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MILITARY TECHNICAL COLLEGE

CAIRO - EGYPT

A METHOD FOR ESTIMATING THE ORDER OF CERTAIN CLASS OF SYSTEM NONLINEARITIES

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ABSTRACT

System nonlinearities are one of the characteristic features of Avionics systems . Desirable nonlinearities appear currently in automatic flight control , assemblies , regulated power supplies , power control of transmitters , mixers , detectors , ... etc . Undesirable nonlinearities receivers transmitters , ... etc , yield signal distortion , intermodulation, spurious interference signal of aanoying effects on board of a/c that is normally crowded with plenty of transceivers using normally crowded spectral band . It is , therefore , essential to identify the Avionics system nonlinearities and supply " good " models for flight control Airborne system designers .

Identification of system nonlinearities involves practically two steps ; estimation of the nonlinearity order and determination of the coefficients of the assumed describing polynomial .

In this work , we give a method for estimating the nonlinearity order for a class of the above-mentioned nonlinearities . Utility of the method is proven through several examlpes .

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I- INTRODUCTION

Some classes of nollinear systems are assumed to work according to certain models , like Wiener , Volterra , bilinear or the memoryless one [6,7]. With an order taken as the highest order of the incorporated non-linearity. During operation of the system , it appears to work at lower order than its real one . As the apparent order depends on the input power level , then for determination of the system order, we must take into account the corresponding operating conditions .

Bad estimation of the system order leads to certain problems in system identification . If the estimated order is higher than the real one , it leads to complication of the model structure besides the un-necessary calculations of unimportant parameters . This is frequently encountered in practice , specially for complicated systems having Wiener or Volterra models . On the other hand , if the estimated order is lower than the real one , it leads to over-simplification of the model structure , and it will yield great errors in estimation of the system parameters [8,9].

We introduce a method for estimating the order of nonlinear system under test using the relative change of power gain ratio . We measure the relative change of output and input power , the estimated system order equals the ratio between these two relative changes.

In section II , the previous methods for estimating the system order are mentioned , In section III , analysis of the proposed method is given for memoyless system characterized by a power series of finite order N . Using different input signals ; DC , sinusoidal,and bandlimited noise , Chosen signal parameters ; e.g. the DC level , amplitude of the first harmonic , ..etc , is measured and used to find N . In section IV the method is described for the bilinear , Wiener,and Volterra models. In section V examples are given through computer simulation showing the potential of the method .

II- METHODS FOR ESTIMATING THE NONLINEARITY ORDER

There are different methods for estimating the nonlinearity order utilizing some special characteristics of nonlinear systems . The most used characteristics are the harmonic generation or the intermodulation distortion,and the broading or whiting effect of the nonliear system for bandlimited input signal . We mention here some known methods for estimating the system order .

1 - Harmonic excitation [1]

If a nonlinear system of order N is excited by a sinusoidal signal of frequency f , the response will be composed of the corresponding harmonics up to the N-th one as shown in Fig.(1). Thus the system order can be found through spectrum analyzer-measurements. In case of bandlimited systems , the higher harmonics may not be detected at the output. This occurs,for example, in systems belonging to simplified Wiener classes [6]and the method fails. To overcome the problem of out-of-band components the folling method is used .

2 - Two-tone method [2]

In this case, the system is excited by two near tones of frequencies f_1 and f_2 . A system of order N , will have a response composed of all possible frequencies : $(\pm n_1 f_1 \pm n_2 f_2)$,such that $|n_1| + |n_2| = N$ (n_1,n_2 are integers) As an example,system of order 3 , will have frequency combinations at the output as shown in Fig.(2) .

The order estimation can be done by measuring these possible frequency combinations by a spectrum analyzer of high resolution to distinguish between different intermodulation components , specially for systems of high order .

3 - Bandlimited noise excitation [3]

The broading effect of the nonlinear system is used to estimate its order. Measurement of the relative increase of the output bandwidth compared to that of bandlimited noise input gives an estimate of the system order, (see Fig.(3)).

