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FULL-WAVE ANALYSIS OF RECTANGULAR MICROSTRIP STRUCTURE USING POTENTIAL APPROACH IN THE SPECTRAL DOMAIN

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ABSTRACT

An accurate and efficient method is presented for analyzing the characteristics of open rectangular microstrip antenna. The characteristic equation is carried out rigorously using the full-wave analysis rather than the quasi-static approximation. The characteristic equation is derived using Galerkin's method applied in the Fourier transform domain using potential approach.
Some numerical results of this method are compared with the results of other method.

1- INTRODUCTION

An accurate and efficient method was developed for analyzing the characteristics of open microstrip rectangular structures. The boundary value problem assciated with the microstrip structure is formulated in terms of a rigorous hybrid-mode representation. The resulting equations are subsequently transformed, via the application of Galerkin's method in the spectral domain, to yield a characteristic equation for the characteritics of the rectangular microstrip antenna. This method is basically a modification of Galerkin's approach adapted for application in the Fourier transform, or spectral domain. One of the advantages of this approach is that, it is numerically more efficient than the conventional methods, that work directly in the space domain. Another important advantage is that the Green's function takes a much simpler form in the transform domain. It has been reported that the results by a fullwave analysis agree extremely well with these measured at high frequencies [1]. This paper presents a full-wave analysis of the open printed circuit structures such as those encountered in microstrip antennas as an eign-value problem with complex eigen value (resonant frequency).

2. FORMULATION OF THE PROBLEM

Fig. 1 shows the cross-section of the rectangular microstrip antenna. The structure is assumed to be uniform and infinite in both x and z - directions. The infinitely thin patch and ground plane are perfect conductors.

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It is assumed that the substrate material is lossless and it's relative permittivity and permeability are ε_r and U_r, respectively. The nature of the mode of propagation is a hybrid mode, which will be considered in the analysis as a superposition of TE (to Z) and TM(to Z) fields, which may, in turn, be expressed in terms of two types of scalar potentials $\varphi(x,y,z)$ and $\psi(x,y,z)$. For instance :

$$E_{zi}(x,y,z) = K_i^2 \varphi_i + \partial^2\varphi_i/\partial z^2 \quad (1\text{-a})$$

$$H_{zi}(x,y,z) = K_i^2 \psi_i + \partial^2\psi_i/\partial z^2 \quad (1\text{-b})$$

$$E_{xi}(x,y,z) = \partial^2\varphi_i/\partial x\,\partial z - j\omega\mu_i(\partial\psi_i/\partial y) \quad (1\text{-c})$$

$$H_{xi}(x,y,z) = j\omega\varepsilon_i(\partial\varphi_i/\partial y) + \partial^2\psi_i/\partial x\,\partial z \quad (1\text{-d})$$

The subscripts i = 1,2 serve to designate the region 1(substrate) or 2 (air).

$$K_1 = (\varepsilon_r \mu_r)^{1/2} K_0 \qquad K_2 = K_o = (\varepsilon_o \mu_o)$$

$$\varepsilon_1 = \varepsilon_r \varepsilon_o \qquad \varepsilon_2 = \varepsilon_o$$

$$\mu_1 = \mu_r \mu_o \qquad \mu_2 = \mu_o$$

where ω is the operating frequency, ε_o and U_o are the free space permittivity and permeability respectively,
It is possible to derive a set of coupled homogeneous integral equations for the boundary value problem associated with the structure shown in Fig.1 using equations 1(a-d) as well as all the boundary conditions, and to numerically solve these equations to determine the resonant frequency. This approach has been avoided as the solution of such integral equations is numerically prohibitively difficult due the convolution integrals involving slowly convergent Green's functions. Instead, a new method is developed where the boundary value problem associated with the structure is solved in the Fourier transform or the spectral domain.
As, a first step, we define the Fourier transform of the scalar potentials, as

$$\tilde{\varphi}_i(\alpha,y,B) = \int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dz \quad \varphi_i(x,y,z)\exp(j(\alpha x+Bz)) \quad (2\text{-a})$$

