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ROBUST GENERALIZED CROSS CORRELATOR FILTERS FOR THE ε- CONTAMINATED SPECTRAL CLASSES

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ABSTRACT

Basically, the solution of estimating the time delay between two signals received at two spatially separated sensors consists of cross correlating the sensor's output and using the time argument that corresponds to the maximum peak as the time delay estimate.

To improve the estimation process, various optimum or heurestic filters have been suggested. Design of these filters requires the exact knowledge of the input spectra which is practically difficult. It would therefore be useful if fixed filters were specifically designed to have good performance over entire classes of input spectral densities. In this paper , robust solutions to the two optimum filters derived by Hassab and Boucher to estimate the time delay are derived. Explicit solutions for the ε-contaminated power spectral densities (PSD's) will be given. Numerical examples are given to illustrate the benefits of using robust filters in time delay estimation (TDE). Simulation results are also given. The results obtained show that robust filters are the saddle point solutions for the ε-contaminated spectral classes and ensure the advantages of robustness.

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I. INTRODUCTION

The problem of estimating time delay is of great importance im many applications such as radar, sonar, communications, acoustics and siesmology [1] Usually localization and tracking of a passive signal source are available by measuring the time delays between the received signals at 3 sensor's with a known distance a part. [2].

Optimum or suboptimum filters are proposed to enhance the performance of the generalized cross carrelator (GCC) used in TD E[3]. The design of these filters requires the exact knowledge of input signal and noises spectra , which is practically difficult. When the filter transfer function do not match the true input spectra, there is a consequent deterioration in the GCC performance. Two approaches to the problem of unknown spectra are of interest. We either estimate the spectra and substitute the estimates in the afore mentioned filters or we search for a robust solution over a range of spectra perturbed from some nominal point [4,5]. In this paper we shall present robust solutions to the two optimum filters proposed by Hassab and Boucher to enhance the GCC performance. For these solutions, t the ε- contaminated spectral classes will be considered. Numerical examples and simulation results will be given to illustrate the theoretical results .

II. HASSAB AND BOUCHER OPTIMUM FILTERS

Hassab and Boucher designed two optimum filters to enhance the estimation of time delay between signals received at two spatially separated sensors Fig. (1) [6]. The mathematical model of the sensor's output is given by:

$$y_1(t) = s(t) + n_1(t) \tag{1}$$

$$y_2(t) = s(t+D) + n_2(t) \tag{2}$$

where $y_1(t)$ and $y_2(t)$ are the sensor's output with power spectral densities (PSD"s) $G_{11}(\omega)$ and $G_{22}(\omega)$, $n_1(t)$ and $n_2(t)$ are real jointly stationary random processes with PSD's $G_{n_1 n_1}(\omega)$ and $G_{n_2 n_2}(\omega)$, $s(t)$ is the signal with PSD $G_{ss}(\omega)$ and D is the time delay . For Gaussian, uncorrelated signal and noises Hassab and Boucher (HB) considered two optimum criterions. HB considered the criterion of maximizing the filter output signal to noise ratio for which the filter transfer function $W_I(\omega)$ was found to be

$$W_I(\omega) = \frac{G_{ss}(\omega)}{G_{11}(\omega)\, G_{22}(\omega)} \tag{3a}$$

$$= \frac{G_{ss}(\omega)}{G_{n_1n_1}(\omega)\, G_{n_2n_2}(\omega)+G_{ss}(\omega)\,(G_{n_1n_1}(\omega)+G_{n_2n_2}(\omega))+G^2_{ss}(\omega)} \tag{3b}$$

$$= \frac{G_{ss}(\omega)}{Q_1(\omega)} \tag{3c}$$

where, $Q_1(\omega) = G_{11}(\omega)\, G_{22}(\omega)$ (3d)

