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MILITARY TECHNICAL COLLEGE
CAIRO - EGYPT

1 # CONVOLUTIONAL CODING VERSUS OPTIMIZED SPACE DIVERSITY
2 # FOR RAYLEIGH FADING CHANNEL

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4 ## ABSTRACT

The conventional remedy for communication over radio frequency channels has always been diversity (time, frequency, space,...etc), and the usefulness of coding in such a context is less widely recognized. In this paper we propose the use of Viterbi decoding technique with non-systematic binary convolutional codes class as an alternative to conventional diversity approaches. This technique falls in the category of computational diversity.

The performance of the proposed Viterbi decoding with the (8,4) convolutional, as an example, is evaluated over Rayleigh fading channel. The results are compared to those obtained from using scanning space diversity optimized according to the switching threshold as well as the results from using both selection and optimum space diversity techniques each optimized according to the optimum usable diversity branches. Also, the performance of a model that combines optimum space diversity and Viterbi decoding algorithm with (8,4) convolutional code is examined.

The comparison show that the proposed scheme is more powerful and less complex than that of the other considered approaches.

I. VITERBI DECODING/THEORY AND RESULTS :

Due to the dependency between the incoded symbol and the preceding coded symbols in convolutional coding, the coded information symbols can be modeled by the first order Markov process. The Viterbi algorithm is a sequential optimization method using these previous modeled information symbols to achieve certain bit error probability. Let:

$Z = (Z_0, Z_1, \ldots, Z_k, Z_{k+1})$, the coded symbols sequence, then the first-order markov process is defined by the transition probability:

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$$P\ (Z/S = X) \qquad (1)$$

where, $X = (X_0, X_1, \ldots, X_k, X_{k+1})$ denotes the sequence of character classes that the state vector:

$S = (S_0, S_1, \ldots, S_k, S_{k+1})$ takes on.

The chance of making a correct decision is increased by selecting the sequence of characters X that maximizes the aposteriori probability $P(S = X/Z)$ for a given sequence of observations Z. According to the nature of decision problem, however, only probabilities $P(Z/X)$ and the apriori probability of the state vector taking the value X, $P(X)$ are available. Therefore, the problem is reduced to a maximization of the following monotonic function over X:

$$W(X, Z) = \log P(Z/X) + \log P(X) \qquad (2)$$

where, we have omitted the term $\log P(Z)$ because it is not a function of X. Assuming that the conditional probability, $P(Z/X)$ for each character is independent, the equation(2) take the form [1]:

$$- W(X, Z) = L\ (X, Z) = \sum_{k=1}^{k} \ell\ (k-1, k) \qquad (3)$$

where:

$$\ell\ (k-1, k) = - \log P(Z_k/X_k) - \log P(X_k/X_{k-1})$$

Moreover; $L\ (X, Z)$ is called the length of the state sequence X and $\ell\ (k-1,k)$ the branch length between states X_{k-1} and X_k.

Now, it becomes obvious that the Viterbi algorithm finds the shortest path over a sequence of states, Thus it is the optimal decoding technique for convolutional coding.

In [2,3] Viterbi describes two upper bounds on the performance of binary convolutional codes:

1. The first event error probability P_E:

$$P_E < \sum_{k=d}^{\infty} a_k\ P_k \qquad (4)$$

2. The bit error probability P_B :

$$P_B < \sum_{k=d}^{\infty} c_k\ P_k \qquad (5)$$

where: P_k is the probability of error of a given path of distance k from the correct path, a_k and c_k are weighting coefficients obtained from the generating function of the code as described in [2].

for Rayleigh fading channel [4] :

$$P_k \leq [(1/2)\ [4P\ (1-P)\ ]\ ]^k$$

$$P = 1/\ (2 + E/N_0)$$

Applying this upper bound to (4) and (5), we can evaluate the performance of the considered (8,4) convolutional code. The results are shown in table

