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APPLICATION OF KALMAN FILTER
IN DIGITAL IMAGE PROCESSING

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ABSTRACT

An application of Kalman Filter , based on state space techniques is presented to solve the enhancement and the restoration of noisy images degraded by motion in the flight direction .

I . INTRODUCTION

During exposure , the resulting image is degraded due to many factors as the optical abberations , the relative motion between an object and the imaging system , the noises due to sensors , the atmospheric effects ... etc. Recently , the digital image processing technique , especially in the domain of remote sensing has the interest for enhancement and restoration of the spatial and aerial images besides to the image analysis ;

The general approach to deal with a problem such as the blur due to image motion is based on characterizing the motion by means of Point Spread Function (PSF) . That will be followed by a more difficult task to process the degraded image for restoring it to the original one , When the degradation is space-invariant with no noise during

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recording , and when the recorded image extends over an infinite or semifinite interval in the space , the inverse filter in the fourier domain is applicable [1] . If the fourier transform of PSF has zeros at some spatial frequencies then the inverse filtring becomes a poorer restoration technique , especially in the presence of noise . The linear mean square error filters are more effective in the presence of noise . When the motion degradation is space-variant , the restoration using the fourier transform approach cannot be used .

In this paper ; we introduce an application of recursive kalman filter to restore the noisy images degraded by motion in one direction (flight direction) . This approach is based on modeling the motion by a linear discret-time dynamical system using the PSF of the image motion in the absence of noise [2] . By using the exact inverse of blurring model , the degraded image can be restored by the recursive model deduced from that inverse one . The problem of zeros that has been mentioned before does not arise because of utilizing an exact model operating over the finite picture . Similarly the initial states in the inverse model express the boundary conditions .

In the presence of noise , that blurring model is still convenient for obtaining the best linear mean square estimates of the image incorporated with scanner modeling by using KALMAN FILTER . This approach has been applied when using the different modeling functions of scanner to show the model that is more convenient for the real world .

II . BLUR DUE TO IMAGE MOTION

The physical description of the degradation due to image motion for a general linear model in the absence of noise is given by :

$$I_d(x_1,x_2) = \iint_{-\infty}^{\infty} h(x_1,x_2,u_1,u_2).I_o(u_1,u_2)\, du_1 du_2 \qquad (1)$$

where

$I_o(u_1,u_2)$ is the object intensity at the spatial point (u_1,u_2) in object coordinates (U_1 , U_2) .

$I_d(x_1,x_2)$ is the degraded image recorded intensity of that spatial point in image coordinates (X_1 , X_2) .

$h(x_1,x_2,u_1,u_2)$ is the response at (x_1,x_2) to a unit impulse at (u_1,u_2) " Point Spread Function " .

If $h(x_1,x_2,u_1,u_2)$ depends on each of the four spatial variables (x_1,x_2, u_1,u_2) independently then the blurring system is linear space variant ; if $h(x_1,x_2,u_1,u_2)$ depends on the spatial differences (x_1-u_1) , (x_2-u_2) then the system is linear space-invariant and equation (1) becomes :

$$I_d(x_1,x_2) = \iint_{-\infty}^{\infty} h(x_1-u_1, x_2-u_2) . I_o(u_1,u_2)\, du_1 du_2 \qquad (2)$$

For images in which the motion blur occurs in straight line (direction of flight for aerial images) e.g. along u_1 , then

$$I_d(x_1) = \int_R h(x_1-u_1) . I_o(u_1)\, du_1 \qquad (3)$$

where

R is the motion trace along U_1 for space-invariant blurring systems .

The discrete form of the previous equation is :

$$I_d(k) = \sum_{r\in R} h(k-r) . I_o(r) \qquad (4)$$

where

k , r are integer values .

The PSF h(i) can be expressed as :

$$h(i) = c_o\Delta(i) + c_1\Delta(i-1) + \dots\dots\dots + c_m\Delta(i-m) \qquad (5)$$

where

$\Delta(.)$ is an impulse function .

m is the value of PSF h(j) at time j , j= o,1,2,...,m .

$$I_d(k) = (c_o + c_1 D + c_2 D^2 + \ldots\ldots + c_m D^m)\, I_o(k) \qquad (6)$$
$$= H(D) \cdot I_o(k)$$

where D is a delay operator .

By applying Z transform , H(z) of PSF h(i) after replacing D by 1/z will be :

$$H(z) = \left[c_o + \frac{c_1}{z} + \frac{c_2}{z^2} + \ldots\ldots + \frac{c_m}{z^m} \right] \qquad (7)$$

H(z) is a proper rational function if the PSF is causal function . H(z) has a finite number of terms .