In some cases,the output spectrum is recorded, and the effective bandwidth B_e is evaluated as :

$$B_e = \sqrt{\int_{-\infty}^{\infty} f^2 S(f)\, df} \Big/ \sigma \quad , \quad S_y(f) \text{ is the psd.} \qquad (1)$$

This measure idicates the system order , for example, a third order system ($y = x^3$) , its effective bandwidth of the output spectrum is higher than the input one by a ratio 1.34 .

4 - Noise-Power-Ratio (NPR) [4]

The noise-loading-method is well known for FDM system characterization. Shown in Fig.(4) , the output power-spectrum-density(psd) measured at a frequency f_o for two cases of input signal:first for a continuous bandlimited input spectrum at the frequency f_o;then with a removed band of the input around the same frequency f_o. Ratio between these two psd in dB is known as the noise-power-ratio NPR given by :

$$NPR(f_o) = 10 \log \left[\frac{\text{output psd at } f_o \text{ for continuous input spectrum}}{\text{output psd at } f_o \text{ for notched input band at } f_o} \right] \qquad (2)$$

The NPR varies with input power level σ^2 as shown in Fig.(5) . The relative change of NPR w.r.t. σ^2 has a maximum value related to the system order N by :

$$- \Delta NPR / (\Delta \sigma^2 \text{ in } dB) = N-1 \qquad (3)$$

This relation has been found by computer simulation in the auther previous study [6],(see Fig.(6)). This method,despite usefulness for system with weak nonlinearity,fails for pure and strong nonlinearities,as the relative change is always zero . This motivates the research for a measure of the system order using the ratio of the relative changes in input and output as reported in this paper. We present a simple method that can be applied to a wider class of nonlinear systems .

III - ORDER ESTIMATION FOR MEMORYLESS NONLINEAR SYSTEM

Memoryless nonlinear system may be characterized (around a chosen operating point) by a power series of finite order N in the form :

$$y(x) = \sum_{n=1}^{N} a_n x^n \qquad (4)$$

To find N , the system response to D.C , sinusoidal and bandlimited Gaussian signals is used as will be seen in the folling :

1 - Using D.C signal excitation

Although the system is of order N ,for small values of input signal x it appears to exhibit lower order than N . The range of these values of x^n considered to be small depends on the magnitude of nonlinear coefficients $\{a_n\}$. If ,for a certain range of input signal ,the nonlinear term $a_n x^n$ is the dominant term , and the other terms is relatively small w.r.t. it ,the corresponding system order will appear to be n, it means we assume that :

$$y \simeq a_n x^n \qquad (5)$$

The change of the output to the input is the derivative of y w.r.t. x,

$$y' \simeq n a_n x^{n-1} \qquad (6)$$

the relative change is

$$y'/y \simeq n / x \qquad (7)$$

then

$$y'/(y/x) \simeq n \tag{8}$$

In the measurement, the small change is denoted by Δx and Δy , let z be the ratio between relative change of output($\Delta y/y$) to relative change of input ($\Delta x/x$), then from eq.(8) we have :

$$z = (\Delta y/y)/(\Delta x/x) \simeq n \tag{9}$$

If in a certain range of input , there are more than one nonlinear dominant terms of approximately the same magnitude , we may assume for this range that :

$$y \simeq a_m x^m + a_\ell x^\ell \quad , \quad \ell > m \tag{10}$$

let

$$a_\ell = \alpha\, a_m \tag{11}$$

then

$$y \simeq a_m (x^m + \alpha x^\ell) \tag{12}$$

and

$$y' \simeq a_m (m x^{m-1} + \ell \alpha x^{\ell-1}) \tag{13}$$

then , ratio of relative changes z will be :

$$z \simeq (m + \alpha \ell x^{\ell-m})/(1 + \alpha x^{\ell-m}) \tag{14}$$

and

$$z \simeq m + (\ell - m)/(1 + 1/\alpha x^{\ell-m}) \tag{15}$$

If the two terms of the same magnitude ,i.e. for :

$$\alpha x^{\ell-m} \simeq 1 \quad \text{or} \quad x \simeq (1/\alpha)^{\frac{1}{\ell-m}} \tag{16}$$

then

$$z \simeq (m + \ell)/2 \tag{17}$$

It is easy to show that the value of z changes from its minimum value m to its maximum value ℓ as x changes from zero to ∞ or as α changes from zero to ∞ as shown in Fig.(7) and Fig.(8) .

