

CONTROL OF CURRENT INSTANTANEOUS VALUE  
TO MINIMIZE TORQUE PULSATIONS IN CURRENT  
FED INDUCTION MOTOR OPERATING AT LOW SPEEDS

F.M. EL KADY (Ph.D.)<sup>\*</sup>, M.A. MORSY SHANAB (Ph.D.)<sup>\*\*</sup>

ABSTRACT

Current inverters apply essentially nonsinusoidal current to induction machines. The fundamental of the current produces the average output torque, and the harmonics produce torque pulsations. An analytical method to determine torque pulsations is established and verified by comparing its results to those obtained using numerical simulation. An optimization technique is proposed to determine the optimal wave form which minimizes torque pulsations. We proved that a significant reduction in torque pulsations can be achieved by controlling the instant current value according to an optimum pattern.

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<sup>\*</sup> Research and development departement, Egyptian Air Force.  
<sup>\*\*</sup> Head, Dept. of El-Power-Military Technical College, Cairo-Egypt.

## I. INTRODUCTION

Torque pulsations are produced in a current fed induction motor by the interaction of the square-wave current with the sinusoidal flux. The frequency of torque oscillations exists at multiples of six times the fundamental frequency of the motor current and flux. Now, increasing number of applications (robotic systems, aircraft drives, machine tools,...) focuses attention on the effects of these oscillations at low speeds.

Estimation of torque pulsations is important to determine the mechanical constraints on the machine, this can be done either analytically or by numerical simulation.

To minimize torque pulsations in a current source inverter induction machine, two approaches were recently presented as follows. 1) the current instantaneous value control [1-4] and 2) the current-pulsewidth modulation (PWM) method [5][6].

In this paper, an optimization technique is proposed to determine the optimal wave form which minimizes torque pulsations.

The approach is verified by a detailed computer simulation over a range of load and speed.

## II. SYSTEM DESCRIPTION

The arrangement of a current source inverter feeding a three-phase induction motor is shown in Fig. 1.

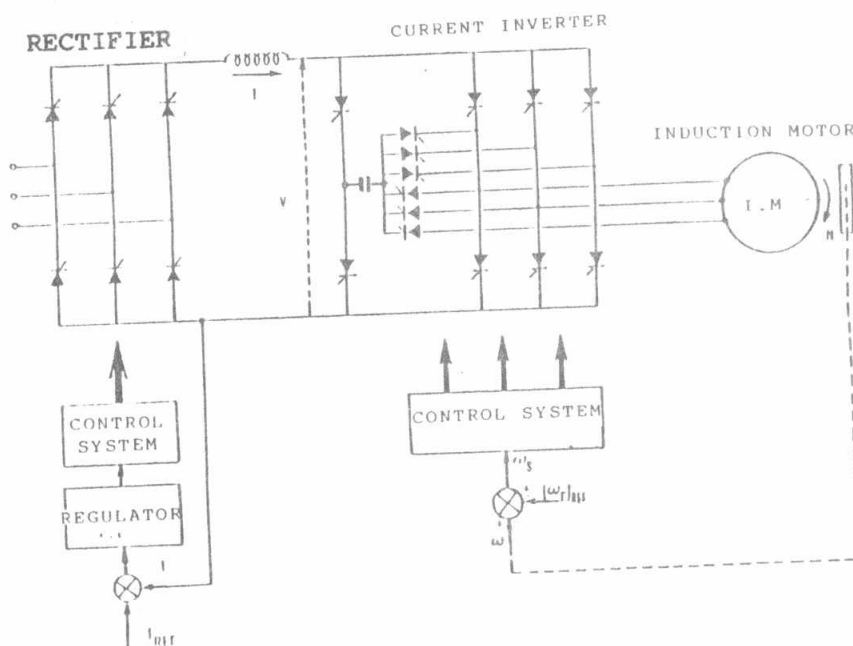


Fig.1. Current source inverter feeding a 3-phase induction motor

## III. ANALYSIS OF TORQUE PULSATIONS

The current supplied by the inverter is decomposed into fundamental value and sum of harmonic components [7].

$$I_s^+ = I_{s0}^+ + \Delta i_s^+ \quad (1)$$

In a system with forced commutation, the commutation process in the inverter can be divided into three parts.

- The period between commutation (this period is very small and can be neglected),
- The first commutation interval,
- The second commutation interval.