III . RESTORATION OF NOISE FREE DEGRADED IMAGE

Firstly , we will obtain a state space model that realizes the motion transfer function H(z) as follows :

$$x(k+1) = A\, x(k) + B\, I_o(k)$$
$$I_d(k) = C\, x(k) + d\, I_d(k) \qquad (8)$$

where

$k = o, 1, \ldots\ldots, N-1$

$x(k)$ is the state vector ; $x(k) \in R^m$.

m is the extent of the blur .

N is the number of pixels contained in line .

A,B,C are constant matrices (mxm),(mxl),(lxm) .

d is a scalar .

(A,B,C,d) represents single-input single-output linear system that realizes the blurring transfer function H(z) , then

$$H(z) = C\,(zI-A)^{-1}B + d \qquad (9)$$

The utilized realization has a canonical structure which will lead to efficient computation . This realization is described for the next two cases [3] :

- For moving objects with background of zero intensity i.e. the initial state vector $x(o) = 0$.

$$W(k+1) = \begin{bmatrix} A & B\gamma \\ 0 & \alpha \end{bmatrix} W(k) + \begin{bmatrix} 0 \\ \beta(k) \end{bmatrix}$$

$$I_d(k) = \begin{bmatrix} C & d\gamma \end{bmatrix} W(k) + V(k)$$

where

$$W^t(k) = \begin{bmatrix} X(k) & Z(k) \end{bmatrix}$$

A recursive estimator has been designed on the basis of the last discrete model to give an estimate $\hat{W}(k)$ of $W(k)$ I I .

$$\hat{W}(k+1) = \left[\bar{A} - F(k)\bar{C}\right]\hat{W}(k) + F(k)\tilde{I}_d(k)$$

$$P(k+1) = \left[\bar{A} - F(k)\bar{C}\right]P(k)\left[\bar{A} - F(k)\bar{C}\right]^t + BK\bar{B}^t + F(k)RF^t(k)$$

$$F(k) = \bar{A}P(k)\bar{C}^t\left[\bar{C}p(k)\bar{C}^t + R\right]^{-1} \qquad (13)$$

where

$$\bar{A} = \begin{bmatrix} A & B\gamma \\ 0 & \alpha \end{bmatrix}; \quad \bar{B} = \begin{bmatrix} 0 \\ \beta \end{bmatrix}; \quad \bar{C} = \begin{bmatrix} C & D\gamma \end{bmatrix}$$

The initial conditions $P(0)$ and $W(0)$ are considered as apart of available data that describe the operation of the estimator .

V . AUTOCORRELATION OF SCANNER OUTPUT

To consider the realistic model of a scanner and because of the periodic nature of its operation , the random process of both image and noise are nonstationary . The determination of such process for the dynamic model is very complicated . The algorithm for the case of stationary process has been calculated [8] , [9] . The scanner output is obtained by scanning horizontally (variable z) and vertically (integer number n) . z is a variable that denotes the position of the scanning spot on the horizontal scanning line and verifies the relation $0 \leq z \leq Z$. The vertical variable $n = 1, 2, \ldots, N$ represents the n^{th} scanning line . The brightness function is defined by $b(z,n)$ that has zero mean . The case of an image 32x32

- For moving objects with background of nonzero intensity i.e. the initial state vector $X(0)$ is nonzero .

Our approach to the noise free restoration problem is to obtain another linear system A, B, C, d that is an exact inverse to blurring system (A, B,C,d) [4][5] .

IV . RESTITUTION OF NOISY DEGRADED IMAGES

During the exposure , the noise degradation is assumed to be additive white noise (zero mean and known variance) added to the motion degraded image . A model that describes both motion and noise degradation is given by :

$$
\begin{aligned}
X(k+1) &= A\,X(k) + B\,I_o(k) \\
\tilde{I}_d(k) &= C\,X(K) + d\,I_o(k) + V(k) \qquad (10) \\
EV(k) &= 0 \\
EV(k_1)V(k_2) &= R\,\Delta(k_1 - k_2) \quad ; \quad k=0,1,\ldots , N-1
\end{aligned}
$$

where

$\Delta(.)$ is an impulse function

$\tilde{I}_d(k)$ is the degraded image due to both noise and motion .

R is the given variance .

Given the correlation function of the line scanner and the original image , a difference model given by white noise can be developed [6] . Such model is given by next equations :

$$
\begin{aligned}
Z(k+1) &= \alpha\, Z(k) + \beta\, U(k) \\
I_o(k) &= \gamma\, Z(k) \qquad (11) \\
EU(k) &= 0 \\
EU(k_1)\acute{U}(k_2) &= K\,\Delta(k_1-k_2)
\end{aligned}
$$

The two models represented by (10) & (11) can be augmented in the following model :

pixels of zero mean that has a square 12x12 pixels of gray level 6.1 surrounded by background grey level -1 will represent the object intensity . The image is a sample function of two dimensional random process with autocorrelation function :

$$R_I(z,i) = 6.1\ e^{-4.35\,|z|-0.136|i|}$$

Because of great difficulties to define the random process exactly also we have introduced the simple representation which considers the scanner as just a delay system .