$$\tilde{\psi}_i(\alpha,y,B) = \int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dz \quad \psi_i(x,y,z)\exp(j(\alpha x+Bz)) \quad (2\text{-b})$$

where α and B are the Fourier transform variables. The transforms of the field quantities are

$$\tilde{E}_{zi}(\alpha,y,B) = (K_i^2 - B^2)\ \tilde{\varphi}_i(\alpha,y,B) \quad (3\text{-a})$$

$$\tilde{H}_{zi}(\alpha,y,B) = (K_i^2 - B^2)\ \tilde{\psi}_i(\alpha,y,B) \quad (3\text{-b})$$

$$\tilde{E}_{xi}(\alpha,y,B) = -\alpha B\ \tilde{\varphi}_i(\alpha,y,B) - j\omega U_i(\partial\tilde{\Psi}_i(\alpha,y,B)/\partial y) \qquad (3\text{-c})$$

$$\tilde{H}_{xi}(\alpha,y,B) = j\omega\varepsilon_i(\partial\tilde{\varphi}_i(\alpha,y,B)/\partial y) - \alpha B\ \tilde{\Psi}_i(\alpha,y,B) \qquad (3\text{-d})$$

The transform of the scalar potentials satisfy

$$\partial^2\tilde{\varphi}_i(\alpha,y,B)/\partial y^2 - \gamma_i^2\ \tilde{\varphi}_i(\alpha,y,B) = 0 \qquad (4\text{-a})$$

$$\partial^2\tilde{\Psi}_i(\alpha,y,B)/\partial y^2 - \gamma_i^2\ \tilde{\Psi}_i(\alpha,y,B) = 0 \qquad (4\text{-b})$$

where,

$$\gamma_1^2 = \alpha^2 + B^2 - K_1^2 \qquad (5\text{-a})$$

$$\gamma_2^2 = \alpha^2 + B^2 - K_2^2 \qquad (5\text{-b})$$

up to this stage, the formulation presented herein is basically the same as that found in Itoh [1]. The essential difference here is the transform variable α which is discrete in [1].
In order that E_z , E_x and hence, $\tilde{E}_z$ and $\tilde{E}_x$ are zero at y=0 and y= ∞ , the solutions of equations (4) are

$$\tilde{\varphi}_1(\alpha,y,B) = A(\alpha,B)\ \sinh\ \gamma_1 y \qquad (6\text{-a})$$

$$\tilde{\Psi}_1(\alpha,y,B) = Q(\alpha,B)\ \cosh\ \gamma_1 y \qquad (6\text{-b})$$

$$\tilde{\varphi}_2(\alpha,y,B) = C(\alpha,B)\ \exp\ (-\gamma_2(y-d)) \qquad (6\text{-c})$$

$$\tilde{\Psi}_2(\alpha,y,B) = D(\alpha,B)\ \exp\ (-\gamma_2(y-d)) \qquad (6\text{-d})$$

where A, Q, C, and D are unknowns.
The second step is to apply the continuity conditions at the interface y=d in the spectral domain

$$\tilde{E}_{z1}(\alpha,d,B) = \tilde{E}_{z2}(\alpha,d,B) \qquad (7\text{-a})$$

$$\tilde{E}_{x1}(\alpha,d,B) = \tilde{E}_{x2}(\alpha,d,B) \qquad (7\text{-b})$$

$$\tilde{H}_{z1}(\alpha,d,B) - \tilde{H}_{z2}(\alpha,d,B) = -\tilde{J}_x(\alpha,B) \qquad (7\text{-c})$$

$$\tilde{H}_{x1}(\alpha,d,B) - \tilde{H}_{x2}(\alpha,d,B) = \tilde{J}_z(\alpha,B) \qquad (7\text{-d})$$

where, $\tilde{J}_x(\alpha, B)$ and $\tilde{J}_z(\alpha, B)$ are the Fourier transforms of unknown patch current components, $J_x(x,z)$ and $J_z(x,z)$ which exist only on the patch where $|x| < W$ and $|z| < L$. Using equations (3) and (5) in (6) linear algebraic equations could be obtained and the coefficients are expressed in terms of unknowns $\tilde{J}_x$ and $\tilde{J}_z$. The third step is to apply the final boundary conditions,

