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1 COMPUTER OPTIMIZATION OF PHASE CODED WAVEFORM PARAMETERS

2 AND PROCESSOR FOR CLUTTER REJECTION

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4 **ABSTRACT**

The problem of elevated temporal and spectral sidelobes, of the large duration pulse signal phase coded by pseudo random binary sequence is considered in this paper . Both the techniques of window function and Lagrange method of multipliers are used to over come this problem. Computer programs are constructed to evaluate the processor form corresponding to the considered class of signals. The results show that an observable reduction of these sidelobes is obtainable with an acceptable mismatched loss.

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1- INTRODUCTION

The use of large time duration pulse signals, phase coded by pseudo random binary sequence provides good properties of ambiguity, resolution, and measurement accuracy in both time-delay (range) and doppler frequency (velocity) domains [1] However these signals suffer from relatively high sidelobes along both time-delay axis and doppler frequency axis. These elevated sidelobes limit consequently the efficiency of phase coded signal in dense environment, composed of more than one target and clutter. In that case the desired target echo is corrupted with many interfering signals, which may tend to suppress the detectability of the desired target. Moreover the elevated sidelobes may lead to false target indication. Therefore it is required to find an optimization method to reduce these sidelobes, in order to suppress clutter responses , while preserving the mentioned good properties of the phase coded signal.

This paper analyzes a two step optimization technique , to reduce the spectral sidelobes along the doppler frequency axis, and the temporal sidelobes along the time-delay axis. The proposed waveform consists of train of K-pulses, each one is phase coded by N-bit binary word. The first step of the optimization technique is to weight each pulse of the transmitted train by a Hann widnow, which results in reduction of the spectral sidelobes. The second step is to design an optimum mismatched filter, which minimizes the mismatched loss, and control the range sidelobes.

The results show that a trench with no response is obtainable, along the ambiguity function, where the out of range clutter is expected to be found.

2- PROBLEM FORMULATION

Let us consider that the transmitted signal is a coherent pulse train, of K-pulses, each one is phase coded by the same pseudo random binary sequence as illustrated in Fig.(1).

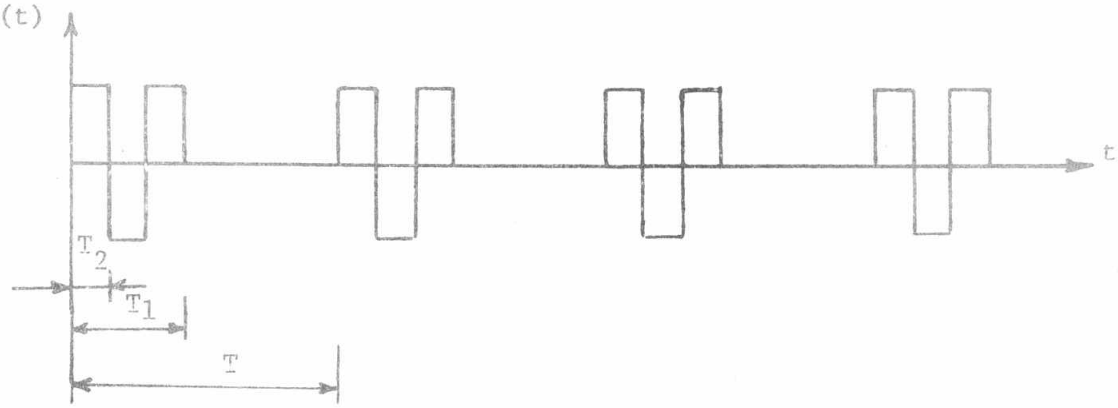

Fig.(1) Train of 4-pulses each one is phase coded by 3-bit binary word.

where

T_2 is the subpulse duration

T_1 is the pulse duration

T is the interapulse repetition period

The behaviour of the ambiguity function [2] for this phase coded waveform, along the time-delay axis is calculated and shown in Fig.(2).