The fundamental value of current is.

$$I_{s0}^+ = \frac{\sqrt{3}}{\pi} I e^{j(\omega_s t - \pi/6)} \quad (2)$$

and

$$\Delta i_s^+ = \frac{I}{\sqrt{3}} \left( 1 - \frac{3}{\pi} e^{j(\omega_s t - \pi/6)} \right) \quad (3)$$

during the first commutation interval,

$$\Delta i_s^+ = \frac{I}{\sqrt{3}} \left[ e^{j\pi/3} + \cos \omega_o (t - t_1) e^{j\pi/3} - \frac{3}{\pi} e^{j(\omega_s t - \pi/6)} \right] \quad (4)$$

during the second commutation interval, with

$$\omega_o = \sqrt{\frac{3}{2 N_s C}}$$

The rotor harmonic current vector can be written as,

$$\Delta i_r^+ = - \frac{M_{sr}}{L_r} \Delta i_s^+ \quad (5)$$

Electromagnetic torque is given by

$$\begin{aligned} T_e &= 2 P M_{sr} J_m (I_s^+ I_r^-) \\ &= 2 P M_{sr} J_m (I_{s0}^+ + \Delta i_s^+) (I_{r0}^- + \Delta i_r^-) \\ &= (T_e)_o + \Delta T \end{aligned} \quad (6)$$

where

$$(T_e)_o = \frac{6 P M_{sr} \omega_r R_r I^2}{\pi^2 (R_r^2 + L_r^2 \omega_r^2)} \quad (7)$$

and

$$\begin{aligned} \Delta T &= 2 PM_{sr} J_m (I_{so}^+ \Delta i_r^- + I_{ro}^- \Delta i_s^+) \\ &= \frac{2\sqrt{3}}{\pi} \frac{PM_{sr}^2 R_r I}{L_r \sqrt{R_r^2 + L_r^2 \omega_r^2}} \operatorname{Re} \left[ \Delta i_s^+ e^{-j(\omega_s t - \pi/6 + B)} \right] \quad (8) \end{aligned}$$

Where

$$B = \tan^{-1} \frac{R_r}{L_r \omega_r}$$

The expressions of  $\Delta T$  in the two commutation intervals are

$$\Delta T_1 = \frac{2 PM_{sr}^2 R_r I^2}{\pi L_r \sqrt{R_r^2 + L_r^2 \omega_r^2}} \left[ \cos(\omega_s t + B - \pi/6) - \frac{3}{\pi} \cos B \right] \quad (9)$$

$$\begin{aligned} \Delta T_2 &= \frac{2 PM_{sr}^2 R_r I^2}{\pi L_r \sqrt{R_r^2 + L_r^2 \omega_r^2}} \left[ \sin(\omega_s t + B) \right. \\ &\quad \left. + \cos(t - t_1) \omega_0 \cos(\omega_s t + B + \pi/6) - \frac{3}{\pi} \cos B \right] \quad (10) \end{aligned}$$

Figures 2 and 3 show the results obtained by the analytical method and the corresponding solution by digital simulation. Fig. 4 gives a comparison between results of the two methods. It is clear that the analytical results are accurate over all the range of rotor frequencies.

#### IV. OPTIMAL CURRENT FORM TO MINIMIZE TORQUE PULSATIONS

Torque pulsations might be eliminated by controlling the instantaneous current value at the input of the inverter in accordance with an optimum current pattern. When the reference frame is rotating with the vector of stator current lies on the axis  $q$ , machine equations can be written as

$$i_{sd}(t) = \sqrt{2} I_s(t) \quad (11)$$

$$i_{sq}(t) = 0 \quad (12)$$

$$0 = \sqrt{2} M_{sr} \frac{d}{dt} I_s(t) + (R_r + L_r \frac{d}{dt}) i_{rd}(t) - L_r \omega_r i_{rq}(t) \quad (13)$$

$$0 = \sqrt{2} M_{sr} I_s(t) + L_r \omega_r i_{rd}(t) + (R_r + L_r \frac{d}{dt}) i_{rq}(t) \quad (14)$$

$$T_{em} = -\sqrt{2} PM_{sr} I_s(t) i_{rq}(t) \quad (15)$$

To obtain the form of  $I_s(t)$  which minimizes torque pulsations at operating point characterized by  $\omega_r$ , the one-sixth cycle ( $S/6$ ) is divided into  $N$  intervals each of length  $\Delta t = S/6 N$ .