$$E_x(x,d,z) = E_z(x,d,z) = 0 \;, \quad |x| < W, \; |z| < L \tag{8}$$

in the spectral domain. Defining;

$$E_z(x,d,z) = \begin{cases} 0 & |X| < W, \; |Z| < L \\ u(x,z) & \text{O.W} \end{cases} \tag{9-a}$$

$$E_x(x,d,z) = \begin{cases} 0 & |X| < W, \; |Z| < L \\ V(x,z) & \text{O.W} \end{cases} \tag{9-b}$$

where, u and V are unknowns, substituting equation (3) and (6) into the Fourier transforms of equation (9), another set of algebraic relations between the transformed field quantities and the unknowns A,Q,C and D is obtained. A,Q,C, and D are then eliminated from this set of relations using the relations derived in the second step. After some mathematical manipulations, the following could be obtained

$$\tilde{G}_{11}(\alpha,B,K_o)\,\tilde{J}_x(\alpha,B) + \tilde{G}_{12}(\alpha,B,K_o)\,\tilde{J}_z(\alpha,B) = \tilde{E}_z(\alpha,B) \tag{10-a}$$

$$\tilde{G}_{21}(\alpha,B,K_o)\,\tilde{J}_x(\alpha,B) + \tilde{G}_{22}(\alpha,B,K_o)\,\tilde{J}_z(\alpha,B) = \tilde{E}_x(\alpha,B) \tag{10-b}$$

Where

$$\tilde{G}_{11} = \left(\frac{\alpha B}{K_1^2 - B^2}\, b_{12} - F_1\, B\, b_{11} \right) / \det \tag{11-a}$$

$$\tilde{G}_{12} = -b_{12} / \det \tag{11-b}$$

$$\tilde{G}_{21} = \left[- \frac{\alpha B}{K_1^2 - B^2} \left(\alpha B b_{12} + \omega \mu_2\, \gamma_2\, b_{11} \right) + F_1 \left(\alpha B^2 b_{11} + \omega \mu_2\, \gamma_2\, b_{21} \right) \right] / \det \tag{11-c}$$

$$\tilde{G}_{22} = -\left(\alpha B b_{12} - \omega \mu_2\, \gamma_2\, b_{11} \right) / \det \tag{11-d}$$

and

$$b_{11} = \alpha \left(\frac{K_2^2 - B^2}{K_1^2 - B^2} - 1 \right) \tag{12-a}$$

$$b_{12} = \frac{\omega \mu_1}{B} \frac{\gamma_1}{} \left(\frac{\gamma_2}{\mu_r \gamma_1} + \frac{K_2^2 - B^2}{K_1^2 - B^2} \tanh \gamma_1 d \right) \qquad (12\text{-b})$$

$$b_{21} = \omega \varepsilon_2 \gamma_1 \left(\frac{\gamma_2}{\gamma_1} + \varepsilon_r \frac{K_2^2 - B^2}{K_1^2 - B^2} \coth (\gamma_1 d) \right) \qquad (12\text{-c})$$

$$F_1 = \frac{\omega \mu_1 \gamma_1}{B(K_1^2 - B^2)} \tanh (\gamma_1 d) , \quad \det = B\, b_{11}^2 + b_{12} b_{21} \qquad (12\text{-d})$$

3- METHOD OF SOLUTION

In this section an efficient method for solving the coupled equations (10) is presented. The method is essentially Galerkin's procedure applied in the Fourier transform domain. It is first noted that the two equations (10) actually contain four unknowns $\tilde{J}_x, \tilde{J}_z, \tilde{E}_x$ and $\tilde{E}_z$. However, by using certain properties of these functions, the two unknowns $\tilde{E}_x$ and $\tilde{E}_z$ can be eliminated from these equations in order to solve equation (10) for $\tilde{J}_x$ and $\tilde{J}_z$ only.
The unknown current components $\tilde{J}_x$ and $\tilde{J}_z$ are expanded in terms of the known basis functions $\tilde{J}_{xm}$ and $\tilde{J}_{zn}$ with unknown coefficients c_m and d_n as follows:

$$\tilde{J}_x(\alpha, B) = \sum_{m=1}^{M} c_m \tilde{J}_{xm}(\alpha, B) \qquad (13\text{-a})$$

$$\tilde{J}_z(\alpha, B) = \sum_{n=1}^{N} d_n \tilde{J}_{zn}(\alpha, B) \qquad (13\text{-b})$$