So for system described by eq.(10),its order appears to be m at the begingging of a range around x given by eq.(16) and ℓ at the end of that range. It means that :

$$m \leq z \leq \ell \tag{18}$$

Thus it is necessary to measure the relative changes (i.e. the ratio z) at the two limits of operating range of the system under test . These two values of z help us in system identification through the assumption that the system has the lowest order m and the highest order ℓ , i.e. the characteristic will be limited to :

$$y(x) \simeq \sum_{n=m}^{\ell} a_n x^n \tag{19}$$

2 - Using sinusoidal signal excitation

Let the input to the system characterized by eq.(4) , to be :

$$x(t) = A\cos(\omega t) \tag{20}$$

the n^{th} power of x(t) will be :

$$x^n(t) = \left(\frac{A}{2}\right)^n \left(e^{j\omega t} + e^{-j\omega t}\right)^n$$

$$= \left(\frac{A}{2}\right)^n \sum_{i=0}^{n} \binom{n}{i} e^{j(n-2i)\omega t} \tag{21}$$

The first harmonic component of that term is found for $n-2i=\pm 1$, it is taken only from odd values of n,then from eq.(21), the total amplitude of first harmonic component B_1 at the output of the nonlinear system will be :

$$B_1 = \sum_{n=1,3,..}^{N} d_n a_n A^n , \quad n .. odd \tag{22}$$

where

$$d_n = n! / \left(\frac{n-1}{2}\right)! \left(\frac{n+1}{2}\right)! \, 2^{n-1} \tag{23}$$

Eq.(22) is a power series of odd terms,then we can apply the same method for evaluation the ratio z of the two relative changes , but the measured parameter here is the amplitude of the first harmonic instead of the total output value for the d.c input case. If we suppose that the dominant term of eq.(22) is the n^{th} order one, then let :

$$B_1 \simeq d_n a_n A^n \tag{24}$$

then

$$(\Delta B_1/B_1) \simeq n(\Delta A/A) \tag{25}$$

and

$$z_1 \simeq n \tag{26}$$

The even order terms do not contribute to the first harmonic,so for an even order system the value of z will indicate a lower order by 1 . To check the exact order we have to measure the amplitude of the second harmonic component or the d.c component if it is possible,i.e. for $z_1=n_1$ (n_1 odd), then for the second harmonic B_2 :

$$z_2 = (\Delta B_2 / B_2)/(\Delta A/A) \simeq n_1 + 1 \qquad (27)$$

and for the d.c component B_o :

$$z_o = (\Delta B_o / B_o)/(\Delta A/A) \simeq n_1 + 1 \qquad (28)$$

If both the d.c and the second harmonic components are not possible to be measured , we propose to multiply the input by the output scaled by factors α_1, α_2 as shown in Fig.(9) , the output y_{+1} will be :

$$y_{+1} = (\alpha_1 x)(\alpha_2 y) = \alpha_1 \alpha_2 \sum_{n=1}^{N} a_n x^{n+1} \qquad (29)$$

For this system , the amplitude of first harmonic component C_1 will be:

$$C_1 = \alpha_1 \alpha_2 \sum_{n=2,4,..}^{N} d_{n+1} a_n A^{n+1} \quad , n.. \text{ even} \qquad (30)$$

and for this case the value of z will be :

$$z_{c_1} = (\Delta C_1 / C_1)/(\Delta A/A) \simeq n_1 + 2 \qquad (31)$$

For example, a 4^{th} order system, will have $z_1=3$, $z_o=z_2=4$, and $z_{c1}=5$ and for a third order system ; $z_1=3$, $z_o=z_2=2$, and $z_{c1}=3$.