VI . APPLICATION OF KALMAN FILTER

Considering the image 32x32 that has been described of a simple model of a scanner of the following representation :

$$Z(k+1) = \alpha Z(k) + \beta U(k)$$
$$I_o(k) = Z(k)$$

Then the augmented model is deduced by using equations (12) , also the kalman filter has been defined by using equations (13) .

Figure 1 is the original image while figure 2 shows the degraded one . The restored image shown in figure 3 is the result using the exact inverse model . The noisy degraded image is shown in figure 4 . During the processing , the variance of the original image , simulteneously with the variance of noise are changed . The results shown in figure 5 represent the restored images for the different cases .

Secondly , we shall use the stationary model of scanner that has been discussed before . This model is represented by differential equation of 5^{th} order [10] , and its numerical results are given by :

$$\alpha = \begin{bmatrix} 0.996 & 0 & 0 & 0 & 0 \\ 0 & 0.983 & 0.031 & 0 & 0 \\ 0 & -1.22 & 0.97 & 0 & 0 \\ 0 & 0 & 0 & 0.926 & 0.03 \\ 0 & 0 & 0 & -4.77 & 0.913 \end{bmatrix}$$

$$\beta k \beta^{T} = \begin{bmatrix} 0.02 & 0 & 0 & 0 & 0 \\ 0 & 0.02 & 0.12 & 0 & 0 \\ 0 & 0.12 & 0.6 & 0 & 0 \\ 0 & 0 & 0 & 0.01 & 0.07 \\ 0 & 0 & 0 & 0.07 & 0.49 \end{bmatrix}$$

$$\gamma = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 \end{bmatrix}$$

Figure 6 shows the noisy degraded image while figure 7 is the restored one.

VII . CONCLUSION

The rôle of recursive linear kalman filtering in image processing either enhancement or restoration has been established . The procedure is applicable to those characterized by mean and correlation functions .

This paper has introduced the application on any image just of zero mean that can be practically realized .

Due to the lack of precise knowledge of the image and/or the noise correlation functions , the introduced processing permits the changing of the variance for each image and noise in order to obtain the optimum result .

Due to the simpler representation , the required time for such processing is comparable with that required in the case of processing for defined correlation functions .

Finally , application of such algorithm on real degraded images will manifist the potential of such processing in enhancement and restoration using kalman filters .

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-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0
-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0
-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0
-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0
-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0
-1.0	-1.0	-1.0	-1.0	6.1	6.1	6.1	6.1	6.1	6.1	6.1	6.1	6.1	6.1	6.1	6.1	-1.0	-1.0	-1.0	-1.0
-1.0	-1.0	-1.0	-1.0	6.1	6.1	6.1	6.1	6.1	6.1	6.1	6.1	6.1	6.1	6.1	6.1	-1.0	-1.0	-1.0	-1.0
-1.0	-1.0	-1.0	-1.0	6.1	6.1	6.1	6.1	6.1	6.1	6.1	6.1	6.1	6.1	6.1	6.1	-1.0	-1.0	-1.0	-1.0
-1.0	-1.0	-1.0	-1.0	6.1	6.1	6.1	6.1	6.1	6.1	6.1	6.1	6.1	6.1	6.1	6.1	-1.0	-1.0	-1.0	-1.0
-1.0	-1.0	-1.0	-1.0	6.1	6.1	6.1	6.1	6.1	6.1	6.1	6.1	6.1	6.1	6.1	6.1	-1.0	-1.0	-1.0	-1.0
-1.0	-1.0	-1.0	-1.0	6.1	6.1	6.1	6.1	6.1	6.1	6.1	6.1	6.1	6.1	6.1	6.1	-1.0	-1.0	-1.0	-1.0
-1.0	-1.0	-1.0	-1.0	6.1	6.1	6.1	6.1	6.1	6.1	6.1	6.1	6.1	6.1	6.1	6.1	-1.0	-1.0	-1.0	-1.0
-1.0	-1.0	-1.0	-1.0	6.1	6.1	6.1	6.1	6.1	6.1	6.1	6.1	6.1	6.1	6.1	6.1	-1.0	-1.0	-1.0	-1.0
-1.0	-1.0	-1.0	-1.0	6.1	6.1	6.1	6.1	6.1	6.1	6.1	6.1	6.1	6.1	6.1	6.1	-1.0	-1.0	-1.0	-1.0
-1.0	-1.0	-1.0	-1.0	6.1	6.1	6.1	6.1	6.1	6.1	6.1	6.1	6.1	6.1	6.1	6.1	-1.0	-1.0	-1.0	-1.0
-1.0	-1.0	-1.0	-1.0	6.1	6.1	6.1	6.1	6.1	6.1	6.1	6.1	6.1	6.1	6.1	6.1	-1.0	-1.0	-1.0	-1.0
-1.0	-1.0	-1.0	-1.0	6.1	6.1	6.1	6.1	6.1	6.1	6.1	6.1	6.1	6.1	6.1	6.1	-1.0	-1.0	-1.0	-1.0
-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0
-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0
-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0
-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0
-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0