In another approach HB considered to criterion of minimizing the mean

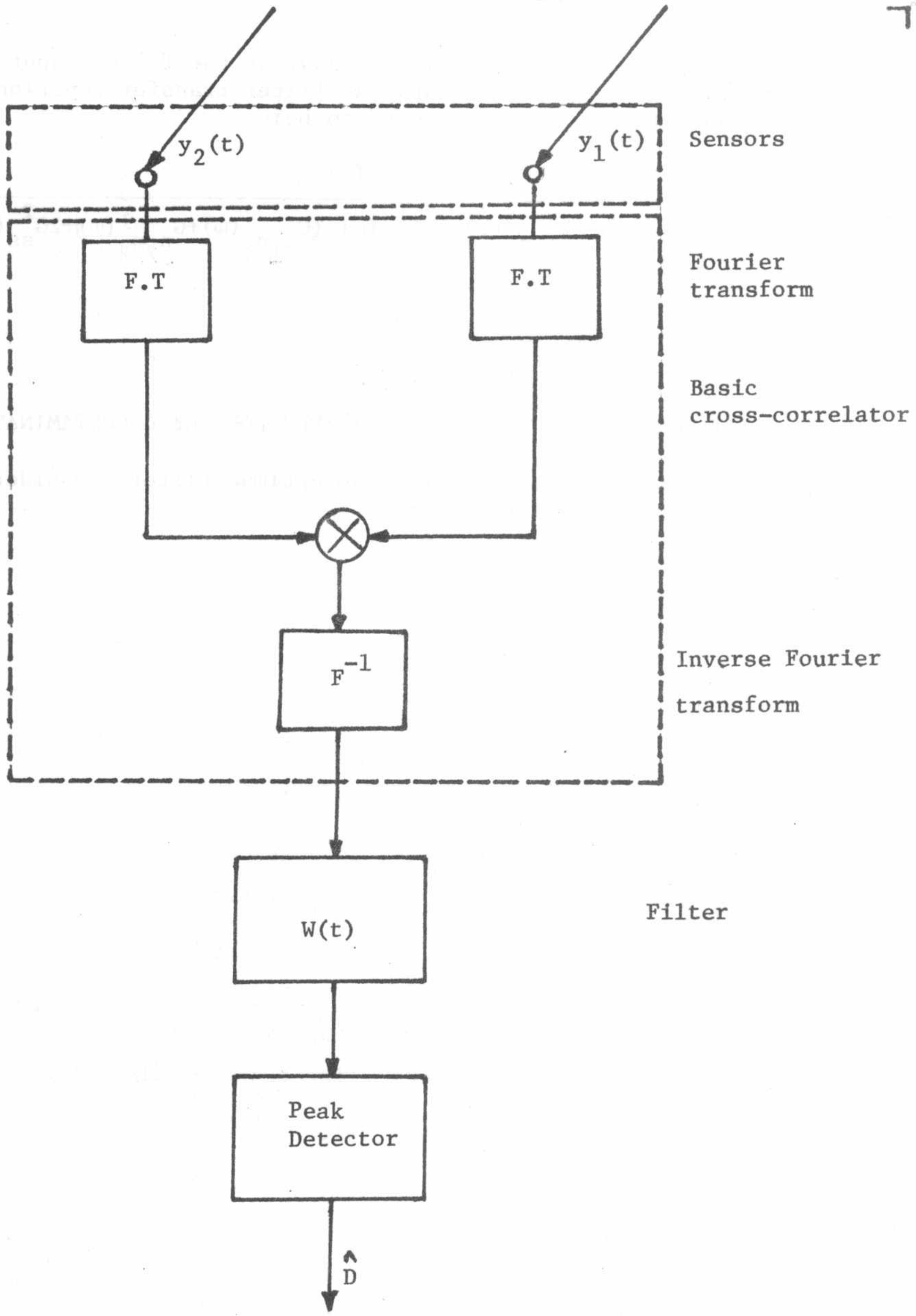
Sensors
$y_2(t)$
$y_1(t)$
F.T
F.T
Fourier transform
Basic cross-correlator
F^{-1}
Inverse Fourier transform
W(t)
Filter
Peak Detector
$\hat{D}$