3 - Using bandlimited Gaussian signal excitation

For many reasons it is better for some systems to be excited by a band-limited Gaussian process. Let x(t) be Gaussian with variance σ^2,the output y of a system characterized by eq.(4) will be expressed as [5] ,

$$y = \sum_{k=0}^{N} b_k H_k(x/\sigma) \qquad (32)$$

where $H_k(x)$ is the Hermite polynomial of order k , b_k is related to the coefficient $\{a_n\}$ by :

$$b_k = \sum_{m=0}^{L} (k+2m)!\, a_{k+2m} \sigma^{k+2m} / k!\, m!\, 2^m \qquad (33)$$

where

$$L = \text{integer}\left(\frac{N-k}{2}\right) \tag{34}$$

The output power spectral density is related to input one by [6] :

$$S_y(f) = \frac{1}{F}\sum_{k=0}^{N} b_k^2\, k!\, \gamma^{*k}(f/F) \tag{35}$$

where F is the cut-off frequency of the input spectrum,and $\gamma^{*k}(\mu)$ is the k -fold convolution of the normalized input power spectral density at normalized frequency μ = (f/F) .

If the system appears to have an order n for certain input power σ^2, then the dominant term will be the n^{th} one ,so we may assume :

$$S_y(f) \simeq \frac{1}{F}\, b_n^2\, n!\, \gamma^{*n}(f/F) \tag{36}$$

at the same input power,the term b_n will also have a dominant term of order n, this is obtained for m=0 in eq.(33),so

$$S_y(f) \simeq \frac{1}{F}\, n!\, a_n^2\, \sigma^{2n}\, \gamma^{*n}(f/F) \tag{37}$$

The relative changes of output psd and input one,gives the measure z as follows :

$$z = [\Delta S_y(f)/S_y(f)] / (\Delta\sigma^2/\sigma^2) \simeq n \tag{38}$$

Measurement using dB instrument

From the previous,the measure z can be generalized to be defined as the ratio between the relative change of output power to the relative change of input power. This is true in all cases of different excitation signals d.c,sinusoidal and random one .

The order estimated depends on our assumption that the system relates the measured output parameter v to the input parameter u around certain operating point as follows :

$$v \simeq \beta u^n \tag{39}$$

taking log of both sides of eq.(39) for two different values of input parameter around the operating point , we have :

$$\log v_1 \simeq \log\beta + n \log u_1 \tag{40}$$

and

$$\log v_2 \simeq \log \beta + n \log u_2 \tag{41}$$

then

$$n \simeq \log(v_2/v_1) / \log(u_2/u_1) \tag{42}$$

Eq.(42) gives us the possibilty to use the dB scale of the measuring intrument. It gives us directly the order N as ,for example , the corresponding change of output parameter in dB due to a change of input by one dB (see examples 3 and 4) .

IV - ORDER ESTIMATION FOR NONLINEAR SYSTEMS WITH MEMORY

Let us consider the bilinear model shown in Fig.(10) where the system is represented by a memoryless nonlinearity preceded and followed by linear filters H(f) and K(f) respectively . The output power spectrum (psd) is given , analogous to $S_y(f)$ in eq.(35),by :

$$S_y(f) = |K(f)|^2 \sum_{n=0}^{N} \frac{n!}{F} \ell_n^2 \, \eta^{*n}(f/F) \tag{43}$$

The n-fold convolution is done for the filtered noise by H(f).Making the same reasoning following eq.(35),we can apply the same method for estimating the system order .

Now, let us consider the nonlinear systems modelled by the Volterra and Wiener models [7] .The output signal y(t) is given by :

$$y(t) = \sum_{n=1}^{N} G_n[k_n ; x(t)] \tag{44}$$

where the functional $G_n(.,.)$ is given by :

$$G_n[k_n ; x(t)] = \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} k_n(\tau_1, \tau_2, \ldots, \tau_n)\, x(t-\tau_1) \ldots x(t-\tau_n)\, d\tau_1 \ldots d\tau_n \tag{45}$$

and $k_n(\tau_1, \ldots, \tau_n)$ is the model kernel that depends exclusively upon the system parameters and not upon x(t) . Evidently any scaling factor σ for x(t) yields σ^n in $G_n[k_n ; x(t)]$,i.e.

$$G_n[k_n ; \sigma x(t)] = \sigma^n G_n[k_n , x(t)] \tag{46}$$

σ^2 is interpreted as the input power if x(t) is a unit power signal and y(t) may be written as :

$$y(t) = \sum_{n=1}^{N} G_n[k_n; \sigma x(t)] = \sum_{n=1}^{N} \sigma^n G_n[k_n; x(t)] \qquad (47)$$

As expression (47) is a power series of the parameter σ ,we can apply the same method using the corresponding measure z for estimating the system nonlinearity order .

