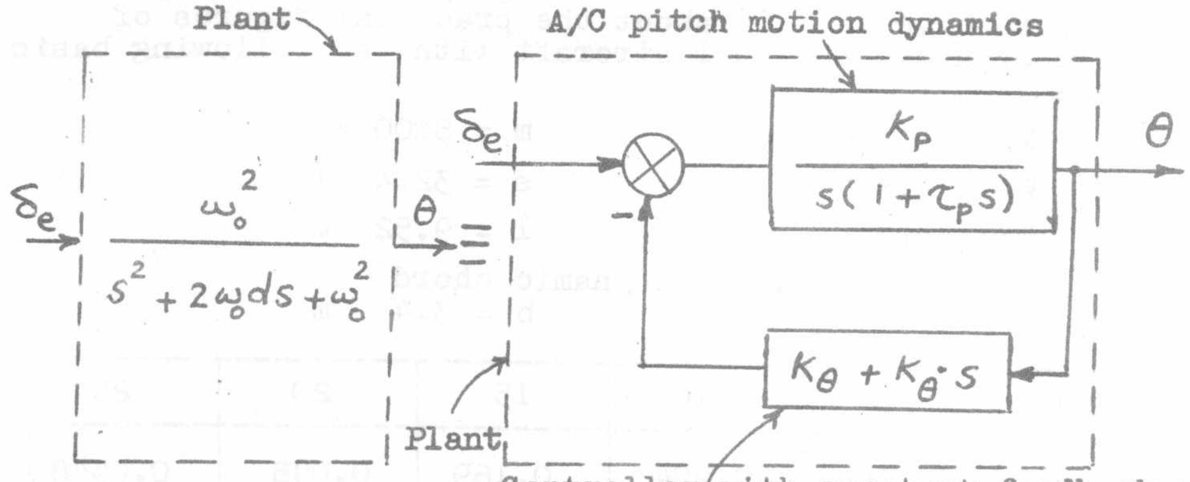

Fig.4. A/C with constant feed-back gain controller (PLANT)

k_θ , $k_{\dot\theta}$ are calculated in order to achieve a second order plant with critical damping of 0.707, and natural frequency of 2 rad/sec., at assumed nominal flight conditions. They are found to be $k_\theta = 3.125$, $k_{\dot\theta} = 1.68$.

The pitch control channel with a classical autopilot, controller with constant state feedback gains, would represent the unadapted second order plant. The negative state feedback gains are chosen to achieve optimal response at normal flight conditions of the typical aircraft (natural freq. ω_n = 2 rad/sec, damping factor $\zeta d = 0.707$).

The transfer function of plant- at certain flight parameters can be represented by

$$H(s) = \frac{4}{S^2 + 2.828\, S + 4} \qquad (13)$$

For a sampling interval of 0.1 seconds, the discrete transformation H(z), can be obtained using techniques described in Franklin and Powell(1980)

$$H(z) = \frac{0.0346\ z^{-1}}{1 - 1.7189\ z^{-1} + 0.7537\ z^{-2}} \qquad (14)$$

The reduced order reference model is chosen, arbitrarily as first order lag with time constant equal to T_m and gain equal unity (Simulation results are presented for

$$T_m = 1 \text{ sec.} \quad , T_m = 2 \text{ sec.})$$

As flight parameters of a fighter a/c (mainly altitude and speed) are always changing in a drastic manner, the plant controlled dynamics (w_o, d), are also changing correspondingly. See Table (1), to have an idea about the practical figures of parameters of a hypothetical aircraft with the following basic parameters [6] ,

Mass of aircraft $m = 8100$ kg

Wing area $s = 32.4$ m^2

Wing span $l = 9.52$ m

Length of mean aerodynamic chord $b = 3.4$ m

H (km)	5	10	15	20	25
d	0.665	0.274	0.169	0.098	0.0548
ω_0	2.61	5.1	5.08	4.32	3.642

Thus, we note that the adaptive controller structure for such a varying plant, is strongly needed.

5. COMPUTER SIMULATION RESULTS

The adaptive design of the aircraft auto-pilot system was subjected to input pulses of amplitude of 0.1 rad., and with 10 seconds duration in one polarity. All adaptive gains (γ_i) were set to 10 and the controller gains (G_i) were initialized to zero in some cases, and to better adapted ones in others.

The output response is shown in Fig.5, Fig.6. for different plant dynamics and, two cases of the chosen reference model.