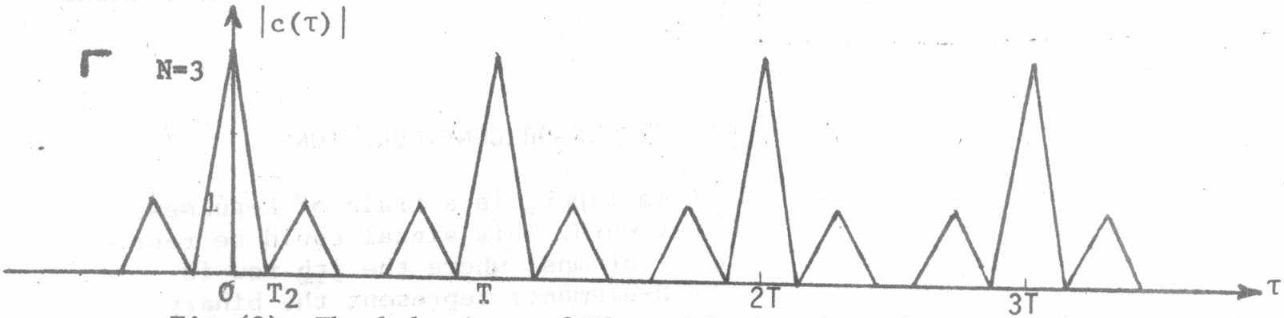

Fig.(2). The behaviour of the ambiguity function ,A.F.,for coherent pulse train, along the time-delay axis.

A computer program is constructed and given in Appendix -I, to evaluate the behaviour along the doppler frequency axis. The result for coherent pulse train of 4-pulses is shown in Fig.(3).

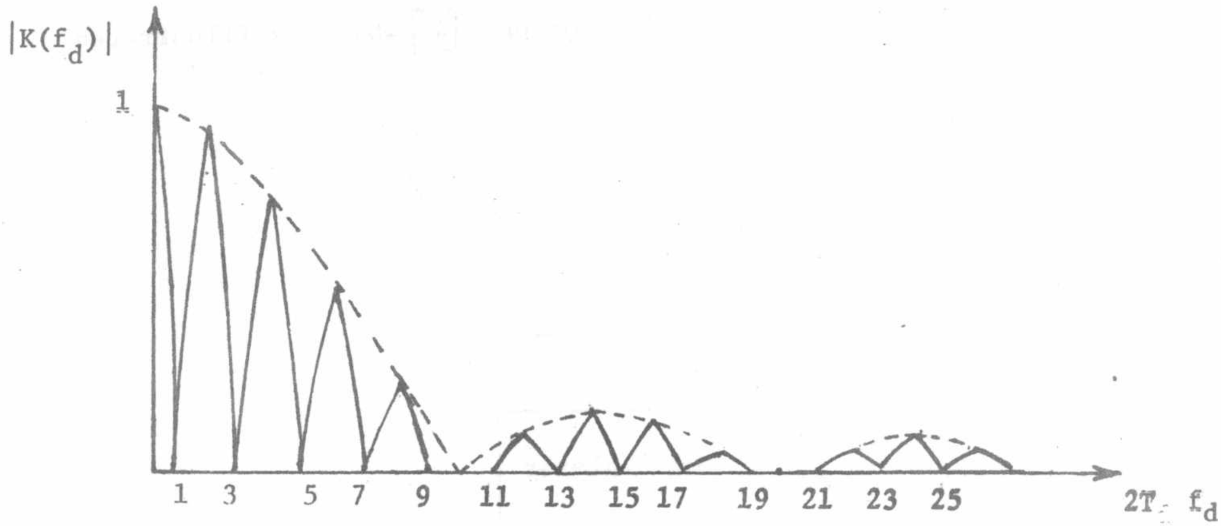

Fig.(3) The behaviour of the A.F, for train of 4-pulses, each phase coded by the same 4-bit binary word, along doppler frequency axis. (T_2=1μ sec, T = 20 T_2).

Clearly the phase coding, by pseudo random binary sequence, of large duration pulse signal, improves the range resolution and measurement accuracy [3]since these depend on the subpulse duration . (T_2). Also the doppler frequency resolution is good, since it depends on the total pulse duration (T_1).

An inspection of the behaviour, reveals that both the range and doppler sidelobes are rather high. This drawback increases the interference, and may tend to false target indications in case of clutter environment.

