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# The Great Pyramid at Giza is a 60-degree Pyramid According to Early Authors' Writings 

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## Keywords

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Pyramid dimensions
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#### Abstract

This paper demonstrates that the Great Pyramid of Giza is originally a 60 -degree pyramid. In general, it is an equilateral triangle, whereas its inclination angle is exactly $60^{\circ}$, not the theoretical angle $51^{\circ} 52^{\prime}\left(51.86^{\circ}\right)$, as postulated by John Taylor in 1859. Therefore, the original vertical height of the pyramid would be 199.47 meters, not 146.6 meters, while the original inclined height of the pyramid would be equal to the length of its base side, equals 230.34 meters. This inclined height of the pyramid, is what Herodotus and many early authors in the Middle Ages meant in their description for the Great Pyramid's dimensions, as they stated a height of the pyramid equals to the base side length, without clarifying weather they meant the vertical height, or the inclined height of the pyramid, but according to the description of the Great Pyramid by Ali ibn Ridwan, a famous Egyptian astronomer in the $11^{\text {th }}$ century $\mathrm{AD} /$ the $5^{\text {th }}$ century AH , it turns out that they meant the inclined height (Apotheme), the line from the pyramid top to the middle of the base side, not the vertical height, that to be equal to the length of the base side of the pyramid. This was confirmed by Pliny's measurement for the Great Pyramid in the classical era, after stating the method of measuring the pyramid vertical height, and emphasized by Charles Perry in 1743. Therefore, the researcher has used the descriptive analytical Approach through this paper to come out to his conclusion.


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## 1. Introduction

It seems clearly that archaeologists and Egyptologists in the $19^{\text {th }}$ century AD differed greatly, in determining the original vertical height of the Great Pyramid and its slope angle, from the early authors of the classical antiquities until the end of the Middle Ages. Whereas it is known in our present time that, the original vertical height of the Great Pyramid is a theoretical height, was determined by John Taylor in 1859 AD, according to his famous theory, which is PI ( $\pi$ ) theory for the shape of the Great Pyramid, where Taylor has assumed that the vertical height of the Great Pyramid is equal to the radius of a circle whose circumference is equal to the perimeter of the Great Pyramid base, or the ratio between twice the vertical height of the pyramid to its base perimeter is equal PI or as the ratio 1:3.14 (Taylor, 1859, 19; Taylor, 1864, 28; Smyth, 1880, 15).

Thus, according to the length of the base side of the Great Pyramid, determined by Flinders Petrie, which is equal to 230.34 meters ( 755.73 feet) (Lehner, 2001, 39; Petrie, 1883, 43), the original vertical height of the pyramid, based on Taylor's theory, is nearly 146.71 meters ( 481.33 feet), whereas the theoretical slope angle of sides of the Great Pyramid, postulated by Taylor in 1859 AD , is $51^{\circ} 51^{\prime} 14.3^{\prime \prime}\left(51.85^{\circ}\right)$, or approximately equals $51^{\circ} 52^{\prime}\left(51.86^{\circ}\right)$ as defined by Petrie (Petrie, 1883, 43; Bartlett, 2014, 301).

As for the dimensions of the Great Pyramid described in the writings of authors in classical times, such as; Herodotus, Pliny, Diodorus of Cisily, and most of authors during the Middle Ages, such as; ibn Abd al-Hakam, Al-Masudi, Ali ibn Ridwan and others, the slope angle of the Great Pyramid's faces would be 60 degree exactly, and the original vertical height of the pyramid would be 199.47 meters approx. ( 654.53 feet), depending on the length of the base side of the Great Pyramid mentioned above.

However, the British colonel Howard Vyse and his engineer John Shae Peering determined the slope angle of the pyramid to be $51^{\circ} 50^{\prime}\left(51.83^{\circ}\right)$, and its vertical height to be 480.75 feet ( 146.53 meters) (Vyse, 1840b, 109), after discovering two pieces of the cladding stones in 1837 AD , still joining their original place at the middle of north side of the pyramid base (Vyse, 1840a, 261), and were destroyed after a short period of the discovery by the hammers of the night-walkers and the Muslim Arabs as stated by Smyth (Smyth, 1874, 19, 489).

After a while, Peering gave another value for the slope angle of the Great Pyramid's faces, as he changed the dimensions of the Great Pyramid, in order to correspond to the value of the royal cubit unit that he had deduced. Therefore, he claimed that the original slope angle of the pyramid's faces, that actually intended by the designer of the pyramid, is $51.34^{\circ}\left(51^{\circ} 20^{\prime} 25^{\prime \prime}\right)$, which differs by about half a degree from the slope angle stated by him in the second volume of Vyse book as mentioned before (Bunson, 1854, 636-637).

On the other side, there are 19 small pieces of the outer casing of the Great Pyramid, completely ignored by archeologists, discovered also by Vyse in 1837, he gave them to the British Museum, but were transferred to London in 1865 AD (Smyth \& Moigno, 1875, 58; Smyth, 1867, 167). Smyth measured the dimensions of all these pieces and their slope angles in 1866 AD, and he found that one of these pieces, found on the northern side of the pyramid, gives a slope angle equal to $53.16^{\circ}$ approx. $\left(53^{\circ} 10^{\prime}\right.$ ) or $180-126^{\circ} 50^{\prime}$, one degree greater than the theoretical
slope angle assumed by Taylor. while in general there are 13 pieces of them whose slope angles are greater than the theoretical slope angle of Tylor, which is $51.85^{\circ}$ ( $51^{\circ} 51^{\prime} 14^{\prime \prime}$ ), and less than $53^{\circ}$ (Smyth, 1867, 169).

In addition, Mr. Francis Ayrton had measured the slope angles of all sides of the Great Pyramid in 1841 AD, and he stated, in a short memorandum in 1865, that he had found the mean slope angle for all sides of the Great Pyramid is $53^{\circ} 31^{\prime} 49^{\prime \prime}$ $\left(53.53^{\circ}\right)$, while the southern side of the Great Pyramid is inclined at an angle of about $56^{\circ}$ (Smyth, 1867, 300), i.e. four degrees more than Taylor's theoretical slope angle of the Great Pyramid's faces.