It is clear from the obtained responses, that the plant output matches the reference model output for different plant dynamics, nearly after 1-2 cycles. Note that model following ability is better for 2 second time-constant model than the other model.

6. CONCLUSIONS

The previous near optimal response of the previous controller design with constant state feedback gains [7] , is maintained only within the prespecified normal operating conditions. But beyond the operating region of the design parameters, the adaptive controller is relied upon to produce a response which is superior to any other response would be possible with constant state feedback gains. This paper presents a possible application of reduced order model reference adaptive control for an aircraft system. The proposed adaptive controller can be implemented using digital computer, to replace many conventional components of an existing classical auto-pilot system.

ACKNOWLEDGMENTS

The outstanding assistance of Maj. Eng. M. Hashaad & Cap. Eng. A.A. Baky , for runing the obtained computer simulations, at Helwan Main A/C Workshops, is greatefully acknowledged.

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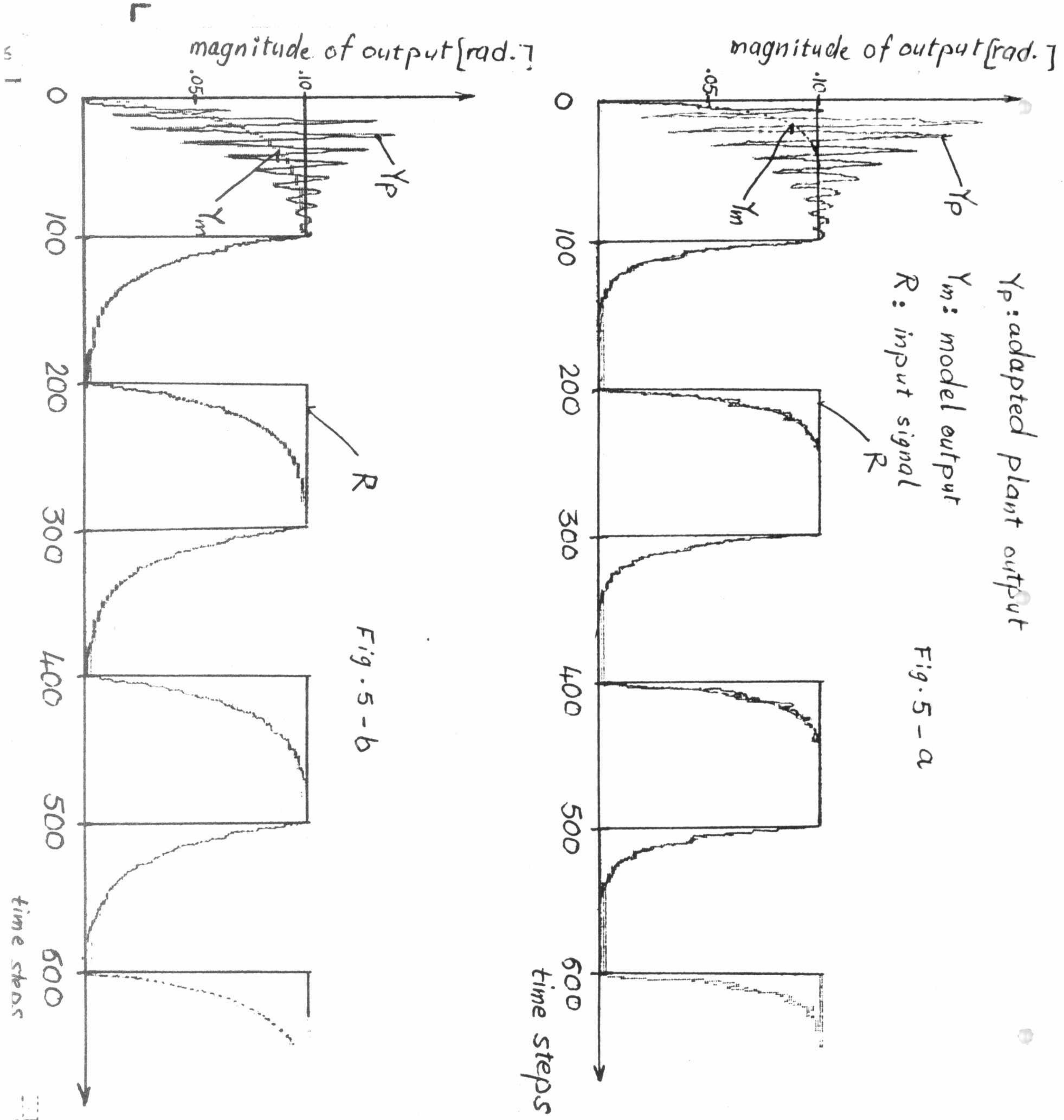

Fig.5 A daptive response for ROMRAC of AIRCRAFT plant
plant : = 5 r/s , d = 0.3 model : first order lag with
a) T= 1 sec , b) T = 2 sec .