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Figure 1 **The model of the original image**

-1.2	-1.2	-1.2	-1.2	-1.2	-1.2	-1.2	-1.2	-1.2	-1.2	-1.2	-1.2	-1.2	-1.2	-1.2	-1.2	-1.2	-1.2	-1.2	-1.2
-1.2	-1.2	-1.2	-1.2	-1.2	-1.2	-1.2	-1.2	-1.2	-1.2	-1.2	-1.2	-1.2	-1.2	-1.2	-1.2	-1.2	-1.2	-1.2	-1.2
-1.2	-1.2	-1.2	-1.2	-1.2	-1.2	-1.2	-1.2	-1.2	-1.2	-1.2	-1.2	-1.2	-1.2	-1.2	-1.2	-1.2	-1.2	-1.2	-1.2
-1.2	-1.2	-1.2	-1.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	-1.2	-1.2	-1.2	-1.2
-1.2	-1.2	-1.2	-1.2	1.6	1.6	1.6	1.6	1.6	1.6	1.6	1.6	1.6	1.6	1.6	1.6	-1.2	-1.2	-1.2	-1.2
-1.2	-1.2	-1.2	-1.2	3.1	3.1	3.1	3.1	3.1	3.1	3.1	3.1	3.1	3.1	3.1	3.1	-1.2	-1.2	-1.2	-1.2
-1.2	-1.2	-1.2	-1.2	4.5	4.5	4.5	4.5	4.5	4.5	4.5	4.5	4.5	4.5	4.5	4.5	-1.2	-1.2	-1.2	-1.2
-1.2	-1.2	-1.2	-1.2	5.9	5.9	5.9	5.9	5.9	5.9	5.9	5.9	5.9	5.9	5.9	5.9	-1.2	-1.2	-1.2	-1.2
-1.2	-1.2	-1.2	-1.2	7.3	7.3	7.3	7.3	7.3	7.3	7.3	7.3	7.3	7.3	7.3	7.3	-1.2	-1.2	-1.2	-1.2
-1.2	-1.2	-1.2	-1.2	7.3	7.3	7.3	7.3	7.3	7.3	7.3	7.3	7.3	7.3	7.3	7.3	-1.2	-1.2	-1.2	-1.2
-1.2	-1.2	-1.2	-1.2	7.3	7.3	7.3	7.3	7.3	7.3	7.3	7.3	7.3	7.3	7.3	7.3	-1.2	-1.2	-1.2	-1.2
-1.2	-1.2	-1.2	-1.2	7.3	7.3	7.3	7.3	7.3	7.3	7.3	7.3	7.3	7.3	7.3	7.3	-1.2	-1.2	-1.2	-1.2
-1.2	-1.2	-1.2	-1.2	7.3	7.3	7.3	7.3	7.3	7.3	7.3	7.3	7.3	7.3	7.3	7.3	-1.2	-1.2	-1.2	-1.2
-1.2	-1.2	-1.2	-1.2	7.3	7.3	7.3	7.3	7.3	7.3	7.3	7.3	7.3	7.3	7.3	7.3	-1.2	-1.2	-1.2	-1.2
-1.2	-1.2	-1.2	-1.2	7.3	7.3	7.3	7.3	7.3	7.3	7.3	7.3	7.3	7.3	7.3	7.3	-1.2	-1.2	-1.2	-1.2
-1.2	-1.2	-1.2	-1.2	5.9	5.9	5.9	5.9	5.9	5.9	5.9	5.9	5.9	5.9	5.9	5.9	-1.2	-1.2	-1.2	-1.2
-1.2	-1.2	-1.2	-1.2	4.5	4.5	4.5	4.5	4.5	4.5	4.5	4.5	4.5	4.5	4.5	4.5	-1.2	-1.2	-1.2	-1.2
-1.2	-1.2	-1.2	-1.2	3.1	3.1	3.1	3.1	3.1	3.1	3.1	3.1	3.1	3.1	3.1	3.1	-1.2	-1.2	-1.2	-1.2
-1.2	-1.2	-1.2	-1.2	1.6	1.6	1.6	1.6	1.6	1.6	1.6	1.6	1.6	1.6	1.6	1.6	-1.2	-1.2	-1.2	-1.2
-1.2	-1.2	-1.2	-1.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	-1.2	-1.2	-1.2	-1.2
-1.2	-1.2	-1.2	-1.2	-1.2	-1.2	-1.2	-1.2	-1.2	-1.2	-1.2	-1.2	-1.2	-1.2	-1.2	-1.2	-1.2	-1.2	-1.2	-1.2
-1.2	-1.2	-1.2	-1.2	-1.2	-1.2	-1.2	-1.2	-1.2	-1.2	-1.2	-1.2	-1.2	-1.2	-1.2	-1.2	-1.2	-1.2	-1.2	-1.2
-1.2	-1.2	-1.2	-1.2	-1.2	-1.2	-1.2	-1.2	-1.2	-1.2	-1.2	-1.2	-1.2	-1.2	-1.2	-1.2	-1.2	-1.2	-1.2	-1.2

```
•—+N**********W+—•
•—+N**********W+—•
•—+N**********W+—•
•—+N**********W+—•
•—+N**********W+—•
•—+N**********W+—•
•—+N**********W+—•
•—+N**********W+—•
•—+N**********W+—•
•—+N**********W+—•
•—+N**********W+—•
•—+N**********W+—•
```