## 2. Literature Review

### 2.1. The Height of the Great Pyramid According to Herodotus \& Early Authors during the Middle Ages:

Herodotus, the Greek historian in the $5^{\text {th }}$ century BC, and many authors in the Middle Ages, whether they were geographers, travelers or historians, mentioned in their writings that the height of the Great Pyramid is equal to the length of its base side, without clarifying whether the intended height mentioned by them, is the vertical height of the pyramid, from the center of the base to the pyramid's top, or they meant the inclined height of the pyramid's face (Apotheme), i.e. the slopping line that connects between the middle of the base side of the pyramid and its top.

For example, Herodotus mentioned in his famous statement, in describing the dimensions of the Great Pyramid, that: "The length of the base side of the Great Pyramid is equal to 800 feet, and the height is equal to it" (Vyse, 1840b, 182, 260). In addition to some of the Muslim authors who had given a height for the Great Pyramid equals to its length of the base side, without determining whether they meant the vertical or inclined height of the pyramid, such as; Abu Ma'asher al-Balki in the $9^{\text {th }}$ century AD (De Jong, 1867, 101), al-Mas'udi in the $10^{\text {th }}$ century AD (المسعودي، 1973، 361), Yakut al-Hamawi in the $13^{\text {th }}$ century AD (البغادي، 1977، 400), and others.

However, these early authors saw the Great Pyramid with its casing stones, smooth, complete and beautiful, and without the heaps of rubble surrounded all the sides of the pyramid (Smyth, 1874, 19), but most of the archaeologists in the $19^{\text {th }}$ century AD have ignored this previous statement of Herodotus and these authors of the Middle Ages, concerning the pyramid height, because it is much longer than the current vertical height of the Great Pyramid, which is theoretical, and is nearly equal to 481.33 feet ( 146.71 meters), while the inclined height of the pyramid side (Apotheme), is equal 611.92 feet approx.. (186.51 meters) (Petrie, 1883, 43; Bartlett, 2014, 301-302). While according to Herodotus and the early authors, the Great Pyramid height, weather vertical or inclined, would be 756 feet ( 230.34 m .).

On the other hand, some of these authors in the $19^{\text {th }}$ century, relied on the writings of the early authors in the Middle Ages to determine the value of the cubit used in the construction of the Great Pyramid, such as; the French engineer Jomard who determined the value of the cubit in ancient Egypt according to the length of the base side of the Great Pyramid stated by ibn Abd al-Hakam in the $9^{\text {th }}$ century AD / the 3rd AH (Jomard, 1818, 181).

But, in the middle of the $11^{\text {th }}$ century $\mathrm{AD} /$ the $5^{\text {th }}$ century AH , Ali ibn Ridwan, an Egyptian astronomer and physician, clarified in his writings, describing the dimensions of the Great Pyramid, that the inclined height of the pyramid side, Apotheme, not the vertical height, is equal to the length of the base side of the pyramid (المقريزي، 1998، 225), and that is mostly what Herodotus and most of authors in the Middle Ages meant for the pyramid's height mentioned in their writings, not the vertical height as modern scholars in the $19^{\text {th }}$ century believed, and this is what the researcher will clarify through this paper in the following pages, by reviewing the dimensions of the Great Pyramid described in the writings of Muslim authors, especially the Egyptian astronomer Ali ibn Ridwan.

However, these authors had mentioned the dimensions of the Great Pyramid with different values for the cubit, whereas the cubit was the measuring unit of length used during the Middle Ages in the countries of these authors, but most of them gave a standard model for the shape of the Great Pyramid, whereas the inclined height of the pyramid's faces (Apotheme) equal to the length of its base side, and the slope angle of the pyramid's faces would be 60 degrees exactly, so in the following tables are the dimensions of the Great Pyramid stated in cubit by a lot of early authors in the Middle Ages; from the $9^{\text {th }}$ century AD/ the $3^{\text {rd }}$ century AH till the $16^{\text {th }}$ century $\mathrm{AD} /$ the $10^{\text {th }} \mathrm{AH}$, as follows: -

## - Early authors in the $\mathbf{9}^{\text {th }}$ century AD / $\mathbf{3}^{\text {rd }}$ century AH:

For Example; the Egyptian historian ibn Abd al-Hakam (Jomard, 1818, 181) (السيوطي، 1967، 71), Abu Ma’asher al-Balkhi (De Jong, 1867, 101; Vyse, 1840b, 319), and ibn Khordadbeh (المقريزي، 1998، 339؛ ابن خرداذبه، 1889، 159), and these are the dimensions of the Great Pyramid stated by them in the following table:-

Table 1. The dimensions of the Great Pyramid by early authors in the $9^{\text {th }}$ century AD $/ 3^{\text {rd }}$ Century AH.

| Author | Work Field | Length of Pyramid Base | Pyramid Height | Notes |
| :---: | :---: | :---: | :---: | :---: |
| Ibn Abd al-Hakam <br> $(803 \mathrm{AD}-870 \mathrm{AD})$ | Egyptian historian | 100 Sultani cubits | 100 Sultani <br> cubits | 100 Sultani Cubits equals 500 <br> cubits of hand at his present time |
| Abu Ma'asher al- <br> Balkhi <br> $(787 \mathrm{AD}-886 \mathrm{AD})$ | Persian Astronomer | 400 cubits | 400 cubits |  |
| Ibn Khordadbeh <br> $(820 \mathrm{AD}-912 \mathrm{AD})$ | Persian Geographer <br> and a historian. | 400 cubits | 400 cubits | King's cubit |

- Early authors in the $10^{\text {th }}$ century AD / $4^{\text {th }}$ century AH:

For Example; Abu Zayd al-Balkhi (الإدريسي، 2002، 60), the geographer ibn al-Faqih al-Hamadani (الهذاني، 1996، 123), Abu al-Hasan al-Mas'udi (المسعودي، 1973، (Vyse, 1840b, 326), Istakhri (الإصطري، 1937، 51 (انصري)، the travel-Author ibn Haukal (ابن حوقل، 1873، 100؛ المقريزي، 1998، 1826), al-
 (444 ،2009, and these are the dimensions of the Great Pyramid stated by them in the following table:-

Table 2. The dimensions of the Great Pyramid by early authors in the $10^{\text {th }}$ century AD $/ 4^{\text {th }}$ Century AH.