Separability of different output components G_n is insured by choosing x(t) to be white Gaussian process , as usually done for the system identification [7]

V - EXAMPLES

We apply the described approach for 4 examples in the following :

Ex. 1

For the polynomial $y = x + 0.1\,x^2 + 0.01\,x^3$
the measure z is evaluated for both D.C and sinusoidal excitation and plotted in Fig.(11) and Fig.(12) for both cases respectively .

Ex. 2

A TWT with amplitude and phase nonlinearities A(r) and φ (r) respectively (see Fig.(13)). The measure z is plotted in Fig.(14) for the case of sinusoidal excitation.

Ex. 3

A "Linear " amplifier with gain 10 is excited by bandlimited noise with power σ^2 and the output spectrum (psd) is measured. The measure z is displayed in Fig.(15) .

Ex. 4

A squarer $y = 0.1\,x^2$ is excited by a bandlimited noise as in Ex.3,and the output psd is measured . Display of the psd and z is given in Fig.(16).

Comments on results and conclusions

Examination of Fig.(11) to Fig.(16) shows that z, which should approximate the integral order N , varies in quasicontinuous manner . This reflects certain sensitivity aspects to the accuracy of the measuring device .

Sensitivity analysis , noise effects and specific measurement mechanization are considered for future extension of the work .

As to limitations relevant to different excitation signals , the separability of the system nonlinearity is of decisive importance . If the nonlinearity is separable , it is possible -at least in principle - to apply any excitation signal , otherwise suitable excitation is to be utilized .

Finally , it is noteworthy to mention that we gave a method allowing quick determination of the system order but not the identification of the individual coefficients . Identification of these coefficients (see eg. [6,9]) is a different problem that is not aimed at this work .

REFERENCES

[1] D.D.and J.F.Spina, "Sinusoidal analysis and modeling of weakly non-linear circuits " , Van Nostrand , N.Y. 1980 .

[2] T.Meseng , "On the characterization of bandpass nonlinearity by two tone measurements " , IEEE-Trans. on Comm.,Vol.COM-26,no.6,June 78, pp.746-754 .

[3] G.L.Wise, " The effect of a zero memory nonlinearity on the band-limitedness of contaminated Gaussian inputs",IEEE Trans.on Inform. theory , Vol. IT-26,May 1980 , pp. 345-347

[4] Y.L.Kuo, " Noise Loading analysis of a memoryless nonlinearity characterized by a Taylor series of finite order ",IEEE Trans. on Instr. Meas.,Vol.IM-20,no.2,May 1971,pp.95-99.

[5] R.J.Fenton, "Noise loading analysis for a general memoryless system", ibid,Vol.IM-26,no.1,March 1977,pp.61-64 .

[6] A.M.HAMAD, " Analysis de systemes a l'aide de signaux multifrequence", These de docteur-ingenieur,Paris,Orsay 1981 .

[7] M.Schetzen , " The Volterra and Wiener theories of nonlinear systems", John Wiley ,N.Y. 1980.

[8] A.Sorba et al ," Identification of weakly nonlinear systems by pseudo noise loading method ", MELECON 85 , Spain , pp. 83-88 .

[9] A.M.Hamad et al , " Modeling of bandpass nonlinear memoryless systems through spectral measurements ", Fouth National Radio Science Symp., Cairo-86 , paper C4 .