Table 1. The performance results

E/N_o	2	2.5	3	4	5
E_b/N_o	4	5	6	8	10
$10 \log E_b/N_o$	6.02	6.99	7.78	9.03	10
Channel error probability $p=1/(2+E_b/N_o)$	$2.50 \; 10^{-1}$	$2.2 \; 10^{-1}$	$2.00 \; 10^{-1}$	$1.67 \; 10^{-1}$	$1.4 \; 10^{-1}$
Uncoded Channel error probability $(E_b = \frac{4}{8} E)$	$3.3 \; 11^{-1}$	$3.08 \; 10^{-1}$	$3.86 \; 10^{-1}$	$2.50 \; 10^{-1}$	$2.2 \; 10^{-1}$
P_E	0.03	0.016	$9.32 \; 10^{-3}$	$3.72 \; 10^{-3}$	$1.73 \; 10^{-3}$
P_B	0.119	0.052	0.026	$8.37 \; 10^{-3}$	$3.38 \; 10^{-3}$

The obtained results are plotted in terms of the ratio (E_b/N_0) where E_b is the available average power per information symbol and N_0 is the one-sided spectral power density of the noise[5]. In other words the energy E per coded symbol, was increased by 10 log (n/k) dB in order to take into account the available energy that shared among information and redundancy bits. A fair comparison can therefore be made of the coded and uncoded cases.
The curves, in fig. 1, show that the performance of the channel using Viterbi decoding is better than the uncoded case and as E_b/N_0 increases the performance becomes much better.

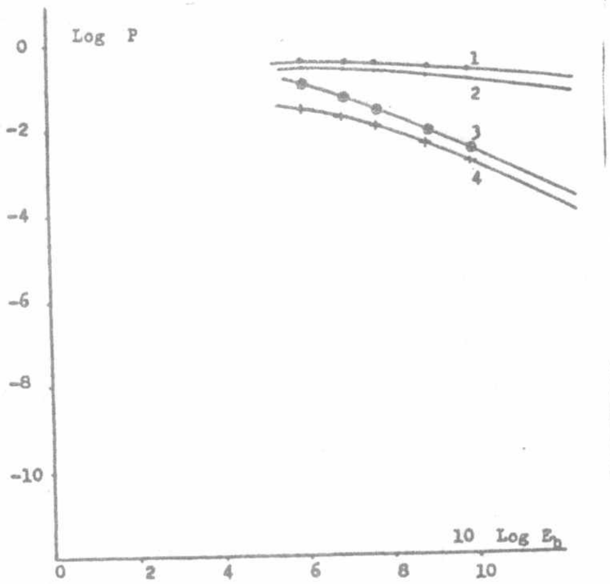

Fig.1 Performance results.
1-uncoded channel P.
2-coded channel P.
3-log P_E.
4-log P_B.

II. OPTIMIZED SCANNING SPACE DIVERSITY:

The scanning space diversity [6] is of the switched type, i.e., at any time a selector device uses only one signal of that available on the diversity branches.

The switching in this technique can be done in many ways [7] , mainly:

- Ideal switched strategy.
- Switch-and-stay strategy.
- Switch-and-examine strategy.

We shall consider the switch - and - stay strategy, as a model of the scanning diversity, and in this strategy the switching is done as follows:

$$Z_i = X_i \quad \text{iff} \quad \begin{cases} Z_{i-1} = X_{i-1} \text{ and } X_{i-1} \geq d \text{ and } X_i \geq d \\ \text{or } Z_{i-1} = X_{i-1} \text{ and } X_{i-1} < d \\ \text{or } Z_{i-1} = Y_{i-1} \text{ and } Y_{i-1} \geq d \text{ and } Y_i < d \end{cases} \tag{6}$$

where:

- Z_i ... the selected sample at instant i .
- X_i and Y_i ... are samples of envelopes of the signals coming from antennas 1 and 2 respectively at instant i.
- d ... is the switching threshold.

