

MILITARY TECHNICAL COLLEGE
CAIRO - EGYPT

MINIMUM PHASE SELECTIVE BANDPASS DIGITAL FILTERS

WITH CONTROLLED PASS-BAND LOSS AND DELAY

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ABSTRACT

A simple numerical procedure is described for the construction of minimum phase selective band-pass digital filters satisfying simultaneous pass-band amplitude and phase constraints for any specified stop-band loss. It is shown that, if the pass-band loss of the filter is expressed as a linear combination of cosine functions, then the filter's group delay can be linearly expressed in terms of these functions together with any stop-band loss constraints. The simplex method of linear programming can then be invoked to minimize the pass-band delay deviation from constancy for any specified peak pass-band loss. The resulting transfer functions are shown to be less sensitive with reference to coefficients truncations. Illustrative examples are also given.

INTRODUCTION

Minimum-phase networks are characterized by having less complex structures together with low sensitivity w.r.t. their elements variations. However, due to the interrelation that exist between their attenuation and phase functions, amplitude selectivity and constant delay are properties which resist attempts to incorporate them simultaneously. This difficulty has been solved in lumped networks by constructing a nonrational transfer function that satisfy both requirements. Then, a physically realizable rational transfer function is constructed that possess a good approximation to the nonrational prototype function,[1-2].

The situation is quite difficult in the digital case. There, we have to find out a multivalued function that changes its character over both of its pass and stop bands. Then, we have to evaluate the resulting, and normally very complex, singular integrals.

In this paper, we propose an alternative simple approach for the construction of selective minimum-phase digital bandpass filters having simultaneous constant passband group delay together with controlable

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passband amplitude response. Initially, the relation between the phase and loss of a digital filter is introduced. Then, it is shown that by expressing the pass-band loss as the sum of a linear combination of cosine functions, the problem of filter design is reduced to a simple linear programming problem. As results demonstrate, this approach is characterized by its flexibility to control both of the bandwidth over which constant delay is achieved, maximum allowable pass-band loss, and the filter stop-band,

DERIVATIONS OF BASIC RELATIONS

Let the transfer function of the minimum-phase digital filter be

$$H(z) = e^{F(z)} = e^{-\alpha(\phi)} \angle -\beta(\phi), \quad z = e^{j\phi}$$

where $\alpha(\phi)$ & $\beta(\phi)$ are Hilbert pairs [3], related by

$$\alpha(\phi) = \alpha_0 - \frac{1}{2\pi} \int_{-\pi}^{\pi} \beta(\theta) \cot\left(\frac{\theta-\phi}{2}\right) d\theta$$

$$\beta(\phi) = \frac{1}{2\pi} \int_{-\pi}^{\pi} \alpha(\theta) \cot\left(\frac{\theta-\phi}{2}\right) d\theta \quad (1)$$

where α_0 is a constant.

Since $\alpha(\phi)$ & $\beta(\phi)$ are even and odd functions in ϕ , respectively, then eqn.(1) reduces to

$$\alpha(\phi) = \alpha_0 - \frac{1}{\pi} \int_0^{\pi} \beta(\theta) \frac{\sin\theta}{\cos\phi - \cos\theta} d\theta$$

$$\beta(\phi) = \frac{\sin\phi}{\pi} \int_0^{\pi} \alpha(\theta) \frac{1}{\cos\phi - \cos\theta} d\theta \quad (2)$$

Now, to have a minimum-phase digital filter with a prescribed minimum stop-band loss together with a constant pass band delay, one have to follow one of two approaches. The first approach is to express $F(z)$ as a multivalued function that changes its character over both of its pass and stop bands. This enables us to express the pass-band loss in terms of the pass-band phase and the stop-band loss. However, this approach normally results in singular integrals that are not easy to evaluate numerically. The second approach is much simpler, and enables us to control both of the filter bandwidth together with its pass-band loss. It relies upon expressing the pass-band loss in the form

$$\alpha(\theta) = \sum_{j=0}^n a_j \cos j\theta \quad (3-a)$$

So, with α_{s1} & α_{s2} to denote the specified minimum stop-band losses, then equation(2) reduces to

$$\beta(\phi) = \frac{\sin\phi}{\pi} \left[\int_0^{\theta_{c1}} \frac{1}{\cos\phi - \cos\theta} \alpha_{s1} d\theta + \int_{\theta_{c2}}^{\pi} \frac{\alpha_{s2}}{\cos\phi - \cos\theta} d\theta + \sum_{j=0}^n \int_{\theta_{c2}}^{\theta_{c1}} \frac{\cos j\theta}{\cos\phi - \cos\theta} a_j d\theta \right] \quad (3-b)$$