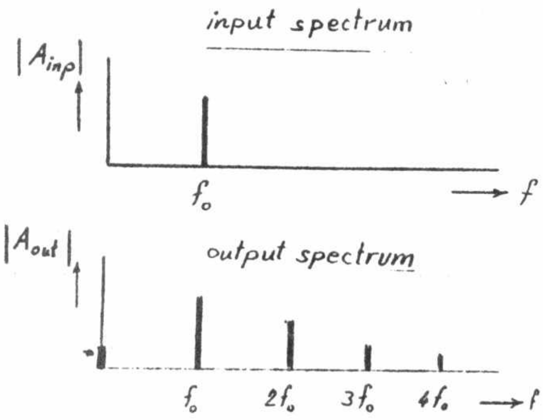

Fig.(1) Harmonic excitation

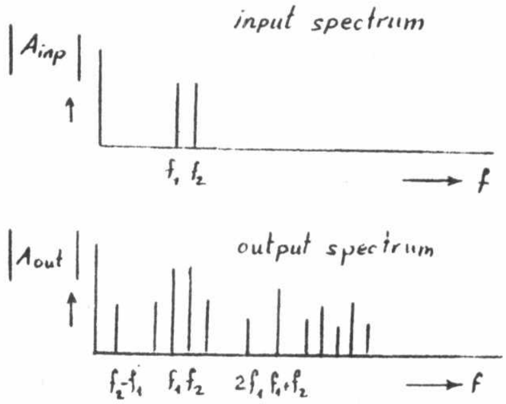

Fig.(2) Two-tone method

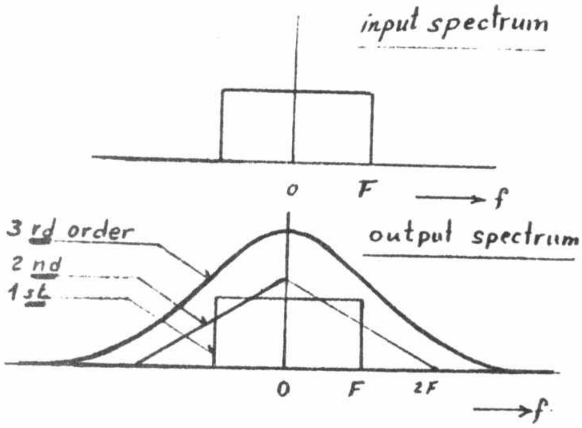

Fig.(3) Bandlimited noise

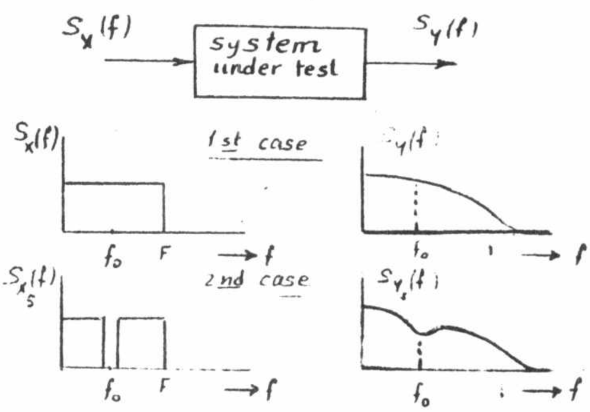

Fig.(4) Noiseloading method

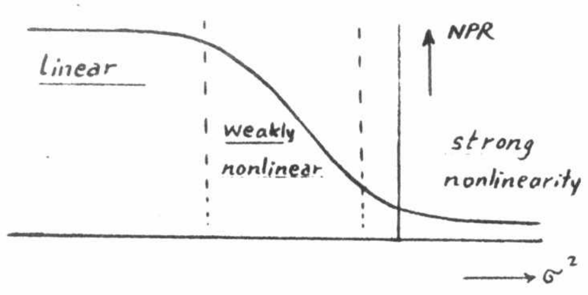

Fig.(5) Noise-power-ratio (NPR)