The average probability of error of this strategy [7] take the form:

$$P_{av} = \left[\frac{1}{2 + b} \right] \left[\frac{1}{1-q + q^2} \right] \left[q + (1-q)^2 \quad e^{-d/2} \right] \tag{7}$$

where:
b... is the energy per bit,
$q = 1 - e^{-d/b}$ for any i

In order to optimize the performance of this strategy, we first not that if we let d=0 in(7) then q=0, and: $P_{av}(0) = 1/(2 + b)$. Also, if we let $d = \infty$ then q=1, $P_{av}(\infty) = 1/(2 + b)$. Since P_{av} (d) 1/(2+b), for every d and b therefore, $P_{av}(d)$ has 1/(2+b) as a least upper bound. As $P_{av}(d)$ assumes this value at d = 0, and $d = \infty$, and it is a continuous function, there exists a minimum point for $0 < d < \infty$.

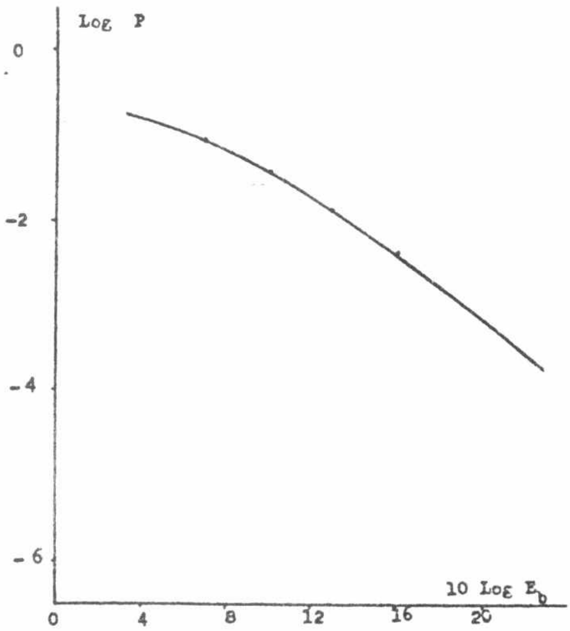

Fig.2. Optimized performance

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This minimum point represents the optimum threshold and it is found numerically following the same steps as[7]:

The optimized performance is shown in fig. 2 and it is clear that the performance becomes much better as the SNR increases.

III. PERFORMANCE OPTIMIZATION OF SELECTION AND OPTIMUM SPACE DIVERSITY:

In diversity systems, the increasing the number of diversity branches provides more diversity and reduces the chance of a bad fade. However, on the other hand, the average signal-to-noise ratio on each branch becomes smaller, and consequently the loss in communication efficiency in each branch becomes larger. Therefore an optimum number of diversity branches should exist.

First, we shall find this optimum number of branches for selection diversity [6]. The bit error probability of this approach is given by :

$$P_{b,sel}(L) = \frac{2^{L-1} \quad L!}{\prod_{k=1}^{L} \left\{ (\gamma_b/L) + 2k \right\}} \qquad (8)$$

where:

L ... number of diversity branches

$_b$... energy per bit

In order to optimize $P_{b,sel}(L)$, Let $B = \ln 1/P_{b,sel}(L)$,

it was numerically proved [8] that dB/dL is a function which (for fixed γ_b) first increases monotonically with L , after $L = \sqrt{0.5 \ \gamma_b}$ it decreases montonically with L. thus the optimum number of diversity branches:

$$L^*_{sel} = \sqrt{0.5 \ \gamma_b} \qquad (9)$$

Substituting this value in (8) we obtain the optimized performance.

Now, for optimum space diversity [7] the bit error probability is given by:

$$P_{b,opt}(L) = P^L \sum_{j=0}^{L-1} \binom{L+j-1}{j} (1-P)^j \qquad (10)$$

, $P = 1/(2 + E/N_0)$

It can be shown [8] that,

$$L^{*}_{opt} \approx \frac{\nu_b}{3} \quad (11)$$

Calculating the magnitudes of L^{*}_{opt} for various ν_b values and subtituting it in (10) we can evaluate the optimized performance of optimum space diversity

The optimized performances of both selection and optimum space diversity are plotted in fig.3. for both various number of diversity branches and different SNR values. It is clear that as either the number of diversity branches or energy per bit increases the performance becomes much better. Also, the performance of the optimized optimum diversity is better than the optimized selection diversity but it is more complex.