Clearly, for $\theta_{c1} \leq \phi \leq \theta_{c2}$, both of the 1st. and 2nd. integrals are regular

and can be evaluated using standard numerical integration formulas. The 3rd. integral is integrable in spite of its singularity at $\theta = \phi$. Its value can be obtained through the recurrence relation

$$I_{j+1}(\phi) + I_{j-1}(\phi) = 2 \cos \phi \cdot I_j(\phi) - \frac{2}{j}(\sin j\theta_{c2} - \sin j\theta_{c1}), (j = 1, 2, \dots)$$

where

$$I_j(\phi) = \int_{\theta_{c1}}^{\theta_{c2}} \frac{\cos j\theta}{\cos \phi - \cos \theta} d\theta \quad \theta_{c1} \leq \phi \leq \theta_{c2}$$

$$I_0(\phi) = \frac{1}{\sin \phi} \ln \left| \frac{\sin \left(\frac{\theta - \phi}{2} \right)}{\sin \left(\frac{\theta + \phi}{2} \right)} \right| \Bigg|_{\theta_{c1}}^{\theta_{c2}}$$

$$I_1(\phi) = \cos \phi \cdot I_0(\phi) - (\theta_{c2} - \theta_{c1}) \tag{4}$$

Consequently, the filter group delay $T(\phi)$ is given by

$$T(\phi) = \frac{1}{\pi} \left[\int_0^{\theta_{c1}} \frac{1 - \cos \phi \cos \theta}{(\cos \phi - \cos \theta)^2} \alpha_{s1} d\theta + \int_{\theta_{c2}}^{\pi} \frac{1 - \cos \phi \cos \theta}{(\cos \phi - \cos \theta)^2} \alpha_{s2} d\theta + \sum_{j=0}^n (\sin \phi \cdot I'_j(\phi) + \cos \phi \cdot I_j(\phi)) a_j \right] \tag{5}$$

where $I'_j(\phi)$ represents the derivative of $I_j(\phi)$ w.r.t. ϕ . Thus, for specified α_{s1} & α_{s2} we have a total of $(n+2)$ unknowns. These are the nominal group delay T_0 and the $(n+1)$ a_j 's of eqn.(3-a). These unknowns can be determined as the least squares solutions of a set of N linear equations describing the delay behaviour over N discrete frequencies distributed over the pass-band. However, we have no control over the pass-band loss behaviour. To have such control, minimization of the pass-band delay deviation should be carried out subject to some constraints over the pass-band loss behaviour. Mathematically, this amounts to

Minimize δ
 subject to $|T(\phi_r) - T_0| \leq \delta$
 $\alpha(\phi_r) \leq \alpha_{\min}$
 $\alpha(\phi_r) \geq 0$
 $r = 1, 2, \dots, N$ (6)

So the form of eqn.(3-a) yields linear constraints and consequently the standard simplex method of linear programming can be used [4], where every a_j of eqn.(3-a) is replaced by $a_j = x_{2j+1} - x_{2j}$, $x_{2j} \geq 0$ & $x_{2j+1} \geq 0$.

Thus, the solution of eqn.(6) gives the relation between α_{\min} & maximum pass-band delay deviation. Fig.(1) shows the relation between α_{\min} & the % pass-band delay deviation, when eqn.(6) is evaluated for different α_s .

The digital filter under consideration has the specifications: B.P.F., with cutoff frequencies at 30° & 60° ; the stop-bands extend from $0-15^\circ$ & $75^\circ - \pi^\circ$; the band of approximation extends from $34^\circ - 56^\circ$ and α_{s1} is taken to be equal to α_{s2} , i.e $\alpha_{s1} = \alpha_{s2} = \alpha_s$. These curves are in agreement with similar results obtained for the low-pass lumped case[2].