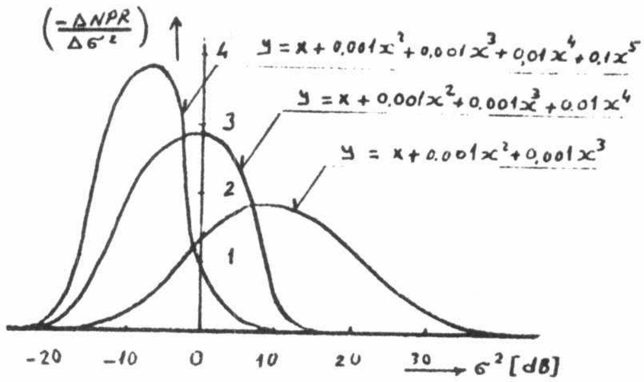

Fig.(6) First derivative of NPR

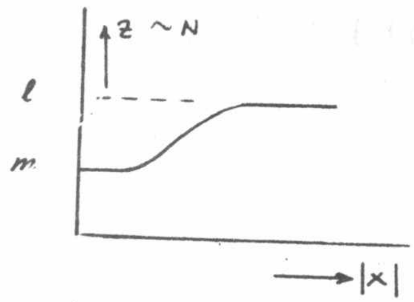

Fig.(7) The measure Z=f(x)

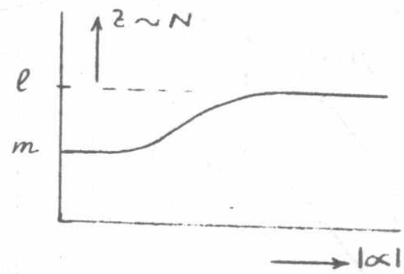

Fig.(8) The measure Z=f(α)

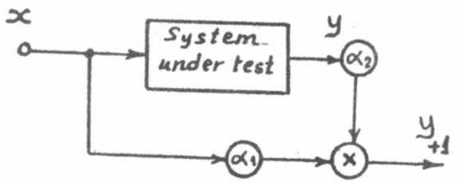

Fig.(9) Increasing the order by one

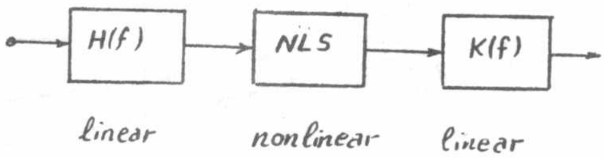

Fig.(10) Bilinear model

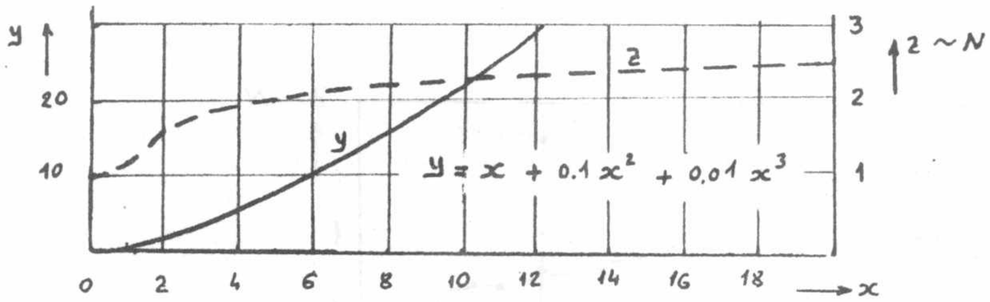

Fig.(11) The measure Z for system excited by d.c signal

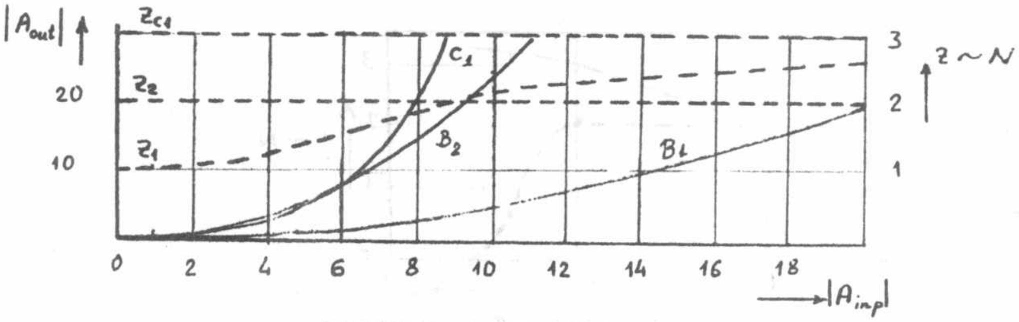

Fig.(12) The measure Z for the above system(sinusoidal excitation)