Figure 2 **The degraded image of the model**

```
-0.6-2.2-2.4 0.6-1.9-3.0-0.5-1.1 0.0-1.1-2.0-3.2-2.1-2.5-5.0 0.6 2.2 0.6 3.9-0.4
-0.9-2.0-1.7-0.9-0.0-0.1-1.1-1.5-0.7-1.8-0.4-2.1-0.3 0.2-1.2-2.8-2.9-4.2-3.4-4.0
 2.4-2.9-0.9-4.3-3.0 0.2-3.0-4.1-1.0 0.0 0.5-0.9-1.0-3.8-1.0-0.2-2.8-2.7-2.3 0.8
-2.2-1.4-1.7-1.2-2.0-1.0-2.7-2.3-2.0-0.2-1.2-3.4-5.0-1.1-0.7-3.0-3.8 1.1-0.3 1.0
 0.2 0.0-0.0 0.4-0.7-1.7-0.9 2.9 2.7 1.3 2.3 2.9-2.2 1.9 3.9-1.2-2.9-2.7 0.1-1.3
-0.7 2.7-0.0-0.3-0.3 1.8-1.0 0.2 1.2-0.5 2.3 1.0 0.2-1.7 2.9 4.5 1.1 0.1 3.2 0.7
-0.5 1.0 2.7-1.9 3.2 1.9 3.5 4.0 2.6 0.5 3.2 2.8 2.8 2.1 2.0 5.1-0.7-1.2-1.4-3.0
-2.0-1.0 1.7 0.2 4.0 0.2 2.0 3.2 0.0 3.7 0.2 3.8 5.1 3.0 5.0 3.1-2.0 1.8 1.0-4.8
-4.7-0.0-0.2-2.3 4.0 2.5 5.2 4.7 4.0 9.2 3.7 5.0 7.4 3.511.010.7-0.8-3.5-4.2-3.3
-0.5-2.0-0.7 1.7 9.0 0.0 0.5 6.2 8.3 7.111.0 7.0 3.8 4.010.5 9.0-2.7-0.2 0.0-0.5
-1.1-1.4 0.5 2.0 0.0 7.7 7.4 3.0 5.2 7.7 0.2 8.8 7.5 7.1 7.0 5.8-0.8-2.3 1.7-1.7
-0.0-5.9-0.7-1.4 0.0 0.1 5.3 2.8 5.8 7.5 8.2 6.712.1 5.0 2.3 3.2-2.1-0.5-0.0-3.5
-1.0-0.5-5.2-1.3 9.0 0.7 5.1 7.012.2 9.7 7.1 0.7 7.8 0.0 4.3 7.2-1.5-1.7-2.7-2.4
-2.3-4.4-1.0-1.0 9.4 2.7 5.1 7.1 7.3 4.010.2 7.2 0.9 4.5 0.4 0.3-1.8-1.5-4.8-0.0
 0.7-2.2 2.5 2.3 9.9 0.8 9.0 2.7 7.4 5.0 3.9 0.3 7.2 0.0 3.0 9.3-1.4-1.4-0.7-0.9
-2.3-0.2-3.2 0.3 0.5 0.0 4.7 6.1 7.0 0.9 9.010.1 0.0 0.4 7.9 0.4-0.4-3.0-4.9-1.0
-3.4-0.7 0.3-0.9 4.4 7.2 2.4 3.5 4.0 2.4 2.3 0.0 8.5 4.9 0.4 0.2-1.3 2.3-3.9-0.0
-2.9-3.0-0.7-2.1 0.5 0.9 3.4 7.3 5.2 4.2 3.2 4.3 5.9 4.9 1.0 0.7-1.0-3.0 2.0-2.3
 0.4-3.3-0.0-1.2 4.1 0.0 2.4 3.0 5.8 3.5-0.0 1.7 1.8 0.3 3.9 3.3-2.2 2.0 1.2 0.2
-1.1 0.2 0.4-2.4 0.0-0.9 2.2 3.0 2.0-2.1-0.7 2.0-0.0 0.5-0.0 0.3-1.1-0.0-0.0-0.3
-3.4-2.2-3.0-0.3-0.3 1.4 2.3 3.5-0.0 2.0 2.0-0.4 0.2 0.4-0.3-2.1-1.1-3.0-1.5-1.0
-0.2-1.7 0.0-1.1-1.9-0.2 0.1-0.0-2.0-0.0-1.0-3.4-2.9-0.5-0.2-2.5 1.1-2.8-1.5-0.9
```