| Author | Work Field | Length of Pyramid Base | Pyramid Height | Notes |
| :---: | :---: | :---: | :---: | :---: |
| Abu Zayd al-Balkhi ( $840 \mathrm{AD}-934 \mathrm{AD}$ ) | Persian Geographer and mathematician | 400 cubits | 400 cubits |  |
| Ibn al-Faqih al-Hamadani ( $893 \mathrm{AD}-947 \mathrm{AD}$ ) | Persian historian and geogra pher | 400 cubits | 400 cubits |  |
| Abu al-Hasan al-Mas'udi (896-956 AD) | An Arab <br> historian, geographer and traveler from Baghdad | 400 cubits | 400 cubits | 100 Royal Cubit equals 500 cubit of his time |
|  |  | 100 Royal cubits | 100 Royal cubits |  |
| $\begin{gathered} \text { Istakhri } \\ \text { (Died at } 957 \mathrm{AD} \text { ) } \end{gathered}$ | travel-author and geographer from Istakhri in Iran | 400 cubits | 400 cubits |  |
| $\begin{gathered} \text { Ibn Haukal } \\ (943-988 \mathrm{AD}) \end{gathered}$ | A Muslim writer, geographer and historian | 400 cubits | 400 cubits |  |
| al-Maqdisi al-Bashari (946-991 AD) | An Arab geographer, travelauthor borm in Jerusalem. | 400 cubits | 400 cubits | Cubit of the king |
| $\begin{aligned} & \text { Ibn Zolaq } \\ & \text { (919-997 AD) } \end{aligned}$ | Egyptian historian | 400 cubits | 400 cubits |  |
| Ibn al-Nadim (Died 990 AD ) | A Muslim bibliographer, biographer and historian bom in Baghdad | 480 cubits | 480 Cubits | Hashemite cubit |

- Early authors in the $11^{\text {th }}$ century AD $/ 5^{\text {th }}$ century AH:

For example; The Persian Abu Mansour al-Tha'alibi (De Jong, 1867, 101), the Egyptian astronomer and physician Ali ibn Ridwan (بن إياس، 1995، 150), whose description of the Great Pyramid will be mentioned in details in the following pages, the Andalusia geographer Abu Ubayd al-Bakery (البكري، 1992، 518), and theirs dimensions are in the following table:-

Table 3. The dimensions of the Great Pyramid by the early authors in the $11^{\text {th }}$ century AD $/ 5^{\text {th }}$ Century AH.

| Author | Work Field | Length of Pyramid Base | Pyramid Height | Notes |
| :---: | :---: | :---: | :---: | :---: |
| Abu Mansur Al-Tha'alibi <br> $(961 \mathrm{AD}-1038 \mathrm{AD})$ | a writer <br> of Arab ethnicity, <br> born in Iran | 400 cubits | 400 cubits |  |
| Ali Ben Ridwan <br> (988-1061 AD) | Egyptian astronomer <br> And physician | 500 cubits | 500 cubits | Vertical height, 430 <br> cubits |
| Abu Ubayd al-Bakery <br> $(1040-1094 \mathrm{AD})$ | Arab Andalusia historian <br> and Geographer | 400 cubits | 400 cubits |  |

- Early authors in the $12^{\text {th }}$ century AD $/ 6^{\text {th }}$ century AH:

For example; The Andalusia traveler-author Abu Hamid Al-Gharnati (الغرناطي، 1993، 102), the Muslim scholar of Persian Al-Zamakhshari (الزمخشري، 1855، 161؛ السيوطي، 1967، 78), the Moroccan historian and geographer Muhammad Al-Idrisi (الإدريسي، 2002، (327-326), and the Persian historian Abu Bakr Al-Hazemy (الحازمي، 1994، 919), and their dimensions are in the following table:-

Table 4. The dimensions of the Great Pyramid by early authors in the $12^{\text {th }}$ century AD $/ 6^{\text {th }}$ Century AH.

| Author | Work Field | Length of Pyramid Base | Pyramid Height |
| :---: | :---: | :---: | :---: |
| Abu Hamid Al-Gharnati <br> $(1080-1169 \mathrm{AD})$ | Andalusia travel-author | 500 cubits | 500 cubits |
| Al-Zamakhshari <br> $(1074 \mathrm{AD}-1143 \mathrm{AD})$ | Muslim scholar of Persian <br> origin | 400 cubits | 400 cubits |
| Muhammad al-Idrisi <br> $(1099-1164 \mathrm{AD})$ | geographer, cartographer <br> and historian, born in <br> Morocco | 400 cubits | 400 cubits |
| Al-Hazemy <br> $(1153-1188 \mathrm{AD})$ | A Persian historian | 400 cubits | 400 cubits |

- Early authors in the $13^{\text {th }}$ century $\mathrm{AD} / 7^{\text {th }}$ century AH:

Some of the most important Muslim authors during this century, who had mentioned the dimensions of the Great Pyramid, which correspond to what Herodotus mentioned in his description of the dimensions of the Great Pyramid, are including; the Persian Ibrahim bin Wasif Shah (المقريزي، 1998، 319) (Vyse, 1840b, 354), Yakut al-Hamawi البغدادي 1977، 400), the famous traveler Abd Allatif al-Baghdadi (الشيخ، 1998، 91), the Iraqi historian ibn al-Athir al-Jazari (Vyse, 1840b, 362), the Egyptian geographer Abu Ga'far al-Idrisi, who is considered the author of the first scientific book about Giza pyramids (الإدريسي، 1991، 67-58), the famous Baghdadi scholar Sibt ibn al-Jawzi (عدالله، 2013، 135), the Persian astronomer and geographer Abu Abdullah al-Qazwini (القزويني، 1960، 267-268), and Abu al-Hasan al-Harawi (الهروي، 2002، 41), and their dimensions are in the following table: -

Table 5. The dimensions of the Great Pyramid by early authors in the $13^{\text {th }}$ century AD $/ 7^{\text {th }}$ century AH.