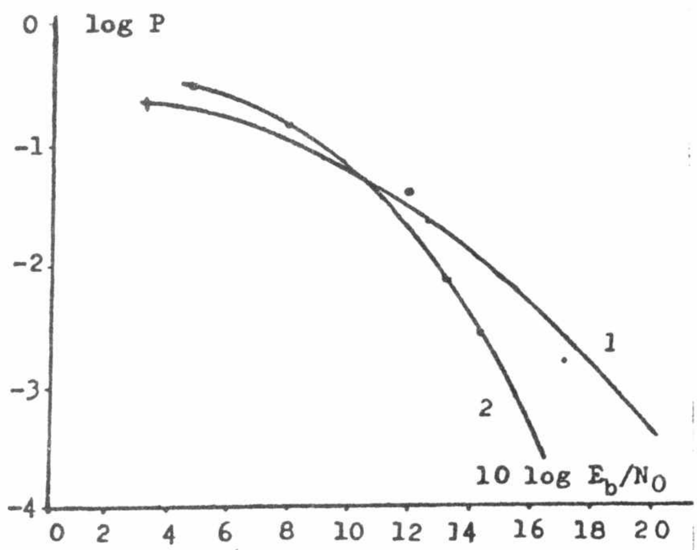

Fig.3. Optimized performances
1.Selection diversity.
2. Optimum diversity.

IV. COMBINING COMPUTATIONAL DIVERSITY AND CONVENTIONAL DIVERSITY:

In [9] Chernoff deduces a tight upper bound on the performance of Rayleigh fading channel with L - th optimum space diversity branches:

$$P_{b,opt}(L) \leqslant (½) \quad [4\,P\,(1-P)]^{L}$$

$$,\ P = 1/(2 + E/N_0)$$

Applying this upper bound to the equations (4) and (5) we can obtain the tight upper bounds for the performance of a model combines between Viterbi decoding and optimal space diversity. Thus for the considered (8,4) convolutional code we have:

$$P_E(L) < \frac{P^5_{b,opt}(L)}{1 - P^5_{b,opt}(L)} \quad (12)$$

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$$\bullet \quad P_B(L) < \frac{P^5_{b,opt}(L)}{\left[1 - P^5_{b,opt}(L)\right]^2} \qquad (13)$$

The performance is evaluated for both various numbers of diversity branches and different SNR values, the results are shown in table (2)

Table (2). Results

	L	2	3	4	5	6
P_E	E/N_o =2	4.02 10^{-3}	7.[illegible] 10^{-4}	1.45 10^{-4}	3.08 10^{-5}	6.79 10^{-6}
	=2.5	1.49 10^{-3}	1.84 10^{-4}	2.52 10^{-5}	3.65 10^{-6}	5.45 10^{-7}
	=3	6.10 10^{-4}	5.24 10^{-5}	4.99 10^{-6}	5.00 10^{-7}	5.14 10^{-8}
	=4	1.27 10^{-4}	5.59 10^{-6}	2.71 10^{-7}	1.37 10^{-8}	7.07 10^{-10}
	=5	3.27 10^{-5}	7.93 10^{-7}	2.09 10^{-8}	5.72 10^{-10}	1.59 10^{-11}
P_B	E/N_o =2	9.19 10^{-3}	1.25 10^{-3}	2.12 10^{-4}	4.04 10^{-5}	8.26 10^{-6}
	=2.5	2.86 10^{-3}	2.75 10^{-4}	3.27 10^{-5}	4.33 10^{-6}	6.12 10^{-7}
	=3	1.03 10^{-3}	7.11 10^{-5}	5.00 10^{-6}	6.00 10^{-7}	5.52 10^{-8}
	=4	1.83 10^{-4}	6.75 10^{-6}	3.00 10^{-7}	1.45 10^{-8}	7.29 10^{-10}
	=5	4.30 10^{-5}	8.99 10^{-7}	2.22 10^{-8}	5.89 10^{-10}	1.61 10^{-11}