The approach analyzed here, for reduction of range and doppler sidelobes, is based on reshapping the transmitted phase coded waveform, and consequently the corresponding processor. This approach for reducing the sidelobes is described in details, in the following two sections 4,5. In section 4 it is clarified the effect of using a Hann window to weight each pulse of the transmitted pulse train, to reduce the spectral sidelobes along the doppler frequency axis. Section 5 explains the effect of applying the Lagrange method of multipliers on the problem of designing the optimum mismatched filter, which control the temporal sidelobes along the time - delay axis . This type of processing reduces the effect of the out of range clutter.

3. SIGNAL REPRESENTATION AND PROCESSOR CONFIGURATION

The considered transmitted signal, in this paper, is a train of K-pulses, each pulse is phase coded by N-bit binary word. This signal could be represented in matrix form, of K-rows , and N-columns, where the jth row is corresponding to the j th pulse and its N-elements represent the binary word used for phase coding of this pulse.
Thus

$$C = \begin{bmatrix} C_{11} & C_{12} \cdots & C_{1N} \\ C_{21} & C_{22} \cdots & C_{2N} \\ \cdots & \cdots & \cdots \\ C_{k1} & C_{k2} & C_{kN} \end{bmatrix} \quad (1)$$

The processor for this signal is a correlator one [4] which is illustrated in Fig. (4).

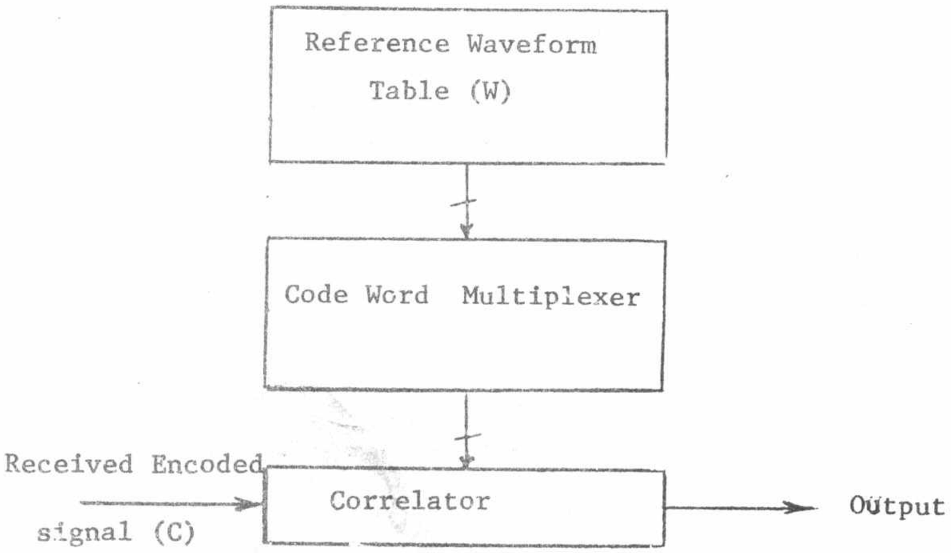

Fig.(4). Simplified multiword correlator processor.

The output of this correlator receiver is given by correlating the received signal array C with the receiver reference array W , which is expressed by (binary or nonbinary) matrix elements in the form:

$$W = \begin{bmatrix} W_{11} & W_{12} \cdots & W_{1N} \\ W_{21} & W_{22} \cdots & W_{2N} \\ \cdots & \cdots & \cdots \\ W_{k1} & W_{k2} \cdots & W_{kN} \end{bmatrix} \quad (2)$$

4- SPECTRAL SIDELOBE REDUCTION USING A WINDOW FUNCTION

The behaviour of the ambiguity function, along the doppler frequency axis, for coherent pulse train was whown in Fig.(3). Of course the variation of the binary code from pulse to pulse can not change this behaviour, since this behaviour depends only on the signal amplitude rather than its phase. This is clear from the definition of the ambiguity function behaviour along doppler axis.

$$\left| X(o,f_d) \right|^2 = \left| k(f_d) \right|^2 = \left| \int_{-\infty}^{\infty} \left| u(t) \right|^2 e^{-j2\pi f_d t} \, dt \right|^2 \qquad (3)$$

If a Hann window, is used to weight each pulse in the transmitted pulse train (or each pulse response on the corresponding processor). Thus the behaviour of the ambiguity function, along the doppler frequency axis, for this case is given by

$$\left| k(f_d) \right|^2 = \left| \int_{-\infty}^{\infty} \left| u(t) \right|^2 h(t) \, e^{-j2\pi f_d t} \, dt \right|^2 \qquad (4)$$

where

$u(t)$ ... is the complex modulation of the transmitted phase coded signal.