| Author | Work Field | Length of Pyramid Base | Pyramid Height | Notes |
| :---: | :---: | :---: | :---: | :---: |
| Ibrahim Ibn Wasif Shah (Died at 1209 AD) | A Persian historian | 100 cubits | 100 Cubits | Which equals 500 cubits of his present time |
| $\begin{gathered} \text { Abu al-Hasan Al- } \\ \text { Harawi } \\ (1147 \mathrm{AD}-1215 \mathrm{AD}) \end{gathered}$ | A traveler and historian, bom in Baghdad | 400 cubits | 400 cubits |  |
| Yakut al-Hamawi (1179-1229 AD) | A traveler, Geographer and historian, borm in Constantinople (Turkey) | 400 cubits | 400 cubits |  |
| Abd Allatif al-Baghdadi (1162-1231 AD) | A traveler, bom in Baghdad | 400 cubits | 400 cubits | Vertical height |
| Ibn al-Athir al-Jazari $(1160 \mathrm{AD}-1233 \mathrm{AD})$ | An Iraqi historian | 100 royal cubits | 100 royal cubits | Royal cubit |
| $\begin{gathered} \text { Abu Ga'far Al-Idrisi } \\ (1172 \mathrm{AD}-1251 \mathrm{AD}) \end{gathered}$ | An Egyptian Geographer | 400 cubits | 400 cubits | Quoted from alMas'udi, Abu Zayd alBalkhi, and Abu Ma 'asher al-Balkhi |
| Sibt Ibn Al-Jawzi (1186-1256 AD) | Muslim scholar in history, bom in Baghdad | 500 cubits | 500 cubits |  |
| Zakariya al-Qazwini <br> (1203-1283 AD) | A Persian physician, astronomer, geographer | 400 cubits | 400 cubits | Quoted from Ibn Zolaq |

- Early authors in the $14^{\text {th }}$ century $\mathrm{AD} / 8^{\text {th }}$ century AH:

They include; the geographer Safi al-Din ibn Abd al-Haq, born in Bagdad, (البغدادي، 1955، 1457), the Egyptian Mamluk historian Abu Bakr ibn Aybak al-Dawadari (الدواداري، 1994، 217), as well as the Algerian writer ibn Abi Halja al-Telmisani (التلمساني، 1288) هـ، 118), and their dimensions are in the following table: -

Table 6. The dimensions of the Great Pyramid by Muslim authors in the $14^{\text {th }}$ century $\mathrm{AD} / 8^{\text {th }}$ Century AH.

| Author | Work Field | Length of Pyramid Base | Pyramid Height | Notes |
| :---: | :---: | :---: | :---: | :---: |
| Ibn 'Abd al-Haq <br> $(1260-1338 \mathrm{AD})$ | Geographer, borm in <br> Baghdad | 400 cubits | 400 cubits |  |
| Ibn Aybak al-Dawadari <br> (his father died in 1313 <br> AD) | An Egyptian historian | 500 cubits | 500 cubits |  |
| Ibn Abi Halja al-Telmisani <br> (1325 AD -1375 AD) | an Arab writer, born <br> in Tlemcen (Algeria) | 400 cubits | 400 cubits | Quoted from al-Mas'udi |

- Early authors in the $15^{\text {th }}$ century $\mathrm{AD} / 9^{\text {th }}$ century AH:

They are including; The Mamluk historian ibn Shaheen Al-Zahiry (الظاهري، 1894، الحميري، 1974، 595), the Andalusia traveler Al-Hamiri (المر), the Egyptian historian Ahmed bin Ali Al-Maqrizi who quoted the dimensions of the pyramid from Al-Masudi, ibn Haukal, ibn Khordadbeh, ibn Al-Nadim, Ibrahim ibn Wasif Shah, and Ali ibn Ridwan (المقريزي، 1998، 319-319), and the Egyptian historian Jalal al-Din al-Suyuti, who quoted the dimensions of the pyramid from ibn Abd al-Hakam Sibt ibn al-Jawzi and al-Masudi السيوطي، (72) (151-147، 1995 ،1967, and these are the dimensions of the Great Pyramid stated by them in the following table:-

Table 7. The dimensions of the Great Pyramid by early authors in the $15^{\text {th }}$ century AD $/ 9^{\text {th }}$ Century AH.

| Author | Work Field | Length of Pyramid Base | Pyramid Height | Notes |
| :---: | :---: | :---: | :---: | :---: |
| $\begin{gathered} \text { Taqī al-Din al } \\ \text { Maqgizi } \\ (1364 \mathrm{AD}-1442 \\ \mathrm{AD}) \end{gathered}$ | Egyptian historian | 400 cubits | 400 cubits | Quoted it from al-Mas'udi, Ibn Khordadbeh, Ibn Haukal |
|  |  | 480 Hashemite cubits | 480 Hashemite cubits | Quoted it from Ibn al-Nadim |
|  |  | 100 royal cubits | 100 royal cubits | Which is equals 500 cubits, as quoted from Ibrahim Ibn Wasif Shah |
|  |  | 500 cubits | 500 cubits | Quoted from Ali Ibn Ridwan |
| $\begin{gathered} \text { Khalil al-Zahiry } \\ (1410 \mathrm{AD}-1468 \\ \mathrm{AD}) \end{gathered}$ | Mamluk historian, born in Jerusalem | 500 cubits | 500 cubits |  |
| $\begin{gathered} \text { Mohammed Al- } \\ \text { Hamiri } \\ \text { (Died at } 1495 \mathrm{AD} \text { ) } \end{gathered}$ | Andalusia travelauthor | 400 cubits | 400 cubits |  |
| Jalal al-Din al- <br> Suyuti <br> ( 1445 AD - 1505 <br> AD) | Egyptian historian | 100 royal cubits | 100 royal cubits | Which equals 500 cubits, quoted from Ibn Abd alHakam and Sibt Ibn Al-Jawzi |
|  |  | 400 cubits | 400 cubits | Quoted it from al-Mas'udi |

Despite of there are many authors in the Middle Ages, according to the previous tables, who mentioned that the length of the base side of the Great Pyramid equals to the pyramid height, but none of these authors had clarified in their description weather they meant the vertical height of the pyramid or the inclined height of the pyramid's faces (Apotheme). Only Ali ibn Ridwan, in the $11^{\text {th }}$ century AD/ the $5^{\text {th }}$ century AH, had accurately described the shape and the dimensions of the Great Pyramid, and in details, and who indicated that the inclined height of the pyramid (Apotheme), the line between the middle of the base side of the pyramid to its top, not the vertical height, is equal to the length of the base side of the pyramid.

### 2.2. Description of the Great Pyramid by Ali ibn Ridwan:

Ibn Ridwan was an Egyptian astronomer, mathematician and physician, born in Giza, lived during the reign of the Fatimid caliphs; al-Hakim bi-Amr Allah, az-Zahir, this means, he saw the pyramids repeatedly. He was described as the scholar of Egypt in his time during the days of Al-Mustansir in the middle of the $11^{\text {th }}$ century AD / the $5^{\text {th }}$ century AH (القفطي، 1908، 288). He also gained a wide fame in the field of astronomy, because of his detailed and accurate description of an important event in astronomy, a great stellar explosion that occurred in 1006 AD when he was 18 years old (Stephenson, 2010, 29,30).