The performance results are plotted for both various

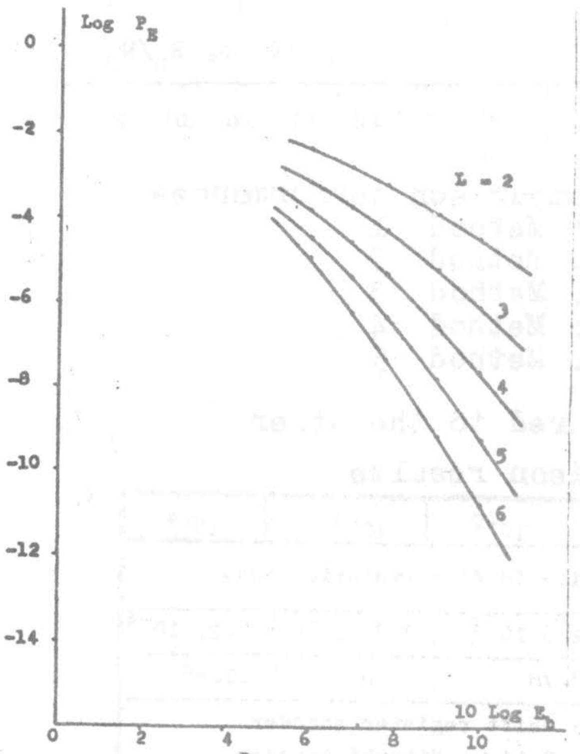

Fig.4. Log $P_E(L)$

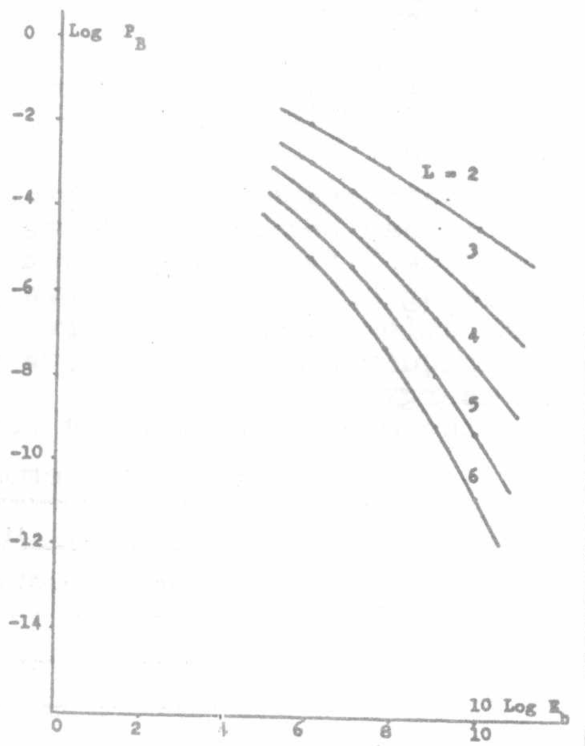

Fig.5. Log $P_B(L)$

number of L and E/N_0 values in fig.4, and in fig.5, the curves show that as E_b/N_0 increases the performance improved but as the number of diversity branches increases the performance becomes much better.

V. COMPARISON AND CONCLUSION:

Table (3) shows the results of a comparison between the previously studied approaches based upon requirements of both energy per bit (E_b/N_0) and system complexity to achieve a certain performance.

The performance results of various considered approaches are plotted in Fig.6.

From the previous comparison the examined approaches can be arranged, ascendingely, in two ways:
First, according to the required energy per bit:
Method 5, method 1, method 4, method 3, and method 2
Secondly, according to the system complexity:
Method 2, method 1, method 3, method 4, and method 5.