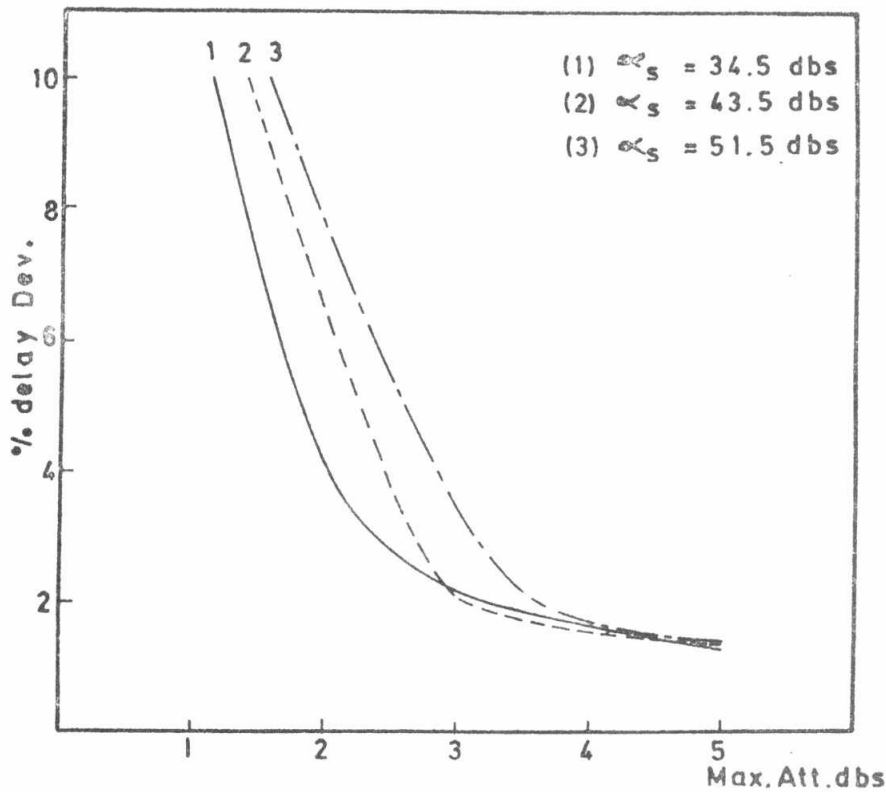


Fig.1 The relation between maximum pass-band loss α_{min} and the % delay deviation from its normalized value.

Having evaluated the filter loss over the whole frequency band, then a least squares rational approximation can be easily obtained. That is, one can determine $H(z)$ such that

$$H(z) = K \frac{N(z)}{D(z)}, \quad K \text{ normalizing factor} \quad (7)$$

approximates - in a least squares sense - the calculated loss response and at the same time satisfy the constant delay property. Note that due to the minimum-phase property, all zeros of $N(z)$ & $D(z)$ lie inside the unit circle. However, to have a numerically stable process, a slight modification to the computation scheme should be adopted, due to the level of $\alpha(\theta)$. Thus, instead we minimize the squared error E given by

$$E = \sum_{j=0}^M [\alpha(\theta_j) - \frac{1}{2} \sum_{i=0}^m \ln(1 - 2r_i \cos(\phi_i - \theta_j) + r_i^2) + \frac{1}{2} \sum_{i=0}^m \ln(1 - 2s_i \cos(\psi_i - \theta_j) + s_i^2)]^2$$

where

$$N(z) = \prod_{i=1}^m (z - r_i e^{j\phi_i}) \quad \& \quad D(z) = \prod_{i=1}^m (z - s_i e^{j\psi_i}) \quad (8)$$