Fig.(1) HB-GCC

square error (MSE) between the desired signal at the filter input and its estimated value at the filter output. The filter transfer function which gives the minimum MSE $W_{II}(\omega)$, was found to be:

$$W_{II}(\omega) = \frac{2\,G^2_{ss}(\omega)}{G_{n_1n_1}(\omega)\,G_{n_2n_2}(\omega) + G_{ss}(\omega)\,(G_{n_1n_1}(\omega)+G_{n_2n_2}(\omega))+2G^2_{ss}(\omega)} \tag{4a}$$

$$= \frac{2\,G^2_{ss}(\omega)}{Q_2(\omega) + 2\,G^2_{ss}(\omega)} \tag{4b}$$

III.ROBUST SOLUTIONS TO HB FILTERS FOR THE ε-CONTAMINATED MODEL

To unify the robust solutions for the two optimum filters considered, $W_I(\omega)$ and $W_{II}(\omega)$ can be written as :

$$W_I(\omega) = \frac{G_{ss}(\omega)}{Q_1(\omega)} = \frac{G_{st}(\omega)}{Q(\omega)} \tag{5a}$$

$$W_{II}(\omega) = \frac{2\,G^2_{ss}(\omega)}{Q_2(\omega)+2G^2_{ss}(\omega)} = \frac{G_{st}(\omega)}{Q(\omega)+G_{st}(\omega)} \tag{5b}$$

where,

$$G_{st}(\omega) = \begin{cases} G_{ss}(\omega) & \text{if } W_I(\omega) \text{ is considered} \\ 2\,G^2_{ss}(\omega) & \text{if } W_{II}(\omega) \text{ is considered} \end{cases} \tag{6}$$

and

$$Q(\omega) = \begin{cases} Q_1(\omega) & \text{if } W_I(\omega) \text{ is considered} \\ Q_2(\omega) & \text{if } W_{II}(\omega) \text{ is considered} \end{cases} \tag{7}$$

Note that $Q_1(\omega)$ is the total statistical variations at the correlator output and $Q_2(\omega)$ is the total noise PSD at the filter input.

The ε- contaminated PSD"s at the filter input are modeled as follows:

$$G_{st}(\omega) = (1-\varepsilon_s)\,G_{sto}(\omega) + \varepsilon_s\,G_{stc}(\omega),$$
$$\int G_{st}(\omega)\,d\omega = 2\,\pi\sigma_s^2 \tag{8}$$

$$Q(\omega) = (1-\varepsilon_q)\,Q_o(\omega) + \varepsilon_q\,Q_c(\omega),$$
$$\int Q(\omega)\,d\omega = 2\pi\,\sigma_q^2 \tag{9}$$

where ε_s and ε_q are degrees of contamination of the nominal PSD's $G_{sto}(\omega)$, $Q_o(\omega)$ and $G_{stc}(\omega)$, $Q_c(\omega)$ are any contaminating PSD's with total powers $2\pi\,\sigma_s^2$ and $2\pi\sigma_q^2$.

The ε- contamination model can be viewed as a special case of the bounded p- point spectral classes [7] with

$$G_{stL}(\omega) = (1-\varepsilon_s)\,G_{sto}(\omega) \tag{10a}$$

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$$Q_L(\omega) = (1-\varepsilon_q)\, Q_o(\omega) \tag{10b}$$

and $G_{stU}(\omega)$, $Q_U(\omega)$ are unbounded every where.
The following two subsets are defined as functions of two finite positive parameters K_s and K_q to obtain the robust solutions,

$$a(K_s) \triangleq \{\ \omega \,|\, G_{stL}(\omega) < K_s\, Q_L(\omega)\ \} \tag{11a}$$

$$b(K_q) \triangleq \{\ \omega \,|\, K_q\, Q_L(\omega) \leq G_{stL}(\omega)\ \} \tag{11b}$$

The robust filters can now be obtained through the following theorem. The proof is similar to that given in [8] for a Wiener filtering problem.