Figure 4 The noisy degraded image of the model (noise variance = 2)

```
 1.7-0.1-1.0-1.0 0.0-0.1-2.8-0.2-0.7 0.1 0.1-0.8-1.8-1.7-1.4-4.2 0.0 2.8-0.6 4.0
-0.0-1.2-1.2-1.4-0.9 1.1-0.6-0.1-1.6-0.5-0.9-0.3-1.6-0.8 0.0-1.6-2.3-1.9-3.7-1.1
-1.2 1.2-1.0-0.2-3.4-1.5 0.6-1.7-3.2-1.4 0.0 0.3-0.4-0.7-2.3-0.2-0.7-3.1-2.5-2.1
 0.7-1.7-0.4-0.5-1.0-2.1-0.7-1.9-1.6-2.0 0.1-0.9-1.9-3.6-0.0 0.5-2.1-3.3 1.6-0.0
-1.5-0.2 0.3 0.0 1.2-0.0-1.0-0.3 3.2 2.3 1.1 1.0 2.7-1.5 2.1 4.0-0.6-1.9-0.5 0.3
 0.0-0.2 2.0-4.1 0.0 0.3 1.5-0.0 1.7 2.0-0.2 2.1 1.9 1.2-0.4 3.0 4.3 2.1 0.7 2.9
-1.0-0.2 1.7 1.9-0.9 3.1 1.9 3.3 2.3 2.2 4.7 2.4 2.2 3.4 1.0 1.4 5.1 1.3-0.5-0.4
-1.5-1.8-1.7 2.5 0.0 3.0 0.3 2.9 3.7 4.9 3.1 5.0 2.4 4.7 2.9 2.4 2.3-0.9 1.2 0.2
-0.0-3.5-1.0 0.4-1.0 3.4 2.0 3.7 2.5 3.0 0.1 2.2 3.5 5.5 2.4 7.1 0.0-1.0-2.0-3.7
 0.5-0.2-2.5-0.9 1.1 0.3 4.0 4.0 3.0 3.1 4.4 0.4 4.8 1.6 2.3 7.2 3.9-2.3-0.7-0.3
-1.9 0.3-1.0 0.0 2.1 3.2 4.3 4.2 1.2 2.2 3.0 3.2 5.4 3.2 4.3 3.9 2.1-0.7-1.4 1.9
-0.0 0.5-3.0-0.9-0.1 2.0 4.1 2.2 3.4 2.5 3.2 3.7 3.4 7.3 3.3-1.3-0.0-0.9 0.0 0.5
-3.1-0.0 0.3-4.1-1.3 4.9 4.4 1.7 4.7 7.7 5.0 3.1 4.1 4.5 3.0-0.2 2.0-0.4-0.0-1.9
-1.7-1.7-1.9-1.3-1.1 5.3-0.0 2.0 4.3 4.7 1.7 5.0 3.2 2.0 1.7 3.0 5.1-0.7-0.0-4.0
 0.2 0.4-0.9 3.0 1.0 5.0 2.5 5.4 3.4 3.3 1.5 1.2 4.0 2.8 3.2 4.2 0.0-0.0-0.8-0.8
 1.7-1.3 0.7-0.9 0.9 2.0 5.1 2.0 2.5 2.0 5.0 4.4 3.7 2.0 3.5 5.7 3.7 0.1-2.0-2.7
-2.0-2.4 0.3 0.4-3.0 0.3 4.5 0.1 0.4 0.4 0.0 0.1 2.8 4.7 2.4 5.5 2.2-0.0 1.9-2.1
-1.4-2.0-2.0-3.4-1.9 2.3 3.7 0.5 3.5 1.7 1.2 0.3 1.1 3.1 2.2-1.2 2.3-1.1-2.0 2.0
-2.1 1.6-3.0-0.3-1.3 1.0-1.4 0.5 1.7 3.3 1.3-1.0-1.1-0.5-1.0 1.3 0.3-1.5 1.4 2.5
-2.7 3.4 0.3 0.2-1.0-0.7-3.4 0.0 1.0 0.8-2.7-1.4 0.1-2.6-1.3-2.3-1.7-0.0 0.5-0.0
-0.0-2.0-0.0-2.1 0.3-1.3-0.0 1.4 1.0-1.7 0.3 1.4-1.0-1.3-0.0-1.7-2.0-0.3-2.3-1.0
 0.0-0.2-0.0 4.3 0.1-2.0 0.1-0.1-1.5-3.2-0.3-0.5-2.8-2.4-0.4-0.2-2.3 1.5-2.5-2.0
```