Fig.2. Torque pulsations

in a current fed  
induction motor  
(Analytical method)

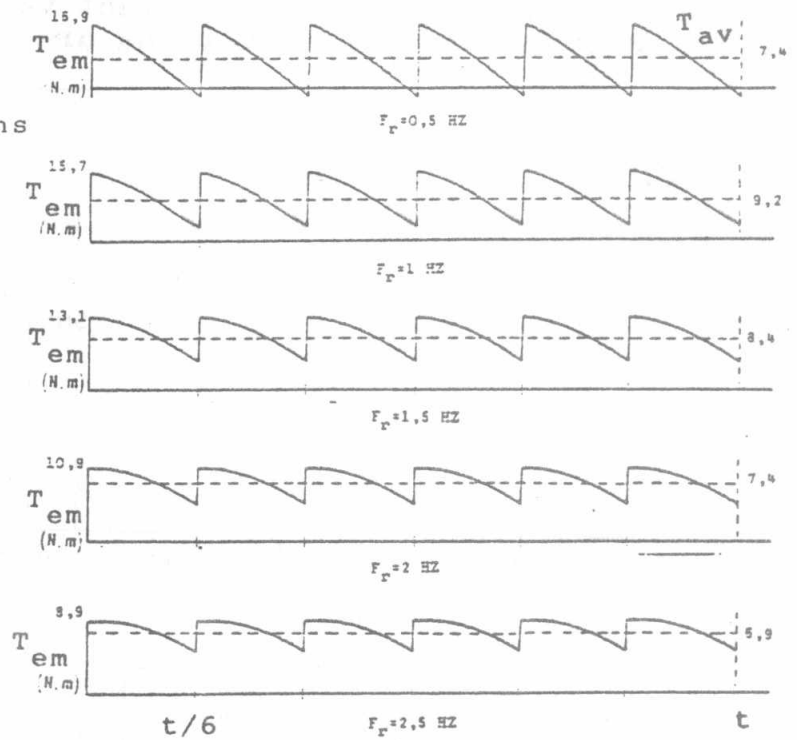
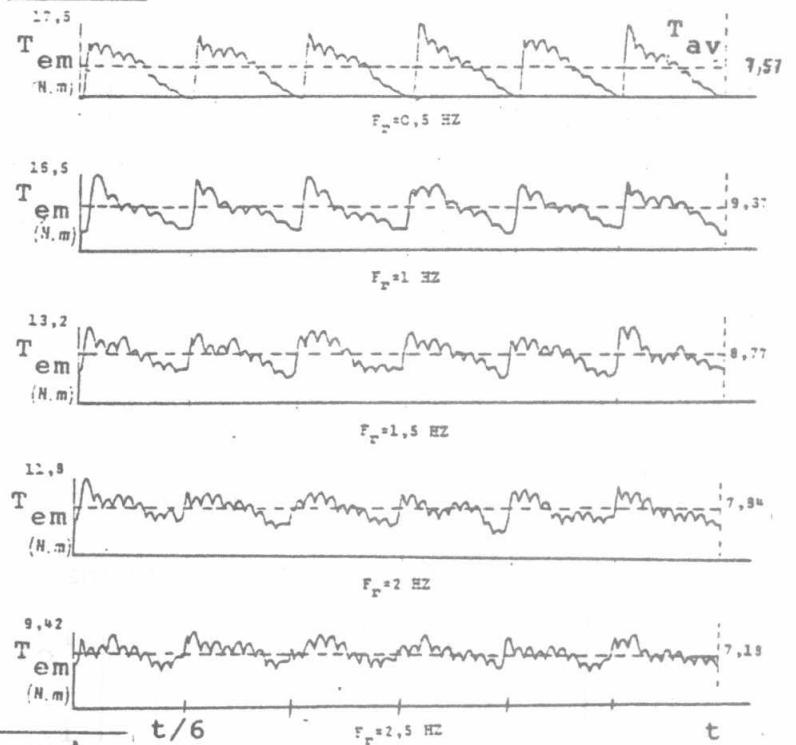
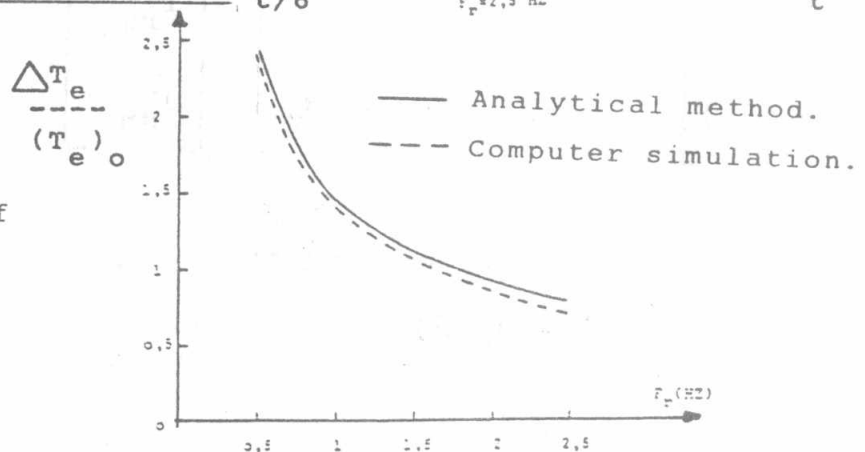
 $F_S = 5 \text{ HZ.}$ 