<u>Theorem :</u> For any total signal PSD' $G_{st}(\omega)$, and total noise PSD, $Q(\omega)$, which are members of the ε- contamination classes defined by equations (8) and (9) respectively, the robust filter $W_{IR}(\omega)$ or $W_{IIR}(\omega)$ satisfying the saddle point condition is given by $W_{IR}(\omega)= G_{stR}(\omega)/Q_R(\omega)$ or $W_{IIR}(\omega)= G_{stR}(\omega)/(G_{stR}(\omega) + Q_R(\omega))$ where the least favourable PSD's $G_{stR}(\omega)$ and $Q_R(\omega)$ are obtained according to the following:

1) If $K_s \leq K$ exist satisfying

$$K_s \int_{a(K_s)} (1-\varepsilon_q)\, Q_o(\omega)\, d\omega + \int_{\bar{a}(K_s)} (1-\varepsilon_s)\, G_{sto}(\omega)\, d\omega = 2\pi\, \sigma_s^2 \tag{12}$$

$$\frac{1}{K_q} \int_{b(K_q)} (1-\varepsilon_s)\, G_{sto}(\omega)\, d + \int_{\bar{b}(K_q)} (1-\varepsilon_q)\, Q_o(\omega)\, d\omega = 2\pi\sigma^2 \tag{13}$$

then

$$G_{stR}(\omega) = \begin{cases} K_s(1-\varepsilon_q)\, Q_o(\omega) & ,\omega\varepsilon\ a(K_s) \\ (1-\varepsilon_s)\, G_{sto}(\omega) & ,\ \omega\varepsilon\ \bar{a}(K_s) \end{cases} \tag{15}$$

$$Q_R(\omega) = \begin{cases} \frac{1}{K_q}\, (1-\varepsilon_s)\, G_{sto}(\omega) & ,\ \omega\varepsilon\ b(K_q) \\ (1-\varepsilon_q)\, Q_o(\omega) & ,\ \omega\varepsilon\ \bar{b}(K_q) \end{cases} \tag{16}$$

2) Otherwise, with $K = \dfrac{\sigma_s^2}{\sigma_q^2}$ (17)

$$G_{stR}(\omega) = \begin{cases} K(1-\varepsilon_q)\, Q_o(\omega) + S_e(\omega) & ,\ \omega\varepsilon\ a(K) \\ (1-\varepsilon_s)\, G_{sto}(\omega) + S_e(\omega) & ,\ \omega\varepsilon\ \bar{a}(K) \end{cases} \tag{18}$$

$$Q_R(\omega) = \begin{cases} \frac{1}{K}\, (1-\varepsilon_s)\, G_{sto}(\omega) + Q_e(\omega) & ,\ \omega\varepsilon\ b(K) \\ (1-\varepsilon_q)\, Q_o(\omega) + Q_e(\omega) & ,\omega\ \varepsilon\ \bar{b}(K) \end{cases} \tag{19}$$

where $S_e(\omega)$ and $Q_e(\omega)$ are arbitrary spectral density functions with

$$S_e(\omega) = KQ_e(\omega) \tag{20}$$

Such that $G_{stR}(\omega)$ and $Q_R(\omega)$ satisfy the power constraints. The saddle point condition for $W_{IR}(\omega)$ is given by :

$$d(W_I, G_{sto}, Q_o) \geq d(W_{IR}; G_{st0}, Q_0) \geq d(W_I; G_{stR}, Q_R) \geq d(W_I; G_{stR}, Q_R) \tag{21}$$

where d(.;.,.) is the SNR at the output of HB-SNR filter. The saddle point condition for $W_{II}(\omega)$ is given by:

$$\begin{aligned} &e(W_{II}; G_{sto}, Q_o) \leq e(W_{IIR}; G_{sto}, Q_o) \leq \\ &e(W_{IIR}; G_{stR}, Q_R) \leq e(W_{II}; G_{stR}, Q_R) \end{aligned} \tag{22}$$

where e(.;.,;) is the mean square error due to the filter for the bracketed values.