$h(t)$ .. is the Hann window [5], which is given by

$$h(t) = \frac{1}{2} + \frac{1}{2} \cos \frac{2\pi t}{T_1} \qquad (5)$$

The integral in equation (4) has been evaluated for the considered signal in section .3. The resultant analytical form, has been evaluated through computer program, given in Appendix (II). Fig.(5) shows the behaviour of the ambiguity function, for the weighted coherent pulse train along the doppler frequency axis.

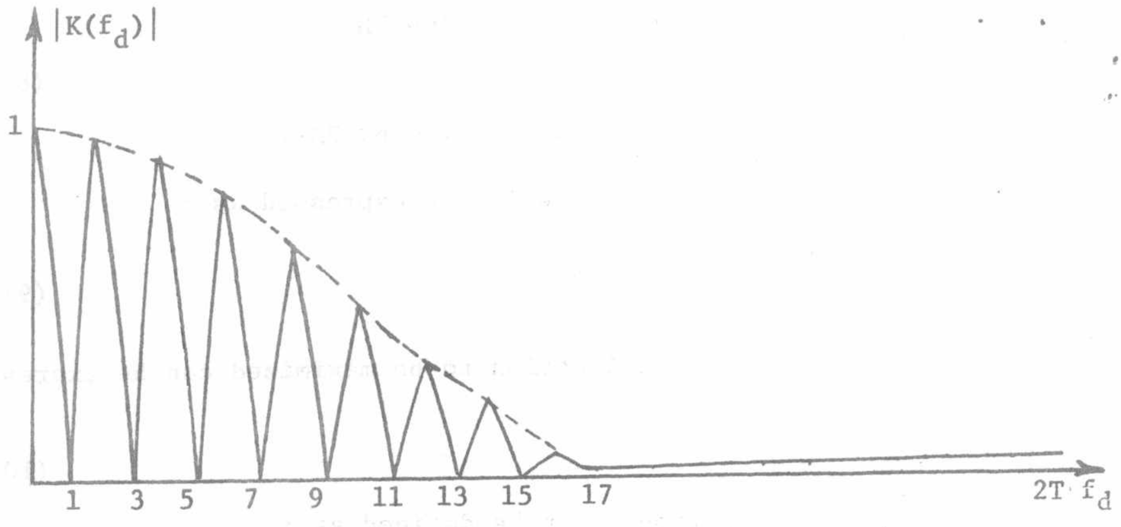

Fig.(5). The behaviour of the A.F, for train of 4-pulses each phase coded by 4-bit binary word, and weighted by "Hann" window, along doppler frequency axis. (T_2=1μ Sec), (T=20 T_2).

A comparison between Fig.(3) and Fig.(5) shows that the use of a Hann window, to weight the transmitted pulses, reduces the spectral sidelobes with inicative reduction. It must be noted that the existance of lobing in the spectrum was expected due to the pulse repetition in the train. However this problem could be solved by increasing the pulse repetition frequency (1/T), in order to exceed the expected maximum doppler frequency shift.

5- RANGE SIDELOBE REDUCTION USING LAGRANGE OPTIMIZATION TECHNIQUE.

The classical lagrange method for constrained optimization is applied to the problem of design of the optimum mismatched reference receiver array(W) corresponding to the considered transmitted array (C).