Figure 5 The restored image using the simple model (for choosed α = 0.8) .

```
-5.4-4.0 -.6-3.0 -.0 -.2 1.0-1.4 -.8 -.8 -.3 -.6-4.1-3.6-4.0-2.0-2.8-5.2-5.9
-6.1-4.4-1.4-4.9-1.9-1.1 1.1  .5  .4  .7 1.9 -.1-2.9-2.2-1.0  .2-1.0-5.9-6.1
-7.8-4.1-3.3-6.3-1.9-1.6 2.5 1.7 2.8 3.5 3.8  .1 1.5 -.1  .9 3.0 1.2-5.6-4.5
-8.6-3.1-5.3-8.1-2.1 -.1 3.4 3.9 5.8 5.5 4.3 2.1 3.8 2.4 4.7 5.6 2.4-4.3-4.4
-9.8-2.5-6.2-6.4 1.4 3.0 5.9 9.2 9.5 9.9 8.0 6.5 6.9 4.8 8.5 6.9 4.1-2.7-1.9
-9.9-5.3-4.7-5.7 2.3 6.6 9.914.613.612.211.4 9.4 9.3 8.713.3 9.8 3.9-2.3 -.6
-7.7-4.5-5.2-4.8 4.6 9.613.518.017.816.116.613.412.812.517.314.6 5.2-1.6-2.3
-5.2-6.5-5.9-2.5 8.313.118.523.423.420.720.317.617.517.422.717.2 6.8-1.2-2.6
-3.4-6.3-5.0 -.510.418.425.228.928.926.226.022.422.423.827.720.5 7.7 -.2-4.6
-4.0-5.6-5.2 2.512.023.129.032.933.430.629.627.625.728.132.120.7 7.2 -.5-4.9
-4.5-5.5-4.3 4.114.426.730.834.334.132.933.332.030.330.634.321.1 8.0  .6-4.5
-5.8-5.1-3.7 5.416.027.732.234.233.933.634.134.431.932.631.519.3 8.0 -.3-5.4
-7.9-3.3-4.7 6.115.527.131.632.832.032.132.435.331.630.931.916.8 6.8-2.1-5.3
-7.5-3.3-4.7 6.416.126.228.729.627.729.130.934.128.530.230.016.3 4.7 -.4-4.4
-7.5-3.2-5.6 5.516.924.626.125.023.925.428.030.225.328.127.213.6 3.9 1.0-4.7
-9.5-2.6-6.4 5.617.921.523.120.018.824.625.324.921.825.423.610.2 2.4 -.1-4.1
****-4.6-6.2 6.914.918.916.916.315.219.720.521.418.621.618.9 7.0 1.4-1.3-4.4
****-6.2-7.4 7.313.613.612.311.011.016.516.815.414.817.014.8 4.1  .1-5.1-4.3
-9.2-8.2-6.0 6.711.5 8.5 8.2 7.6 7.614.113.610.2 8.110.6 8.6 1.3-2.2-5.9-3.5
-7.8-9.4-4.2 3.4 8.9 4.2 6.4 4.7 5.310.0 9.9 5.6 3.5 5.3 4.2-2.1-4.0-8.5-4.3
-6.7-7.8-3.7 2.4 5.2 2.6 2.4  .9  .0 5.0 7.3  .3  .9  .7  .1-3.3-3.5-8.3-4.3
-6.4-7.6-1.4 1.5 1.7  .9-1.3-2.6-1.4  .3 2.5-3.1-2.9-1.9-2.8-3.9-4.9-7.1-6.4
```