Only the two Egyptian historians; al-Maqrizi in the $15^{\text {th }}$ century $\mathrm{AD} /$ the $9^{\text {th }}$ century AH , and ibn Iyas in the early $16^{\text {th }}$ century $\mathrm{AD} /$ the $10^{\text {th }}$ century AH , who quoted this description from ibn Ridwan in their writings, whereas ibn Ridwan stated that:-

The measurement of the first pyramid in cubit, which is used to measure buildings today in Egypt, is that; each side of the pyramid base is 400 cubits, by the black cubit, whose length is 24 fingers, is 500 cubits. Its base is a square and the four sides are equilateral; two sides of it are on the meridian line, while the other two sides are on the line of east to west. The length of each side of the base with the black cubit is 500 cubits, while the sloping line, from the top of the pyramid to the middle of its base side, is 470 cubits, if it were completed, it would be 500 cubits.

The pyramid is surrounded by four triangles with a square base, each triangle is isosceles, and each of the two equal sides of the triangle would be 560 cubits, if it were completed, but in its present state only 470 cubits. These four triangles meet at one point, which is the top of the pyramid, and in a perfect state, the vertical height of the pyramid would be 430 cubits. Therefore, the area of each triangle is 125,000 square cubits, and the sum of all areas of these four triangles is equal to 500,000 square black cubits. (المقريزي، 1998، 325؛ بن إياس، 1995، 150)

So according to the description of ibn Ridwan, the shape of the Great Pyramid would be surrounded by four triangle surfaces with a square base, each triangle is isosceles, and each of the two equal sides of each triangle, the line between the base corner of the pyramid and its top (Arris), would be 560 cubits, i.e. longer than the length of the base side of the pyramid, which is equal to 500 cubits, and the vertical height of the pyramid would be about 430 cubits, while the inclined height of the pyramid (Apotheme), the slopping line between the center of the base side to the pyramid's top would be equal to the length of the base side of the pyramid, would be 500 cubits.

While in general, the shape of the Great Pyramid, i.e. the cross section, would be an equilateral triangle, whereas the length of the pyramid base side would be equal to the inclined height of the pyramid, and not the vertical height,
so according to the account of ibn Ridwan, it turns out that the pyramid height mentioned by Herodotus and the early authors in the Middle Ages, is the inclined height of the Pyramid (Apotheme), not the vertical height as believed by the archeologists in the $19^{\text {th }}$ century AD until the present time.

Therefore, according to the length of the base side of the pyramid known in our present time, which is equal to 230.34 meters ( 755.73 feet), and depending on the previous description of ibn Ridwan for shape of the Great Pyramid, therefore, the dimensions of the Great Pyramid would be as follows (Figure 1):-

- The slope angle of the pyramid's faces would be $60^{\circ}$ exactly.
- The vertical height of the pyramid would be nearly 199.47 meters ( 654.52 feet), that is nearly 53 meters ( 174 feet) higher than the current theoretical height of the pyramid.
- The inclined height of the pyramid (Apotheme) would be 230.34 meters ( 755.73 feet), equals to the length of the base side of the pyramid.
- The height of the pyramid edge (Arris), the line between the base corner to the pyramid top would be 257.53 meters ( 844.9 feet), while the current height of the pyramid edge is 219.2 meters ( 719.17 feet).


Figure 1. The dimensions of the Great Pyramid, as described by the Egyptian astronomer Ali ibn Ridwan in the middle of the $11^{\text {th }}$ century $\mathrm{AD} / 5^{\text {th }}$ century AH .

### 2.3. Description of the Great Pyramid by Abu al-Salt al-Andalusi:

This previous description of ibn Ridwan for the Great Pyramid can be considered the first model for the shape of the Great Pyramid as indicated in (Figure 1), and which was briefly stated by Herodotus and in the writings of most of early authors in the Middle Ages. While there is another model for the shape of the Great Pyramid mentioned by Abu al-Salt al-Andalusi, an astronomer who lived in Egypt about 20 years and died: $1134 \mathrm{AD} / 528 \mathrm{AH}$, and quoted by some later authors since the $12^{\text {th }}$ century AD/ $6^{\text {th }}$ century AH, Abu al-Salt described the two pyramids at Giza in his epistle, he mentioned that:-

What is more amazing and strange, after the capabilities of God Almighty and his artifacts, than the ability to build a massive body (the pyramid), from great stones, square-base, cone-shaped, its vertical (perpendicular) height is 319 cubits, surrounding it, four surfaces of equilateral triangles, the length of each side of which is 460 cubits, and it is with the greatness of perfect workmanship, perfect grooming, and good discretion, so that it was not affected by the wind storms, rainfall and earthquakes, and this is the character of each of the two pyramids opposite to Fustat, on the western side of the river, as we have seen them. (Vyse, 1840b, 355-356) (الإدريسي، 1991، 17-18؛ المقريزي، 1998، 335)

Some authors, subsequent to Abu al-Salt, in the Middle Ages since the $12^{\text {th }}$ century AD/ the $6^{\text {th }}$ century AH , had quoted these dimensions of the Great Pyramid, or the two Great pyramids at Giza, from Abu al-Salt, and they are as follows:-

- Early authors in the $\mathbf{1 3}^{\text {th }}$ century $\mathrm{AD} /$ the $\mathbf{7}^{\text {th }}$ century AH :
- Al-Assad bin Mamatti (1149 AD - 1209 AD) (بن مماتي، 1991، 70-79).
- Yakut al-Hamawi (1179 AD - 1229 AD) (البغدادي، 1977، 401), as well he mentioned the dimensions of the first model for the shape of the Great Pyramid.
- Abd al-Latif al-Baghdadi (1162 AD - 1231 AD) (الشيخ، 1998، 92-91), who also mentioned the dimensions of the pyramid according to the first model of the shape of the Great Pyramid.
- Abu Ja'afar al-Idrisi (1172 AD - 1251 AD) (الإدريسي، 1991، 17-18), also he mentioned the dimensions of the first model for the shape of the Great Pyramid.
- Al-Qazwini (1203 AD - 1283) (القزويني، 1960، 267), as well he mentioned the dimensions of the first model for the Great Pyramid shape.
- Early authors in the $\mathbf{1 4}^{\text {th }}$ century AD / the $\mathbf{8}^{\text {th }}$ century AH:
- Jamal al-Din al-Watwat (1235 AD - 1318 AD) (السيوطي، 1967، 267).