The objective function is chosen as the mismatched loss (L_w) , and the set of constraints are taken as the peak response and zero range sidelobes. Defining the mismatched loss, which is to be minimized as :

$$L_w = \frac{\left[\sum_{j=1}^{k} \sum_{i=1}^{N} C_{ji} W_{ji}\right]^2}{KN \sum_{j=1}^{k} \sum_{i=1}^{N} W_{ji}^2} \qquad (6)$$

The constraints indicate (2N-2) range sidelobes of zero value defined by

$$g_p = 0 \quad , \qquad p = 1,2,3,\ldots,2N-1 \quad , \quad P \neq N \qquad (7)$$

We have derived a generalized formula to express the range sidelobes which is given by :-

$$g_p = \sum_{j=1}^{k} \sum_{i=1}^{p} C_{j,i+N-p} \; W_{ji} \qquad 1<p<N$$

$$= \sum_{j=1}^{k} \sum_{i=(P-N)+1}^{N} C_{j,i+N-P} \; W_{ji} \qquad N<p<2N-1 \qquad (8)$$

The main lobe value is given by g_N, which is expressed as :

$$g_N = \sum_{j=1}^{K} \sum_{i=1}^{N} C_{ji} \; W_{ji} = KN \qquad (9)$$

Without loss of generality, the function to be maximized can be expressed as :

$$f = \sum_{j=1}^{k} \sum_{i=1}^{N} W_{ji}^2 \qquad (10)$$

Therefore the Lagrange function F can be defined as :

$$F = f + \sum_{p=1}^{2N-1} \lambda_p \, g_p \qquad (11)$$

where

F is a function of KN reference variables from the array W and (2N-1) multiplier variables (λ) . To find the extremum of F, a set of (kN+2N-1) equations could be derived from:

$$\frac{\partial F}{\partial W_{ji}} = 0 \qquad , j = 1,2,\ldots k, \qquad i=1,2,\ldots N \qquad (12)$$

$$\frac{\partial F}{\partial \lambda_p} = 0 \quad , P = 1,2, \ldots, (2N-1). \quad (13)$$

Computer program , constructed and shown in app. III generates the coefficient matrix of the system of linear equations, hence using the Gauss method, it is possible to solve this system of equations, leading to the evaluation of the elements of the reference array W. Furthermore the program evaluates the cross correlation function between the transmitted array C. and the receiver one W.

Consider the transmitted array as a train of 4-pulses each pulse is phase coded by 4-bit Barker code. The code is shifted from pulse-to-pulse by one bit. Thus the transmitted array C. is represented as :-

$$C = \begin{bmatrix} 1 & 1 & -1 & 1 \\ 1 & 1 & 1 & -1 \\ -1 & 1 & 1 & 1 \\ 1 & -1 & 1 & 1 \end{bmatrix} \quad (14)$$

This transmitted array is applied, as an input data, to the constructed program, and the results show that the corresponding receiver reference array W is typically equal to the transmitted one, thus

$$C = W \quad (15)$$

This result means that the mismatched loss is equal to zero. The behaviour of the ambiguity function along the time-delay axis is shown in Fig.(6).

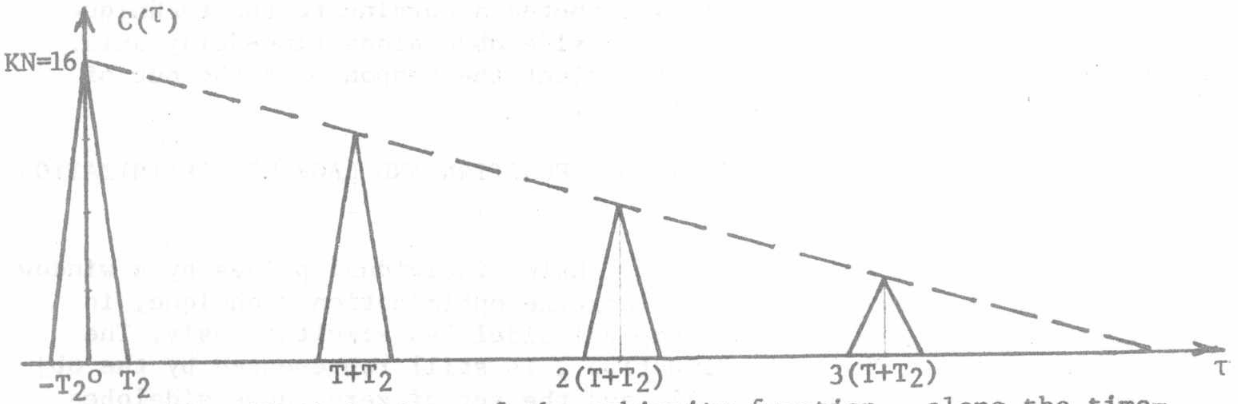

Fig.(6) . The behaviour of the ambiguity function , along the time-delay - axis, for the considered signal in (14) (k=4,N=4). (T_2=1 μ Sec).