This basis functions $\tilde{J}_{xm}$ and $\tilde{J}_{zn}$ must be chosen such that their inverse Fourier transforms are zero except for the region $|x| < W$ and $|z| < L$. After substituting equation (13) into (10) the inner products with the complex conjugate of the basis functions $\tilde{J}^*_{xp}$ and $\tilde{J}^*_{zp}$ are taken for different values of P. This yields the matrix equation:

$$\sum_{m=1}^{M} K_{pm}^{(1,1)} c_m + \sum_{n=1}^{N} K_{pn}^{(1,2)} d_n = 0, \quad p = 1,2,, \ldots , N \qquad (14\text{-a})$$

$$\sum_{m=1}^{M} K_{pm}^{(2,1)} c_m + \sum_{n=1}^{N} K_{pn}^{(2,2)} d_n = 0, \quad P = 1,2,\ldots,M \qquad (14\text{-b})$$

where, from the definition of the inner products associated with the Fourier transform defined by equation (2), the matrix elements are:

$$K_{pm}^{(1,1)} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \tilde{J}^*_{zp}(\alpha, B)\, \tilde{G}_{11}(\alpha, B)\, \tilde{J}_{xm}(\alpha, B)\, d\alpha\, dB \qquad (15\text{-a})$$

$$K_{pn}^{(1,2)} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \tilde{J}^*_{zp}(\alpha, B)\, \tilde{G}_{12}(\alpha, B)\, \tilde{J}_{zn}(\alpha, B)\, d\alpha\, dB \qquad (15\text{-b})$$

$$K^{(2,1)}_{pm} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \tilde{J}^{*}_{xp}(\alpha,B)\, \tilde{G}_{21}(\alpha,B)\, \tilde{J}_{xm}(\alpha,B)\, d\alpha\, dB \qquad (15\text{-c})$$

$$K^{(2,2)}_{pn} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \tilde{J}^{*}_{xp}(\alpha,B)\, \tilde{G}_{22}(\alpha,B)\tilde{J}_{zn}(\alpha,B)\, d\alpha\, dB \qquad (15\text{-d})$$

One can verify via an application of Parseval's theorem that the right-hand sides of equation (10) are indeed eliminated by this procedure, because the inverse transforms of $\tilde{E}_x, \tilde{E}_z$ and $\tilde{J}_x$, $\tilde{J}_z$ are non zero only in the complementary regions in the (x,z) plane at y=d. Since $K^{(1,1)}_{pm}$, etc., are functions of frequency,a non trivial solution of the simultaneous equations (14) is derived by seeking a complex frequency that makes the determinant of the coefficient matrix of equation (14) zero. The corresponding eign-vectors(c_m, d_n) specifies the current distribution on the patch. Equation (14) is exact if $M=N \rightarrow \infty$. However, in practice, M and N must be finite, and such truncation introduces an approximation. If individual basis functions $\tilde{J}_{xm}$ and $\tilde{J}_{zn}$ are chosen such that their inverse Fourier transforms include qualitative nature of the true unknown current distributions. It is possible to use only a few basis functions to obtain good results, and the computation time can be reduced. Another important feature for time saving is to chose the basis functions which are expressed in closed forms. Although this is not always possible with patchs that have general shapes, in the present rectangular patch one may use $J_{xm}(x,z)$ and $J_{zn}(x,z)$ which reasonably represent qualitative natures of the true components and still whose Fourier transforms are analytically obtainable. Choice of the basis functions have been studied in a number of recent publications [3],[4]. The accuracy of the solution can be systematically improved by increasing the number of basis functions (M + N) and by solving larger size matrix equations.
However, if the first few basis functions are chosen so as to approximate the actual unknown current distribution reasonably well, the necessary size of the matrix can be held small for a given accuracy of the solution.
One possible choice for the dominant mode,J_{z1} and J_{x1} have been chosen as [5]:

$$J_{x1}(x,z) = \frac{1}{w}\left(\sin \frac{\pi x}{w}\right) \cdot \frac{z}{2L^2} \qquad (16\text{-a})$$

$$J_{z1}(x,z) = \frac{1}{2w}\left(1 + \left|\frac{x}{w}\right|^3\right) \cdot \frac{1}{L}\cos\left(\frac{\pi z}{2L}\right) \qquad (16\text{-b})$$

4. NUMERICAL RESULTS

A Fortran program has been made to perform the calculations described above. The integration over the α -B plane has to be done numerically.

The integration involved in equations (15) is carried out on the grounds that the continuous spectrum which belongs to the radiation field from the patch is not expected in the solution, so the eign field has only a discrete spectrum which belongs to the trapped modes. Numerical computations have been carried out for the resonant frequency of open rectangular microstrip antenna using a ICL computer which is too slow such that each matrix element in equation (14) such as K_{pm} has been computed accurately up to four significant digits or better in about one hour. Numerical evaluations have been done with RT-Duriod substrate with $\varepsilon_r = 9.6$ and a thickness of d= 0.64 mm. Table 1.summarizes computed resonant frequencies for different microstrip structures. The results agree extremely well with that using immitance approach as depicted in Fig.2.

$$b_{12} = \frac{\omega\mu_1 \gamma_1}{B}\left(\frac{\gamma_2}{\mu_r \gamma_1} + \frac{K_2^2 - B^2}{K_1^2 - B^2} \tanh \gamma_1 d\right) \quad (12\text{-b})$$

$$b_{21} = \omega \varepsilon_2 \gamma_1\left(\frac{\gamma_2}{\gamma_1} + \varepsilon_r \frac{K_2^2 - B^2}{K_1^2 - B^2} \coth (\gamma_1 d)\right) \quad (12\text{-c})$$

$$F_1 = \frac{\omega\mu_1 \gamma_1}{B(K_1^2 - B^2)} \tanh (\gamma_1 d) \;, \; \det = B\, b_{11}^2 + b_{12}\, b_{21} \quad (12\text{-d})$$

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This basis functions $\tilde{J}_{xm}$ and $\tilde{J}_{zn}$ must be chosen such that their inverse Fourier transforms are zero except for the region $|x| < W$ and $|z| < L$. After substituting equation (13) into (10) the inner products with the complex conjugate of the basis functions $\tilde{J}^*_{xp}$ and $\tilde{J}^*_{zp}$ are taken for different values of P. This yields the matrix equation:

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where, from the definition of the inner products associated with the Fourier transform defined by equation (2), the matrix elements are:

$$K_{pm}^{(1,1)} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \tilde{J}^*_{zp}(\alpha, B)\, \tilde{G}_{11}(\alpha, B)\, \tilde{J}_{xm}(\alpha, B)\, d\alpha\, dB \quad (15\text{-a})$$

$$K_{pn}^{(1,2)} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \tilde{J}^*_{zp}(\alpha, B)\, \tilde{G}_{12}(\alpha, B)\, \tilde{J}_{zn}(\alpha, B)\, d\alpha\, dB \quad (15\text{-b})$$

$$K^{(2,1)}_{pm} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \tilde{J}^{*}_{xp}(\alpha,B)\, \tilde{G}_{21}(\alpha,B)\, \tilde{J}_{xm}(\alpha,B)\, d\alpha\, dB \qquad (15\text{-c})$$

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However, if the first few basis functions are chosen so as to approximate the actual unknown current distribution reasonably well, the necessary size of the matrix can be held small for a given accuracy of the solution.
One possible choice for the dominant mode, J_{z1} and J_{x1} have been chosen as [5]:

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The integration involved in equations (15) is carried out on the grounds that the continuous spectrum which belongs to the radiation field from the patch is not expected in the solution, so the eign field has only a discrete spectrum which belongs to the trapped modes. Numerical computations have been carried out for the resonant frequency of open rectangular microstrip antenna using a ICL computer which is too slow such that each matrix element in equation (14) such as K_{pm} has been computed accurately up to four significant digits or better in about one hour. Numerical evaluations have been done with RT-Duriod substrate with ε_r = 9.6 and a thickness of d= 0.64 mm. Table 1.summarizes computed resonant frequencies for different microstrip structures. The results agree extremely well with that using immitance approach as depicted in Fig.2.