- Shehab Al-Din Al-Nuwari (died at 1333 AD) (النويري، 2004، 359-358، 133 ).


## - Early authors in the $\mathbf{1 5}^{\text {th }}$ century $\mathrm{AD} /$ the $9^{\text {th }}$ century AH:

- Abu al-Abbas al-Qalqashindi (1355 AD - 1418 AD) (القلقشندي، 1914، 324).
- Taqī al-Din al-Maqrizi (1364 AD-1442 AD) (المقريزي، 1998، 335), as well he mentioned the dimensions of the first model of the Great Pyramid shape.
- Jalal Al-Din Al-Mahali (1489 AD-1459 AD) (Pinkerton, 1814, 823; Vyse, 1840b, 339).
- Jalal Al-Din Al-Suyuti (1445 AD - 1505 AD) (السيوطي، 1967، 74، 75), also he mentioned the dimensions of the first model for the Great Pyramid shape.
- Ibn Taghribirdi (1410 AD - 1470 AD) (49 ، الأتابكي، 1992).

According to the last description of Abu al-Salt for the shape of the Great Pyramid, the pyramid would be a square base, surrounded by four surfaces of equilateral triangles, the length of each side of the triangles would be equal to the length of the base side of the pyramid, which is 460 cubits, and the vertical height of the pyramid would be nearly 325.27 cubits, in its perfect state, not 319 cubits, the inclined height of the pyramid's faces (Apotheme) would be 398.37 cubits ( 400 cubits nearly), if completed (Pinkerton, 1814, 823), while the height of the pyramid edge (Arris) would be equal to the length of the base side, equals to 460 cubits.


Figure 2. The dimensions of the Great Pyramid described by Abu al-Salt al-Andalusi in the $12^{\text {th }}$ century $\mathrm{AD} / 6^{\text {th }}$ century AH.

Therefore, according to the previous description of Abu al-Salt, which can be considered as the second model for the shape of the Great Pyramid mentioned by early author in the Middle Ages, and depending on the current length of the base side of the pyramid, which is equal to 230.34 meters ( 755.73 feet), the dimensions of the Great Pyramid would be as in (Figure 2):-

- The slope angle of the pyramid's faces would be nearly $54.74^{\circ}\left(54^{\circ} 44^{\prime} 8^{\prime \prime}\right)$.
- The vertical height of the Great Pyramid would be nearly 162.86 meters ( 534.32 feet), nearly 16 meters ( 53 feet) higher than the current theoretical height of the pyramid.
- The inclined height of the pyramid (Apotheme) would be 199.47 meters ( 654.52 feet), nearly 13 meters ( 43 feet) longer than the current inclined height of the pyramid.
- The height of the pyramid edge (Arris) would be 230.34 meters ( 755.73 feet), equals to the length of the base side of the pyramid.


### 2.4. Al-Baghdadi correcting the Great Pyramid' dimensions of Abu al-Salt:

It should be noted that Abd al-Latif al-Baghdadi, in the $13^{\text {th }}$ century AD, commented on this previous account of Abu al-Salt, he stated that this measurement is incorrect, and then he corrected it; whereas he stated that the vertical height of the pyramid should be 400 cubits, instead of being 317 cubits as mentioned by Abu al-Salt (الثيخ، 1998 ، 92-91). So that, according to this previous height corrected by Al-Baghdadi, and the length of the base side of the pyramid stated by Abu al-Salt, which is 460 cubits, the slope angle of the pyramid's faces would be nearly $60^{\circ}$, while the inclined height (Apotheme) would be 460 cubits, i.e. equals to the length of the base side, and this largely corresponds to the first model for the shape of the Great Pyramid described by ibn Ridwan in the $11^{\text {th }}$ century AD.

### 2.5. Palerne Emphasizing the Pyramid Height as Described by ibn Ridwan:

What confirms the description of ibn Ridwan and the inaccuracy of the description of Abu al-Salt for the dimensions of the Great Pyramid, what was stated by Jean Palerne, a French traveler visited the pyramids of Giza in 1581 AD, he determined the length of the base side of the Great Pyramid to be 1680 feet, and its height be 1440 feet (Palerne, 1606, 146; Vyse, 1840b, 194). So depending on these dimensions, the slope angle of the Pyramid's faces would be nearly $59.74^{\circ}$, considering the mentioned height is the vertical height of the pyramid.

While in a different way by comparison, Palerne confirmed that his dimensions of the Great Pyramid are compatible with the description of ibn Ridwan, by stating that the height of the pyramid is more than three times the height of Notre Dame church de Paris, whereas the height of the church, or church towers' height, is about 400 steps (Palerne, 1606, 148-149).

Therefore, according to the height of the towers of Notre Dame church, which is 217 English feet or 203.5 French feet (66. 14 meters) (Board of Regents of the Smithsonian Institution, 1890, 730), the vertical height of the Great Pyramid would be equal to nearly 651 English feet or 610.5 French feet (198.35 meters), and this height
largely corresponds to the vertical height of the Great Pyramid according to the first model for the shape of the Great Pyramid that equals 199.5 meters approx..

### 2.6. Greaves used the Description of Abu al-Salt for the Great Pyramid:

However, al-Baghdadi corrected the dimensions of the Great Pyramid mentioned by Abu al-Salt, but John Greaves, a Professor of Astronomy in the University of Oxford in the $17^{\text {th }}$ century AD, after measuring the Great Pyramid in 1637 AD, he had stated the dimensions of the pyramid, that are in line with the description of Abu al-Salt for the pyramid, he claimed that the pyramid is a square base, surrounded by four equilateral triangles, whereas he made the base side equal to 693 feet, which would be equal to the height of the pyramid edge (Arris), and the pyramid vertical height, if completed to its top, would be nearly 490 feet, while the inclined height (Apotheme) would equal nearly 600 feet (Greaves, 1737, XIX- XX).

Greaves was harshly criticized by some authors, so he revised his measurements several times, then he announced that he gave a short height to the Great Pyramid, as he discovered that the vertical height of the pyramid is equal to 499 feet, not 490 feet as he previously mentioned (Thevenot, 1663, VIII; Greaves, 1737, XIIIXIV). Therefore, the slope angle of the pyramid's faces, according to the pyramid height corrected by Greaves after reviewing his calculations, would be $55.22^{\circ}$, not $54.74^{\circ}$ according to his first publication for the dimensions of the Great pyramid, which was compatible with the second model for the shape of the Great Pyramid or the description of Abu al-Salt.