Fig.3. Torque pulsations

in a current fed  
induction motor.

(Computer simulation)

 $F_S = 5 \text{ HZ.}$ 

 Fig.4. variation of  
torque pulsations.


During each interval we assume constant value of current (Fig. 5), therefore machine equations are

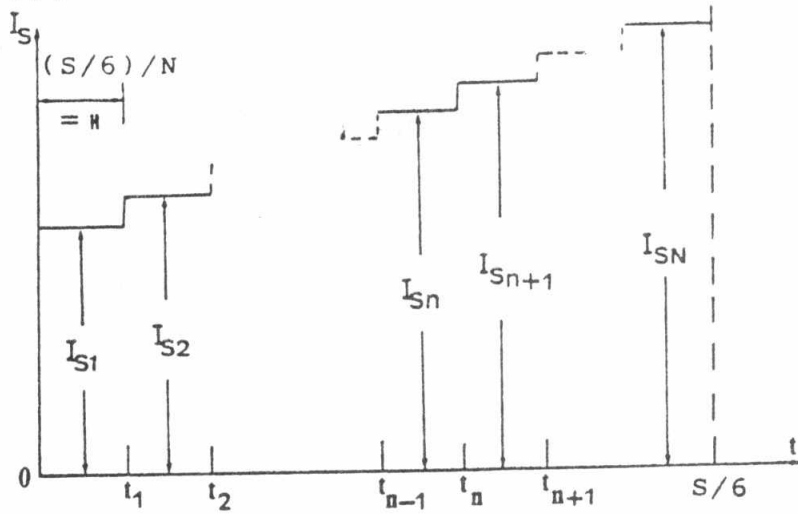


Fig.5.

$$\frac{d}{dt} \begin{bmatrix} i_{rd}(t) \\ i_{rq}(t) \end{bmatrix} = \begin{bmatrix} A \end{bmatrix} \begin{bmatrix} i_{rd}(t) \\ i_{rq}(t) \\ I_{sn} \end{bmatrix} \tag{16}$$

where

$$n = 1, 2, \dots N$$

$$\begin{bmatrix} A \end{bmatrix} = \begin{bmatrix} -R_r/L_r & w_r & 0 \\ -w_r & -R_r/L_r & -\frac{\sqrt{2} M_{sr} w_r}{L_r} \end{bmatrix} \tag{17}$$

The solution of differential equations (16) gives.

$$\begin{bmatrix} I_{rd}(t) \\ I_{rq}(t) \end{bmatrix} = \begin{bmatrix} X_1 & X_2 & X_3 \\ -X_2 & X_1 & X_4 \end{bmatrix} \begin{bmatrix} I_{rd0} \\ I_{rq0} \\ I_{sn} \end{bmatrix} \tag{18}$$

where

$$X_1 = 1 - \frac{w_r^2 H}{2} - k_o H + \frac{k_o^2 H^2}{2}$$

$$X_2 = w_r H (1 - k_o H)$$

$$X_3 = - \frac{w_r^2 H^2 M_{sr}}{\sqrt{2} L_r}$$

$$X_4 = \frac{\sqrt{2} M_{sr} w_r}{L_r} H \left( \frac{k_o H}{2} - 1 \right)$$

$$K_o = R_r / L_r$$

$$H = S/6 N$$

From equations (18) and (15), electromagnetic torque ( $T_{em}$ ) is expressed as a function of instantaneous value of current  $I_{sn}(t)$ .