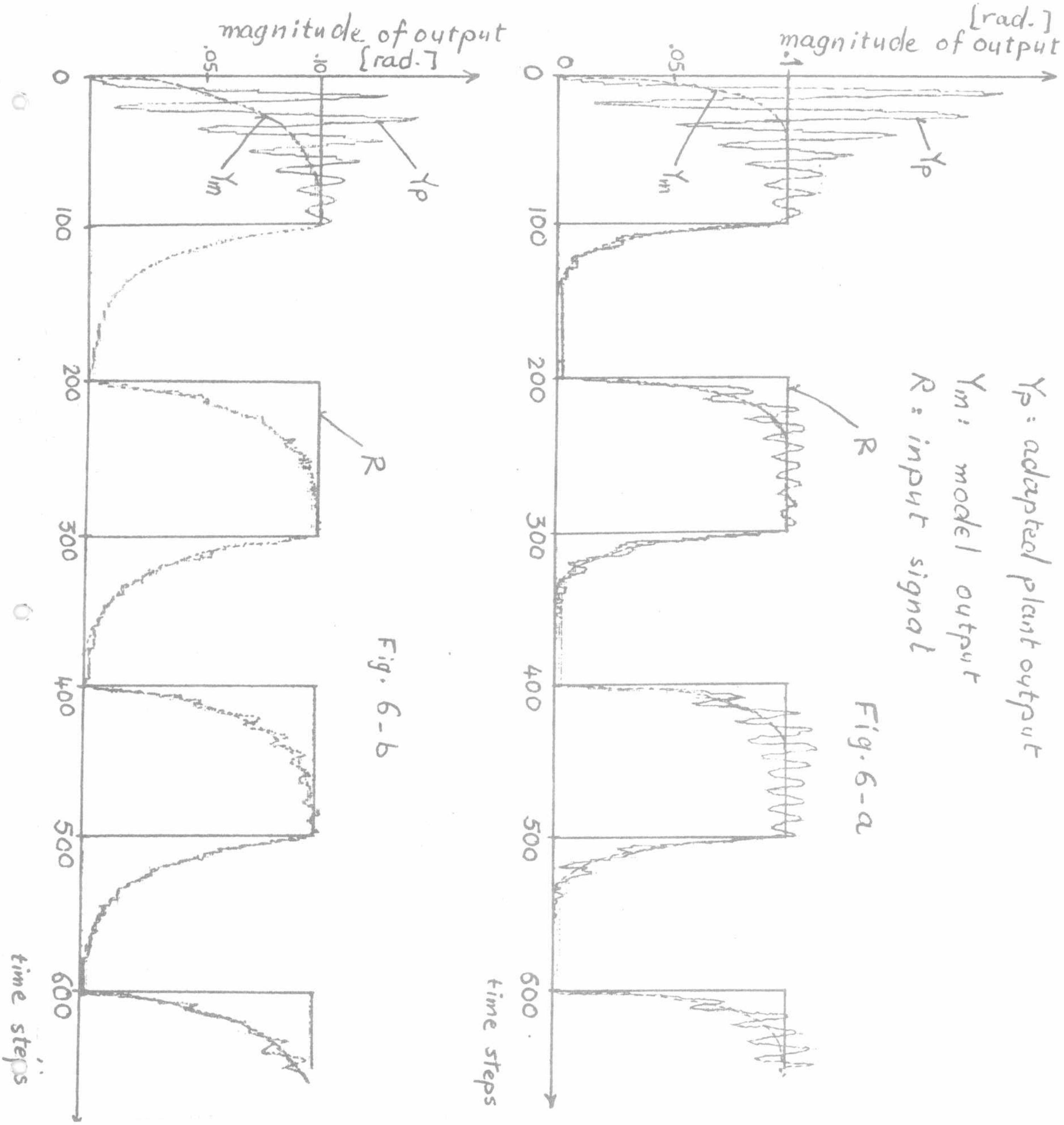

Fig.6 A daptive response for ROMRAG of aircraft plant

plant : = 3 r/s ; ζ = 0.6 model : first order lag with

a) T = 1 sec . b) T = 2 sec .