### 2.7. Charles Perry Proving that the Great Pyramid is a $60^{\circ}$ Pyramid:

In addition to the intense criticism of Greaves at his time, Charles Perry, a Norwegian traveler and scholar, confirmed in 1743 AD that the measurements taken by Greaves are incorrect, whereas Perry strongly criticized and excluded Greaves' measurements, and he was conceived that Greaves had an implicit belief in the Great Pyramid shape that he wanted to prove (Perry, 1743, 317).

After Perry measured and studied the dimensions of the Great Pyramid, he found the ratio between the length of the base side of the Great Pyramid and its vertical height is equal to $8: 7$ (Perry, 1743, 316), so Perry believed that, based on the length of the pyramid base determined by Greaves, 693 feet, and according to this previous ratio, the vertical height of the pyramid should be nearly 607.41 feet, not 490 feet as determined by Graves, that means, the vertical height of the pyramid determined by Greaves is about 117 feet shorter than the height should be according to the length of the base side of the pyramid determined by Greaves (Perry, 1743, 317).

On the other hand, Perry explicitly stated that he was certain that the vertical height of the Great Pyramid is not equal to the length of its base side, whereas he found that the length of the southern side of the base to be 789.25 feet,
and the western side equals to 780 feet, so according the previous ratio between the pyramid base and its vertical height, which is $8: 7$. Perry determined the vertical height of the pyramid to be nearly 687 feet, depending on the length of the short base side that equals 780 feet (Perry, 1743, 316; Vyse, 1840b, 249). Consequently, the slope angle of the Great Pyramid's faces would be nearly $60.41^{\circ}$, slightly greater than the slope angle of the first model for the Great Pyramid shape.

Furthermore, Perry stated that the four inclined surfaces of the Great Pyramid are equal to each other, tend to each other at an angle of $30^{\circ}$, and with these four inclined surfaces on each other, it appears to the viewer who is at a certain distance and at an appropriate height, as if it were a true equilateral triangle, and this means that the cross section of the Great Pyramid shows it as an equilateral triangle, this means, the pyramid's faces make an angle of $60^{\circ}$ with the base of the pyramid (Perry, 1743, 316).

### 2.8. Pliny \& Diodorus of Cisily had determined the Exact Vertical Height of the Great Pyramid:

As for the early authors in the classical era, it should be noted that the dimensions of the Great Pyramid set by both Pliny and Diodorus of Sicily, do not contradict Herodotus' description of the Great Pyramid, on the contrary, their measurements are consistent with the first model for the shape of the Great Pyramid. In a different way than Herodotus, Pliny and Diodorus mentioned a height of the Great Pyramid, which often meant the vertical height, quite different from the length of the base side of the pyramid, but according to their measurements, the slope angle of the pyramid's faces would be $60^{\circ}$.

For example, Diodorus of Sicily, in the first century BC, had determined that the length of the base side of the pyramid is about 700 feet (or seven plethora), and the height of more than 600 feet (or six plethora), whereas the Plethron is a measuring unit of length equivalent to 100 feet (Oldfather, 1946, 217; Wilkinson, 1843, 324) (كامل، 2013، 109. Therefore, the slope angle of the Great Pyramid would be nearly $59.74^{\circ}$ ( $60^{\circ}$ approx.), if Diodorus meant the vertical height of the pyramid not the inclined height of the pyramid's faces (Apotheme), while the inclined height (Apotheme) would be about 694.7 feet, and that would be close to the length of the base side of the pyramid mentioned by Diodorus.

But if the mentioned height of Diodorus is the inclined height of the pyramid's faces (Apotheme), in this case, the slope angle would be $54.31^{\circ}$, which largely corresponds to the slope angle of the second model for the shape of the Great Pyramid, whereas the four inclined surfaces of the pyramid are equilateral triangles, while the height of the pyramid edge (Arris) would be equal to the length of the base side of the pyramid, equals to 700 feet. Therefore, John Greaves considered Diodorus' account the most accurate and correct account for the dimensions of the Great Pyramid among the early authors (Greaves, 1737, XX, 92), because he believed that Diodorus meant the inclined height, not the vertical height.

On the other hand, Pliny the Elder, a Roman historian and geographer in the first century AD , confirmed that the intended height by Diodorus is the vertical height, and that the Great Pyramid is a true 60 degree pyramid, Whereas Pliny recorded the method of Thales of Miletus ( 624 BC - 546 BC), a Greek astronomer and mathematician, to measure the vertical height of the pyramids, by measuring the shadow length of objects at the time when the shadow is equal to the height of the bodies (Pearson, 2020, 61; Riley, 1857, 338; Hicks, 29).

Accordingly, Pliny accurately determined the vertical height of the Great Pyramid, as he stated that the pyramid height is equal to 725 feet, and the length of its base side equals to about 833 feet, as stated in the Latin version of the book of the German researcher Lulius Sillig in 1851 AD, as mentioned also by the English scientist John Bostock in 1857 AD (Secundi, 1851, 329; Riley, 1857, 337), so according to the previous dimensions, the slop angle of the Great Pyramid's faces would be about $60.12^{\circ}$ (approx. $60^{\circ}$ ), therefore, these dimensions of Pliny, as well would be coincide with the first model for the shape of the Great Pyramid, not the second model.

## 3. Discussion \& Conclusion:

According to the description of the Great Pyramid of Giza by the Egyptian astronomer Ali ibn Ridwan, in the $11^{\text {th }}$ century $\mathrm{AD} / 5^{\text {th }}$ century AH , the researcher has concluded that the mentioned height in the descriptive statement of Herodotus for the dimensions of the Great Pyramid, and by most of the early authors in the Middle Ages, is the inclined height (Apotheme), not the vertical height as believed by the scholarly authors in the $19^{\text {th }}$ century AD, whereas Herodotus and these early authors give a height to the Great Pyramid equals to the length of its base side, without explaining whether they meant the vertical height, or the inclined height of the pyramid's face (Apotheme), the slopping line from the middle of the base side of the pyramid to its top.