Table 1 Calculated resonant frequency

Structure 2L(cm)	2w(cm)	No.of basis functions Jx	Jz	Resonance freqency (GHZ)
0.45	0.32	1	1	9.52215 + J0.0090114
0.50	0.32	1	1	9.5200 + J0.0187484
0.65	0.32	1	1	9.51044 + J0.020530
0.75	0.32	1	1	9.5099 + J0.0188895
0.80	0.32	1	1	9.50213 + J0.0019434
0.80	0.40	1	1	9.5079 - J0.00059
0.80	0.50	1	1	9.504 - J0.00069
0.80	0.64	1	1	9.5074 - J0.0046
0.80	0.70	1	1	9.497 - J0.0044
0.80	0.80	1	1	9.495 - J0.0052

$\varepsilon_r = 9.6$ d=0.64 mm

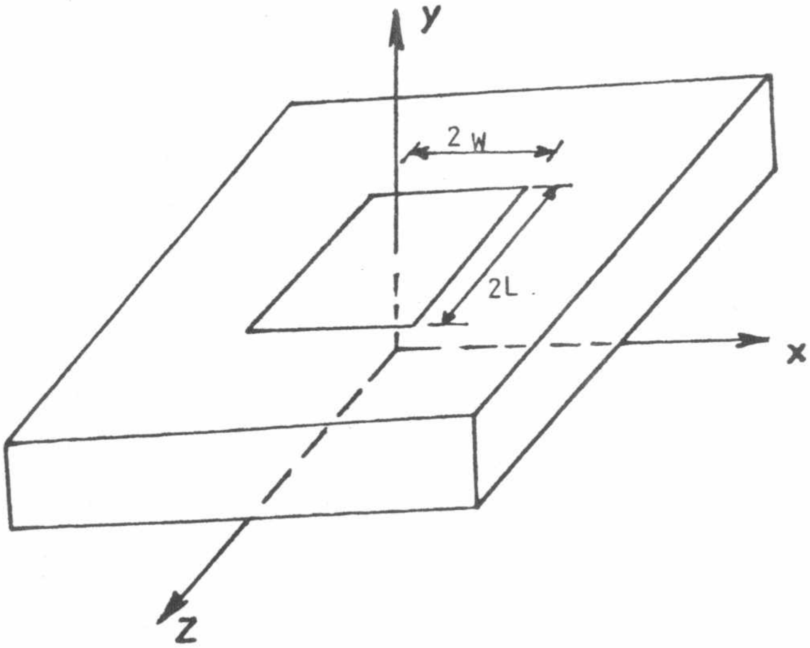

Fig.1 Open microstrip structure

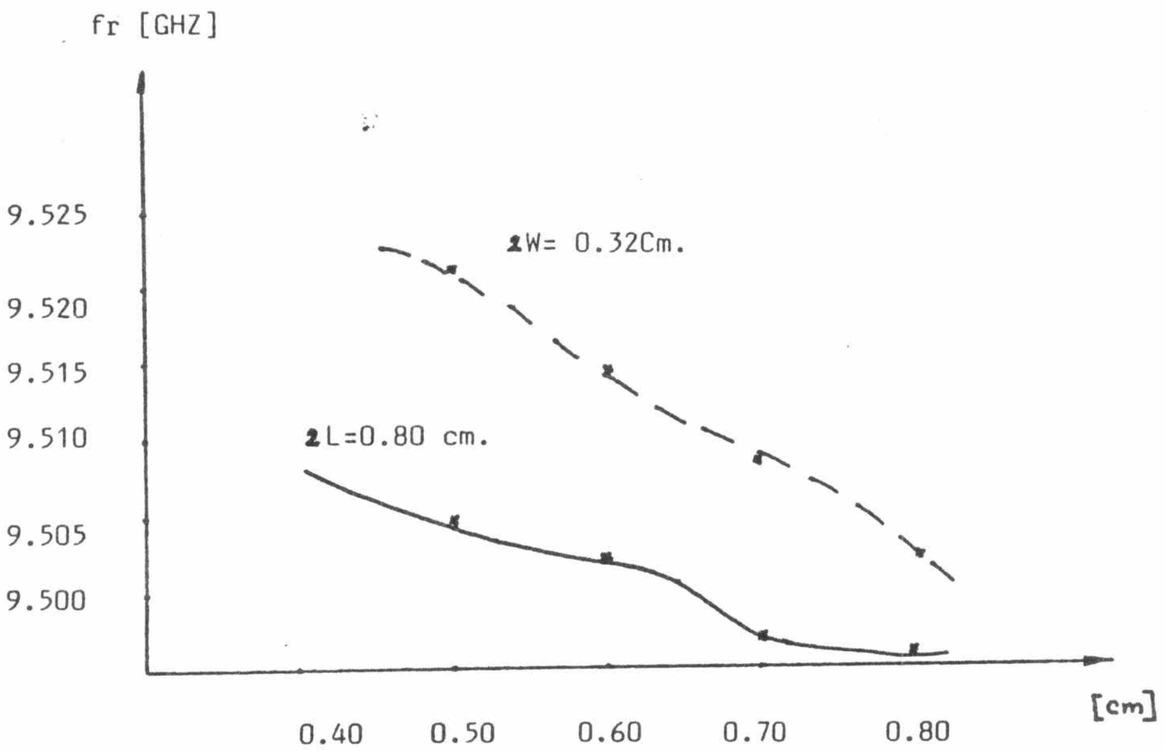

Fig.2. Variation of resonant frequencies as function of patch structure

--- Resonant frequency versus patch Length

—— Resonant freqency versus patch width

x Resonant frequency using imitance method

The imaginary part of the computed resonant frequency is due to the energy lost by radiation from the fringing fields at the open ends, and it is extremely small.

5. CONCLUSION

Full-wave method for analyzing open printed circuit structures is presented. The formulation is based on the rigorous full-wave analysis using the potential approach. The characteristic equation has been obtained via the application of Galerkin's method in the spectral domain.