Another example for the use of Barker code, is considered, where the transmitted pulse train is given by:

$$C = \begin{bmatrix} 1 & 1 & -1 & 1 \\ 1 & 1 & 1 & -1 \\ -1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \quad (16)$$

The corresponding receiver reference array for this transmitted signal is computed as : -

$$W=\begin{bmatrix} 1.248 & 1.058 & -2.023 & 1.399 \\ 0.470 & 0.790 & 1.089 & -1.583 \\ -1.597 & 0.928 & 0.759 & 0.649 \\ 0.819 & 0.361 & 0.393 & 0.833 \end{bmatrix} \qquad (17)$$

This refrence array represents a mismatched receiver for the transmitted signal and the mismatched loss is computed, and equal to (0.08 dB). The behaviour of the cross ambiguity function, between C, and W, along the time-delay axis is shown in Fig.(7).

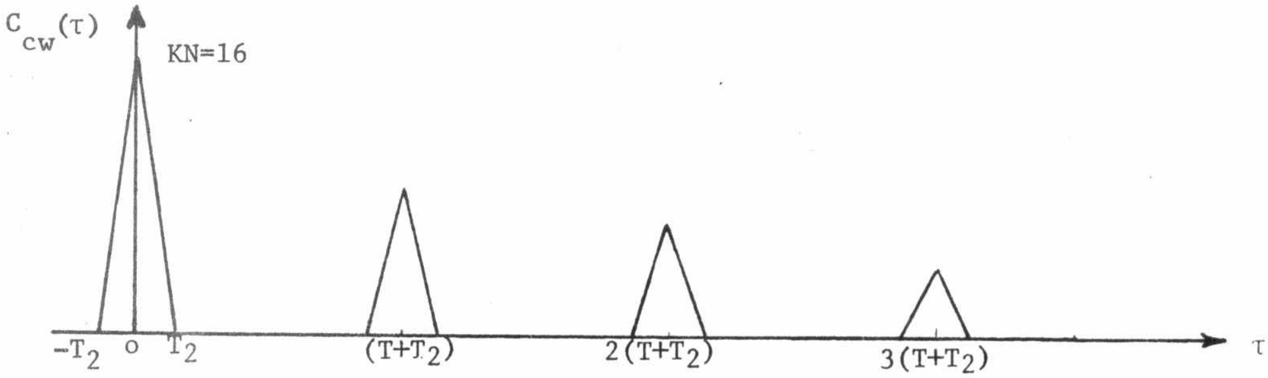

Fig.(7) The behaviour of the cross ambiguity function, along time-delay axis, for the considered signal in (16) (k=4, N=4) (T2 =1 μ Sec).

A comparison of Figs (6),(7) shows that the use of train of pulses ,each phase coded with different binary word, and the corresponding matched (or mismatched) receiver reference array computed according to the technique illustrated above, reduces the temporal sidelobes along time-delay axis. Thus this technique is useful one, to reject the response of the out of range clutter near to the desired target.

6- COMBINED EFFECTS OF WINDOW FUNCTION AND LAGRANGE OPTIMIZATION TECHNIQUE

It must be noted that the effect of weighting individual pulses by a window function could be combined with the Lagrange optimization technique, in order to reduce the spectral and temporal sidelobes simultaneously. The optimization problem, for this situation , is still represented by the objective function, given by (6), (11), and the set of zero range sidelobes constraints in (7), (8).

The difference in this case is that the main-lobe constriant in (9) is replaced by K- constraints for the individual pulses with weighted responses. Thus the optimization problem is stated as follows:

maximize F

Subjected to : $g_p = 0$

and

$$\sum_{i=1}^{N} C_{ji} W_{ji} = H_j \quad , j=1,2,\ldots K \qquad (18)$$

where

H_j ... is the j th pulse weighted response .