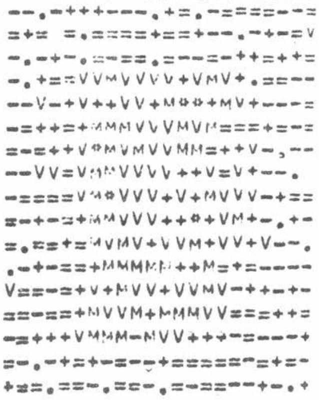

Figure 6 **The noisy degraded image with noise variance = 3 .**

```
-1.3 1.0  .2-2.1 1.6-2.3-6.2-5.6-5.0-3.0-1.1 2.4-3.2 1.7  .5-2.1 1.1-3.1 1.6
 -.9 -.6  .1-3.1-5.9-3.6 -.9  .8-3.1-1.9 2.7-2.6-1.5-1.4 2.1-1.3 1.7-5.2-2.7
-4.4 1.1-3.6-2.1-1.8-4.3  .9-2.6-1.3  .2  .5-5.0 6.2 -.3-2.0 -.6 2.7-2.8 2.4
-3.1 3.1-4.9-4.6-3.6 -.3-2.2-1.9 -.8-3.4-5.3-1.9 -.8 -.4  .8-1.8 -.6 -.1-1.9
-5.3 1.6-3.0 4.9 5.9 2.9  .4 6.0  .1 2.3 1.9 3.6 -.1-1.9 2.9-6.7  .7 1.0 4.9
-3.3-8.2 2.9 1.3-2.0 3.2 3.5 6.3 1.2-4.4  .1-1.9-2.9 1.7 4.3-2.3-5.1-2.3 1.5
 3.0 1.4-3.9 1.8 2.6 1.2 2.1  .9 1.9  .7 4.9  .9 -.8 1.3 2.7 3.6 -.8-1.5-5.8
 4.5-6.9-4.9 5.6 5.7 2.7 6.9 8.1 7.5 3.8 1.4 1.8 3.5 5.1 8.0-1.0  .1-2.0-2.8
 3.1-1.1-1.2 5.5 3.1 9.313.611.510.5 9.1 8.8 5.0 5.411.3 9.4 2.9-1.4 -.0-6.6
-2.5  .4-4.6 9.2 3.5 9.9 9.011.111.9 9.7 6.2 8.8 3.9 8.211.1-3.4-4.8-3.0-2.7
-1.3-1.0 -.9 6.2 7.610.3 7.0 8.0 5.9 8.210.310.811.2 7.0 8.6  .1 -.0 1.3 -.6
-2.9  .2 -.9 6.9 8.1 6.710.1 8.0 7.7 7.8 6.8 9.2 7.5 9.2-2.2-3.8 -.9-3.7-4.6
-4.7 4.9-4.8 6.1 4.2 5.4 8.3 8.3 7.0 5.9 3.410.0 6.3 2.8 9.9-3.3-3.4-6.3-1.5
 2.4 1.0-1.2 5.8 9.5 7.8 5.3 6.4 3.3 4.8 7.9 8.8 2.8 9.5 6.3 4.3-5.0 4.4  .9
 1.1 1.6-3.0 2.712.3 8.8 8.7 4.6 6.9 5.2 6.9 4.5 5.6 6.6 6.0 -.1 -.3 4.4-2.4
-5.3 3.6-2.9 5.814.5 6.1 9.2 4.2 4.515.110.0 2.8 7.1 9.3 5.0 -.9-1.6-1.8  .7
-2.1-3.4  .5 9.2 4.0 8.6  .4 7.9 8.6 4.0 4.7 9.0 9.6 7.9 2.5 -.4  .1-1.8-1.4
 -.9-2.5-4.3 6.9 9.6  .4 3.6 1.9 6.0 8.4 7.5 1.8 7.8 6.1 4.1  .6 -.4-9.2 -.3
 -.4-5.1 3.2 3.3 6.1 -.6 2.8 5.6 6.4 9.7 8.0 2.8-1.3 -.0-3.7 -.2-3.4-1.1 2.2
  .6-3.9 3.7-5.4 3.7 -.7 7.0 4.1 7.3 2.9 4.6 1.8 2.1  .8 -.5-3.0-2.8-7.0-2.2
 -.9 2.7  .1  .2-1.4 4.3-2.2-1.4-3.4-2.0 5.6-3.5 5.2  .2-2.1 1.5 3.3  .4  .3
-3.3-2.3 5.0 -.9-3.6 1.6-4.9-3.6 4.0-4.7-4.0-1.5-1.8 2.6-1.4 2.3-2.8 2.9-6.0
```

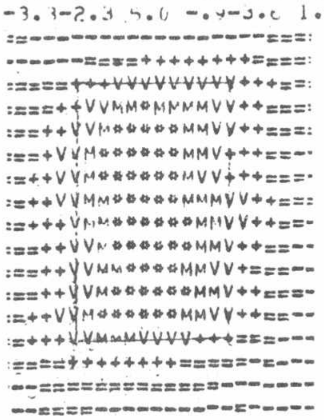

Figure 7 **The restored image using the stationary model of the image .**