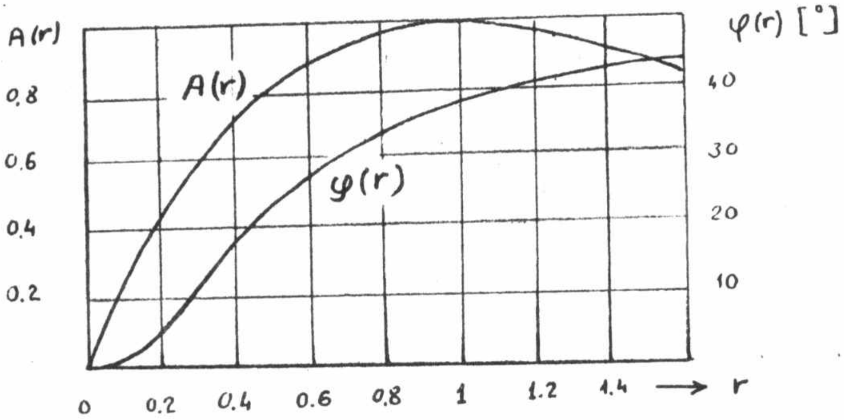

Fig.(13) a TWT characteristics

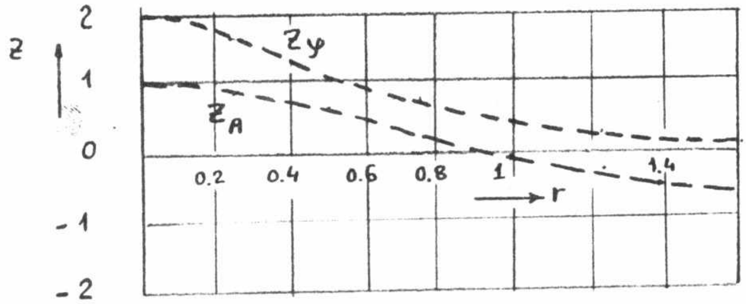

Fig.(14) The measure Z for the above TWT characteristics

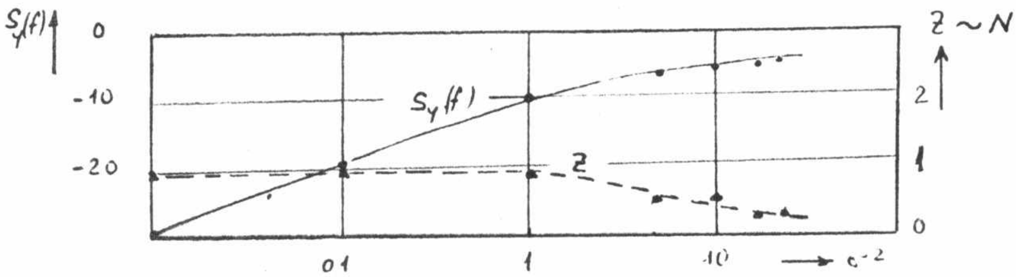

Fig.(15) Power spectral density of a linear amplifier

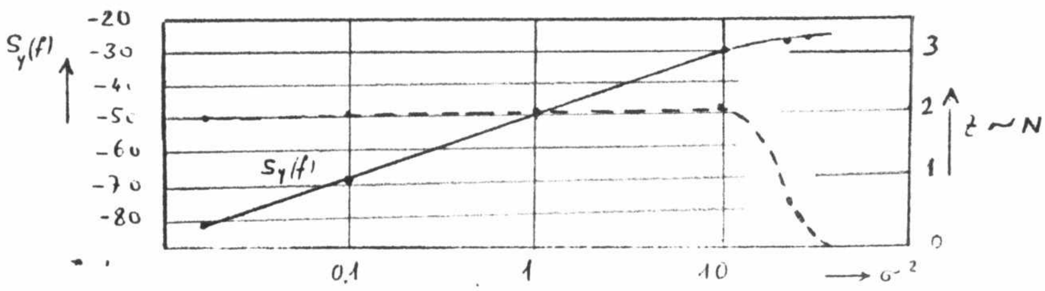

Fig.(16) Power spectral density of a squarer