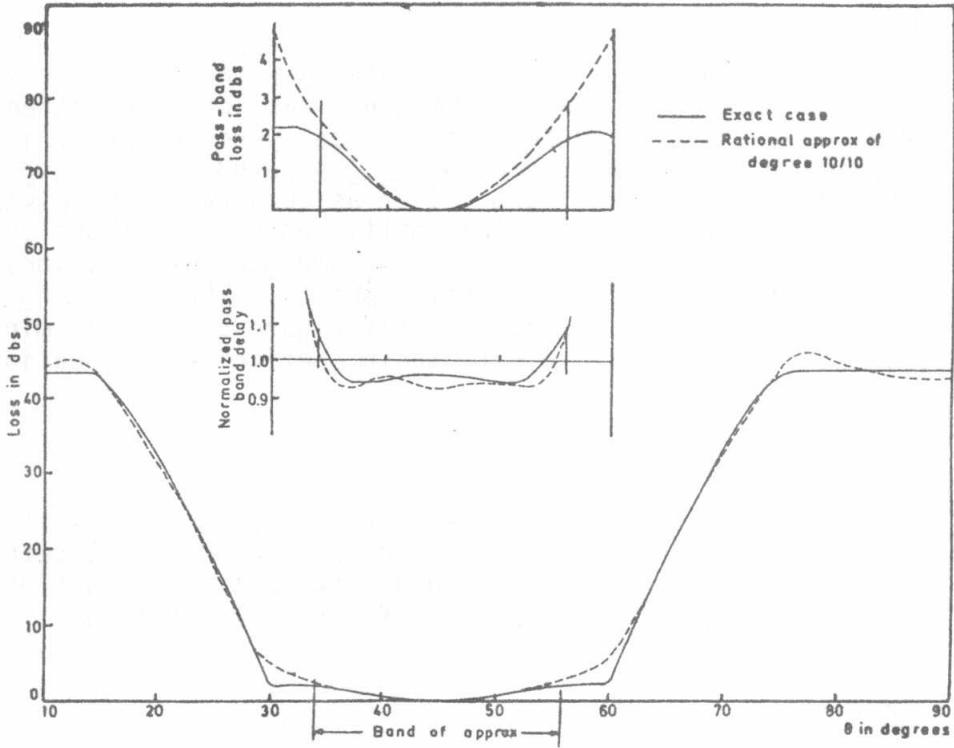


Fig.2 Loss and delay responses of the exact Hilbert B.P.F. and its 10/10 equivalent rational least squares approximation.

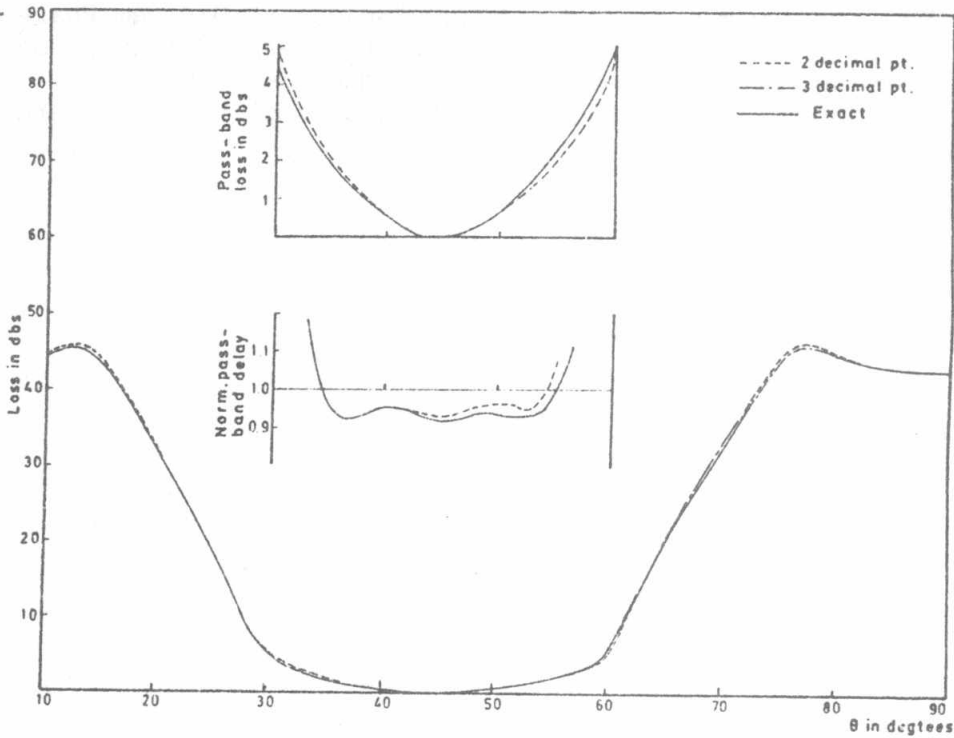


Fig.3 The effect of truncations on the frequency responses of the 10/10 rational approximation of Fig.2.

and M is the total no. of approximating points over the whole frequency band. In all of the cases studied, convergence was achieved within less than 10 iterations. Fig.2 shows the loss and delay responses of a 10/10 rational approximation when evaluated for $\alpha_s \geq 43.5$ dBs and $\alpha_{min} \leq 2$ dBs, together with their prototype Hilbert filter as obtained from solving of eqn.(6). Finally, to see the effect of coefficients truncations on the filter responses, the poles and zeros of the computed 10/10 rational approximation were truncated to 3&2 decimal points. The resulting frequency responses are shown in Fig.3. This Figure shows that minimum-phase filters are less sensitive w.r.t. their coefficient truncations.

CONCLUSIONS

A simple numerical procedure is described for the design of selective minimum-phase digital filters with controlled pass-band loss together with constant group delay. The results obtained show that the resulting filters are less sensitive w.r.t. the filter's coefficients truncations.

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