IV. NUMERICAL EXAMPLES AND SIMULATION RESULTS

The signal and noise spectra at the sensors input are considered to be of the ε- contaminated spectral classes, that is .

$$G_{ss}(\omega) = (1-\varepsilon_s)\, G_{sso}(\omega) + \varepsilon_s\, G_{ssc}(\omega) \qquad \text{and}$$

$$\begin{aligned} G_{n_1n_1}(\omega) &= G_{n_2n_2}(\omega) \\ &= (1-\varepsilon_n)\, G_{nno}(\omega) + \varepsilon_n\, G_{nnc}(\omega) \end{aligned}$$

where,

$$G_{sso}(\omega) = \left\{ \frac{A}{(.111+\omega^2)^2} \right. \qquad , |\omega| \leq \pi$$

$$G_{nno}(\omega) = \begin{cases} 1 & , |\omega| \leq \Pi\pi \\ 0 & , |\omega| > \pi \end{cases}$$

Now if $\varepsilon_s = \varepsilon_n = 0.2$, then $\varepsilon_q = 0.36$. For input SNR= 0 dB, A = 0.1482 at the nominal input spectra. This example was solved numerically to compare the performances of the GCC when either $W_I(\omega)$ or $W_{IR}(\omega)$ is used. It was solved a second time to compare the performances of the GCC when either $W_{II}(\omega)$ or $W_{IIR}(\omega)$ is used. The results obtained are shown in tables (1) and (2) respectively

<u>Simulation:</u> To simulate the behaviour of the filters $W_I(\omega)$ and $W_{IR}(\omega)$, twenty five independent trials were run for each case considered. For each case, three independent sequences of uncorrelated Gaussian variates were generated. Different spectral characterestics were generated by linear filtering of these uncorrelated sequences. Two separate signal plus noise sequences were formed by adding relatively delayed versions of one sequence to each of the remaining two, using a delay of 20 units. The total length of each sequence was 512 units. The optimum and robust HB-SNR filters were implemented using a 512 point FFT, and the dominant peak was located at the processor output using parabolic fitting. For each different case the same set of 25 pairs of signal

plus noise sequences were used to get a fair comparison. The results of the simulation processes are tabulated in table (1).

Similar simulation procedures were made to simulate the behaviour of the GCC when either $W_{II}(\omega)$ or $W_{IIR}(\omega)$ is used. The results of simulation for these filters are tabulated in table (2).

Table (1)
Results of simulation and numerical solutions to the ε- contaminated spectra. Input SNR = 0 dB. $W_I(\omega)$ or $W_{IR}(\omega)$ is used .

Case	Output SNR	$\hat{D}$	$\hat{\sigma}^2$	$\sigma^2_{th.}$
$(W_I;\ G_{sto}, Q_o)$	0.738	20.227	0.025	0.102
$(W_{IR};\ G_{sto},\ Q_o)$	0.6008	20.656	0.006	0.735
$(W_{IR};\ G_{stR},\ Q_R)$	0.596	20.005	0.0006	0.042
$(W_I;\ G_{stR},\ Q_R\ )$	0.437	20.087	0.045	0.137

Table (2)
Results of simulation and numerical solutions to the ε-contaminated spectra. Input SNR= 0 dB. $W_{II}(\omega)$ or $W_{IIR}(\omega)$ is used.

Case	Theoretical MSE	$\hat{D}$	$\hat{\sigma}^2$	$\sigma^2_{th.}$
$W_{II};\ G_{sto},\ Q_o$	1.764	20.13	0.065	0.095
$W_{IIR};\ G_{sto},\ Q_o$	2.367	20.65	0.006	0.729
$W_{IIR};\ G_{stR}, Q_R$	2.989	20.0001	10^{-6}	0.0004
$W_{II}, G_{stR},\ Q_R$	3.525	19.944	0.072	0.036

V. CONCLUSIONS AND FUTURE EXTENSIONS

The theoretical , numerical and simulation results show that the robust filters $W_{IR}(\omega)$ and $W_{IIR}(\omega)$ are the saddle point solutions for the optimum HB filters $W_I(\omega)$ and $W_{II}(\omega)$ respectively when the ε- contaminated spectral classes at the sensor's input are considered. For future extensions M.ary sensors may be considered or correlation between noises or between signal and noises at the sensor's input may be taken into consideration.

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