The researcher has come out to the conclusion that the Great Pyramid is a true 60 degrees pyramid, whereas the original shape for the Great Pyramid, according to the description of the early authors in the classical antiquities and the Middle Ages, is about a square-base, surrounded by four inclined surfaces of isosceles triangles, each of the two equal sides of the four triangles is longer than the length of the base side of the pyramid. While in general, the pyramid is an equatorial triangle, i.e. the cross section, whereas its slope angle of its four surfaces is exactly 60 degrees, not $51^{\circ} 52^{\prime}\left(51.86^{\circ}\right)$ as defined by Petrie, and its inclined height (Apotheme), the slope line from the pyramid top to the center of its base side, equals to the length of the base side of the pyramid.

Therefore, the original dimensions of the Great Pyramid at Giza according to ibn Ridwan, and depending on the current length of the base side of the pyramid known
in our present time, which is equal to 230.34 meters ( 755.73 feet), would be as follows:-

- The slope angle of the pyramid's faces would be $60^{\circ}$ exactly.
- The vertical height of the pyramid would be nearly 199.47 meters ( 654.52 feet), that is nearly 53 meters ( 174 feet) higher than the current theoretical height of the pyramid.
- The inclined height of the pyramid (Apotheme) would be 230.34 meters ( 755.73 feet), equals to the length of the base side of the pyramid, and this is what Herodotus meant and early authors in the middle era.
- The height of the pyramid edge (Arris), the line between the base corners to the pyramid top, would be 257.53 meters ( 844.9 feet), while the current height of the pyramid edge is 219.2 meters ( 719.17 feet).

In this paper, the researcher has found that there were two models for the shape of the Great Pyramid, the first model was mentioned in details by ibn Ridwan at the $11^{\text {th }}$ century AD , whereas the shape of the Great Pyramid like the previous description in the upper paragraphs, which was briefly described by Herodotus in the $5^{\text {th }}$ century BC, and most of authors in the Middle Ages, only mentioned the pyramid height and length of the base side in their description, whereas they stated a height for the pyramid equals to the length of its base side, such as; ibn Abd al-Hakam and ibn Khordadbeh in the $9^{\text {th }}$ century AD, ibn al-Nadim, and al-Mas'udi in the $10^{\text {th }}$ century $A D$, Abu Ubayd al-Bakery, and al-Tha'alibi in the $11^{\text {th }}$ century AD , Abu Hamid Al-Gharnati, and Muhammad Al-Idrisi in the $12^{\text {th }}$ century AD , ibn Wasif Shah and Sibt ibn al-Jawzi in the $13^{\text {th }}$ century, al-Dawadari and al-Telmisani in the $13^{\text {th }}$ century, as well as Al-Hamiri and Al-Maqrizi in the $15^{\text {th }}$ century AD.

While the second model for the shape of the Great Pyramid was mentioned by the astronomer Abu al- Salt Andalusi in the $12^{\text {th }}$ century AD/ $6^{\text {th }}$ century $A H$, whereas the pyramid surrounded by four surfaces of equatorial triangles, with a square base, each side of the triangles equals to the length of the base side of the pyramid, and this description was quoted by few authors in the Middle Ages, such as, Al-Qazwini and ibn Mamatti in the $13^{\text {th }}$ century, al-Dimashqi and Al-Nuwari in the $14^{\text {th }}$ century AD, as well as ibn Taghribirdi and al- Mahali in the $15^{\text {th }}$ century.

Therefore, according to the previous description of Abu al-Salt, and depending on the current length of the base side of the pyramid, which is equal to 230.34 meters (755.73 feet), the dimensions of the Great Pyramid would be as follows:-

- The slope angle of the pyramid's faces would be nearly $54.74^{\circ}\left(54^{\circ} 44^{\prime} 8^{\prime \prime}\right)$.
- The vertical height of the pyramid would be nearly 162.86 meters ( 534.32 feet), nearly 16 meters ( 53 feet) higher than the current theoretical height of the pyramid.
- The inclined height of the pyramid (Apotheme) would be 199.47 meters ( 654.52 feet), nearly 13 meters ( 43 feet) longer than the current inclined height of the pyramid.
- The height of the pyramid edge (Arris) would be 230.34 meters ( 755.73 feet), equals to the length of the base side of the pyramid.

The researcher has concluded that the first model of the shape of the Great Pyramid according to the description of ibn Ridwan, not the second model of Abu al-Salt, was the original shape of the Great Pyramid, whereas in the $13^{\text {th }}$ century AD , Abd al-Latif al-Baghdadi stated that the dimensions of Abu al-Salt for the Great Pyramid are inaccurate, then he corrected it to be comply with the first model for the shape of the pyramid of ibn Ridwan, whereas the slope angle of the pyramid's faces would be $60^{\circ}$, not $54.74^{\circ}$.

However, John Greaves, in the $17^{\text {th }}$ century, he used the second model to determine the shape and the height of the Great Pyramid, but Charles Perry in 1843 AD, he criticized Greaves' dimensions, and he confirmed that the slope angle of the pyramid is nearly $60^{\circ}$, by mentioning that the ratio between the vertical height and the base side of the pyramid is equal to $7: 8$. While in the classical era, the measurements of Pliny and Diodorus also emphasize the first model of the shape of the Great Pyramid assumed in this paper.

According to the original shape of the Great Pyramid, that concluded through this paper, the area of each of the four inclined faces of the Great Pyramid would be half the area of the Great Pyramid, whereas the area of the base equals $53,051.9$ square meters ( $571,045.99$ square feet), while the area of each inclined face would be $26,525.95$ square meters ( $285,522.94$ square feet).

According the original vertical height of the Great Pyramid, deduced by the researcher in this paper, which equals 199.47 meters ( 654.42 feet), the volume of the Great Pyramid would be $3,527,450.265$ cubic meters ( 3.5 million cubic), not 2.6 million cubic meters as estimated by Andre Pochan in 1978 and other authors.

Finally, the researcher recommends measuring the slope angle of all four sides of the Great Pyramid, especially the south side, as Francis Ayrton, after measuring the slope angles of all sides of the pyramid in 1841 AD , he found that the slope angle of the southern side of the pyramid is $56^{\circ}$, while the mean slope angle of all sides of the Great Pyramid is $53^{\circ} 31^{\prime} 49^{\prime \prime}\left(53.53^{\circ}\right)$.

The researcher also recommends shedding light on the 19th small pieces of the casing stone of the Great Pyramid, discovered by Howard Vyse in 1837 AD, and transferred to the British museum in 1865, whereas Smyth found one of these pieces gives an angle greater than $53^{\circ}$, i.e. one degree more than the current theoretical slope angle of the Great Pyramid, which is equal to $51^{\circ} 52^{\prime}\left(51.86^{\circ}\right)$.

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