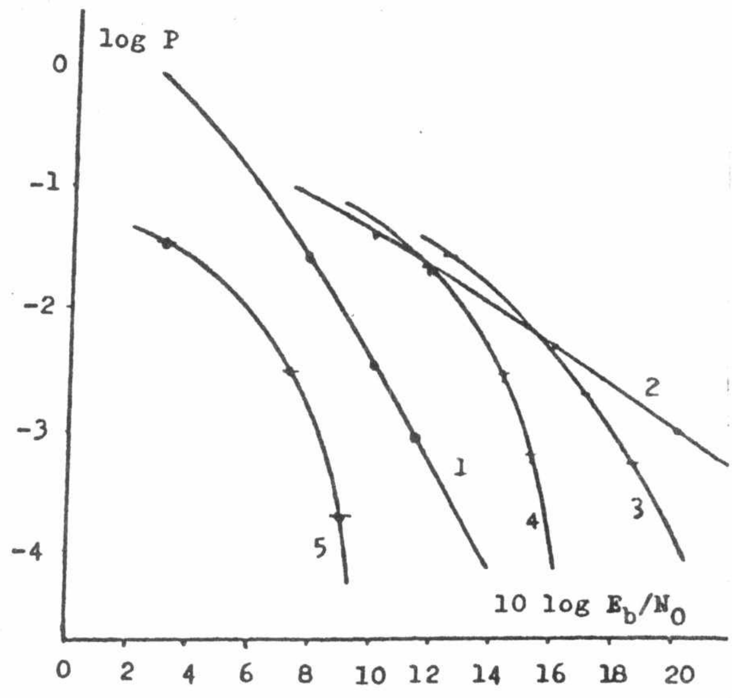

Fig.6. Comparison performances
Curve 1 : Method 1
Curve 2 : Method 2
Curve 3 : Method 3
Curve 4 : Method 4
Curve 5 : Method 5

From this comparison we can conclude that the Viterbi decoding algorithm compromises between the requirements of energy per bit and the system complexity compared to the other

Table 3. Comparison results

Order of error probability	10^{-2}	10^{-3}	10^{-4}
Method 1 : Viterbi decoding with (8,4) convolution code			
The achieved P	2.6 10^{-2}	3.39 10^{-3}	8.22 10^{-4}
10 Log E_b/N_0	7.78	10	11.46
The complexity requirements	- Shift register encoder - Optimum Viterbi decoder		

considered approaches.

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Table 3. Continue

Order of error probability	10^{-2}	10^{-3}	10^{-4}
Method 2 : Optimized switch-and-stay diversity			
The achieved P	$3.9\ 10^{-2}$	$4.69\ 10^{-3}$	$9.71\ 10^{-4}$
$10 \text{ Log } E_b/N_o$	10	16.02	20
Number of branches	2	2	2
The Complexity requirements	- Two antennas and one receiver only - Optimum switching threshold		
Method 3 : Optimized selection diversity .			
The achieved P	$2.4\ 10^{-2}$	$1.95\ 10^{-3}$	$5.35\ 10^{-4}$
$10 \text{ Log } E_b/N_o$	12.55	16.99	18.57
Number of branches	3	5	6
The complexity requirements	- Optimum number of antennas, and one receiver only, which is large.		
Method 4 : Optimized optimum diversity			
The achieved P	$2.2\ 10^{-2}$	$2.79\ 10^{-3}$	$6.32\ 10^{-4}$
$10 \text{ Log } E_b/N_o$	11.76	14.31	15.56
Number of branches	5	9	12
The complexity requirements	- Optimum number of antennas, and the same number of receivers, which is very large.		
Method 5 : Viterbi decoding with (8,4) convolution code coupled with optimum diversity			
The achieved P	$3.5\ 10^{-2}$	$2.94\ 10^{-3}$	$1.88\ 10^{-4}$
$10 \text{ Log } E_b/N_o$	3.01	6.99	9.03
Number of branches	2	2	2
The complexity requirements	- Shift-register encoder. - Optimum Viterbi decoder. - Large number of, both antennas and receivers, diversity branches.		