The full-wave analysis for open microstrip structure was developed by Itoh [5] using imitance matrix approach which was developed recently [6]. The agreement between the results of the two methods is extremely well. This method using the potential approach has attractive feature, that it gives a complete representation of the field components inside and outside the structure.

REFERENCES

[1] Tatsuo Itoh, "Analysis of Micro-strip Resonators," IEEE Trans. Microwave Theory Tech., vol.MTT-22, No.11, PP 946-952. Nov.1974.

[2] T.Itoh, "Spectral-Domain Approach for Calculating the Dispersion Characteristic of Microstrip lines," IEEE Trans. Microwave Theory Tech., PP.496-499, July.1973.

[3] R.H.Jansen, "High-Speed Computation of Single and Coupled and Finite Strip Thickness," IEEE Trans. Microwave Theory Tech., Vol.MTT-26, PP.75-82, Feb. 1978.

[4] E.F.Kuester and D.C.Chang,"An appraisal of Methods for Computation of the Dispersion Characlerlstics of Open Microstrip," IEEE Trans. Microwave Theory Tech., Vol. MTT-27. PP.691-694, July 1979.

[5] T.Itoh, "A full-wave Analysis Method for Open Microstrip Structures," IEEE Trans. Antennas Propagat., Vol. AP-29, No.1, PP.63-67, January 1981.

[6] T.Iton, "Spectral domain imitance approach for dispersion characteristics of shielded microstrip with tuning septums," Presented at the 9th European Microwave Cong. Brigton England, Sept. 17-21, 1979.