The system of simultaneous equations, derived for this situation, consists of $[K(N+1) + 2(N-1)]$ linear equations in the set of reference variables (W_{ji}) and multiplier variables (λ_p). As an illustration for this case, consider the transmitted array as given by:

$$C=\begin{bmatrix} -1 & -1 & 1 \\ 1 & -1 & -1 \\ -1 & 1 & -1 \\ 1 & 1 & 1 \end{bmatrix} \quad (19)$$

and the vector H is given by :

$$H=\begin{bmatrix} 2.5 \\ 3 \\ 3 \\ 2.5 \end{bmatrix} \quad (20)$$

The results show that the corresponding reference array (W), which minimizes the mismatched loss (reaching a value of Lw =0.07 dB) and meets the zero range sidelobe constraints is given by :

$$W=\begin{bmatrix} -0.958 & -0.833 & 0.708 \\ 0.875 & -1.000 & -1.125 \\ -1.042 & 0.917 & -1.042 \\ 0.792 & 0.917 & 0.792 \end{bmatrix} \quad (21)$$

The behaviour of the cross correlation function between C and W is shown in Fig.(8), which clarifies the reduction of the range sidelobes around the response of the desired target

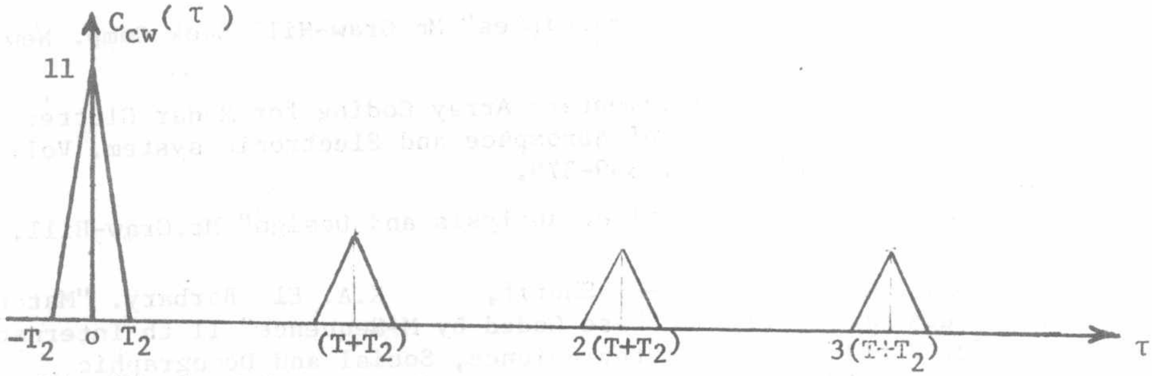

Fig.(8) The behaviour of the cross correlation function for the train of pulses with weighted responses in (19). (k=4) (N=3, $H = \begin{bmatrix} 2.5 \\ 3 \\ 3 \\ 2.5 \end{bmatrix}$) ($T_2$= 1 μ Sec).

7- CONCLUSION

The pulse signal phase coded by pseudo random binary sequence is very suitable waveform to obtain good resolution, measurement accuracy and ambiguity properties, simultaneously in range and velocity domains . This phase coded signal suffers from the relatively elevated sidelobes in both domains, which increase the interference and probability of false alarm. [6,7]. This drawback is solved by reshapping the transmitted signal, in the form of train of pulses, each phase coded by different binary word. Moreover each pulse response is weighted by a Hann window . The results, of the computer solution for these problems, show that, it is possible through using the optimum mismatched receiver corresponding to this signal, with an acceptable mismatched loss, to get out with the following remarks .

1- Weighting the individual pulse response by Hann window leads to reduction of the spectral sidelobes.

2. By mismatched processing of the received signal, a trench along the time-delay axis of the ambiguity function is obtainable, which suppresses the out of range clutter, with acceptable mismatched loss.
3. The drawback of spectrum lobing due to the use of train of pulses, could be tolerated by increasing the pulse repetition frequency to the order exceeding the maximum expected doppler frequency. This can be done, but not in contradiction with the desired maximum unambiguous range. The necessary compromise have to be considered
4. It must be pointed out that, the considered optimization techniques could be applied to the case of pulse signals, phase coded by maximum length sequence which offer higher compression ratios than the Barker codes do. In this case only simple changes of the input data for the computer programs, given in appendices I,II,III, are needed to evaluate the optimum mismatched filter reducing the sidelobes for these signals.

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