At each interval the value of  $I_{sn}(t)$  to minimize torque pulsations is obtained by minimizing the objective function

$$F = [T_{em} - T_o]^2 = f [I_{sn}(t)] \tag{19}$$

Subjected to

$$I_{smin.} \leq I_{sn} \leq I_{smax.}$$

For (N) intervals, the optimal form of current to minimize torque pulsations is

$$I_{sn} = A_0 + A_1 t + A_2 w_r + A_3 t^2 + A_4 w_r^2 + A_5 t w_r \tag{20}$$

where  $A_j, j = 1, 2, \dots, 5$  constants.

Figure 6 gives the optimal form of current to obtain minimum torque pulsations at different values of  $w_r$ .

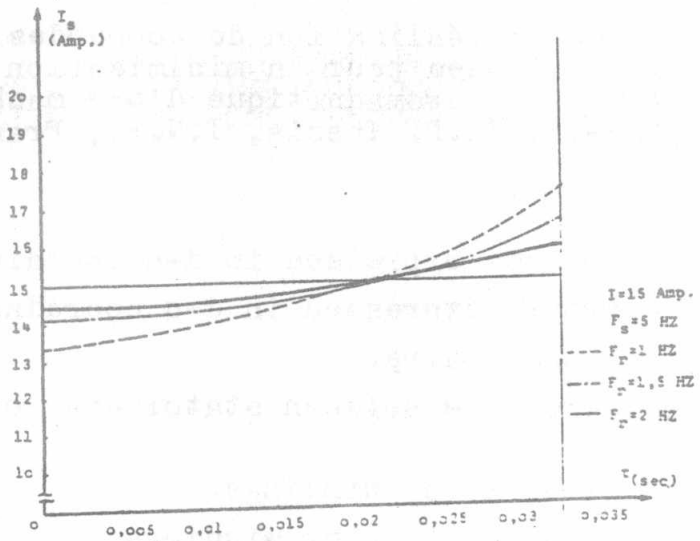


Fig. 6. S/6

Figures 7 and 8 show the simulation results in steady state operation with and without optimal current. A large reduction in torque pulsations is clearly evident when feeding with optimal current form.

V. CONCLUSIONS

A method of determining torque pulsations acting on the rotor of an induction machine when supplied from a current source inverter has been analysed.

We have investigated a method for determining the optimal current form to minimize torque pulsations which occurs when an induction motor is supplied from a current source inverter. We proved that a significant reduction in torque pulsations can be achieved by controlling the instant current value according to an optimum current pattern without reducing the average torque and efficiency.

REFERENCES

1. Lipo, T.A., "Analysis and control of torque pulsations in current fed induction motor drives" IEEE power electronics specialists conference, 89-96 (1978).
2. Chin, T.H., "A new controlled current type inverter with improved performance", IEEE. IAS International semiconductor power converter conference Record, 185-192 (1977).
3. Chin, T.H., and Tomita, H., "The principles of eliminating pulsating torque in current source inverter induction motor systems", IEEE Transactions, Vol. IA-17, No. 2, 160-166 (1981).
4. Loren, H.W., and Paul M.E., "A high performance controlled-current inverter drive", IEEE Transactions, Vol. IA-16, 193-202, (1980).
5. Bowes, S.R., and Mech, M.I., "computer-aided design of PWM inverter systems", Pro. IEE, 129, 1-17, (1982).
6. Bose, B.K., and Sutherland, H.A., "A high-performance pulse-width modulator for an inverter fed drive system using microcomputer", IEEE Transactions, Vol. IA-19, No. 2, 235-243 (1983).
7. F.M. El Kady, "Etude et réalisation de commandes optimales de convertisseurs statiques pour la minimisation des ondulations du couple électromagnétique d'une machine asynchrone triphasée", Ph.D. Thesis, I.N.P., France (1983).

NOMENCLATURE

$I_{sd}, I_{sq}$	Stator currents expressed in d-q coordinates.
$I_{rd}, I_{rq}$	rotor currents expressed in d-q coordinates.
$P$	number of pole pairs.
$M_{sr}$	mutual inductance between stator and rotor windings.
$R_r$	resistance of rotor windings.
$L_r$	self inductance of rotor windings.
$w_r$	angular frequency of motor revolution.
$U, I$	voltage and current in the d.c. link.
$V_s^+, I_s^+$	symmetrical components of instantaneous values of voltage and current in the stator.



$V_r^+, I_r^+$

symmetrical components of instantaneous values of voltage and current in the rotor.

$V_{so}^+, I_{so}^+, I_{ro}^+$

symmetrical components of fundamental values of voltages and currents in stator and rotor.

$T_e$

Electromagnetic torque.

C

Capacity of condenser of commutation.

$N_s$

Leakage inductance per phase of stator.

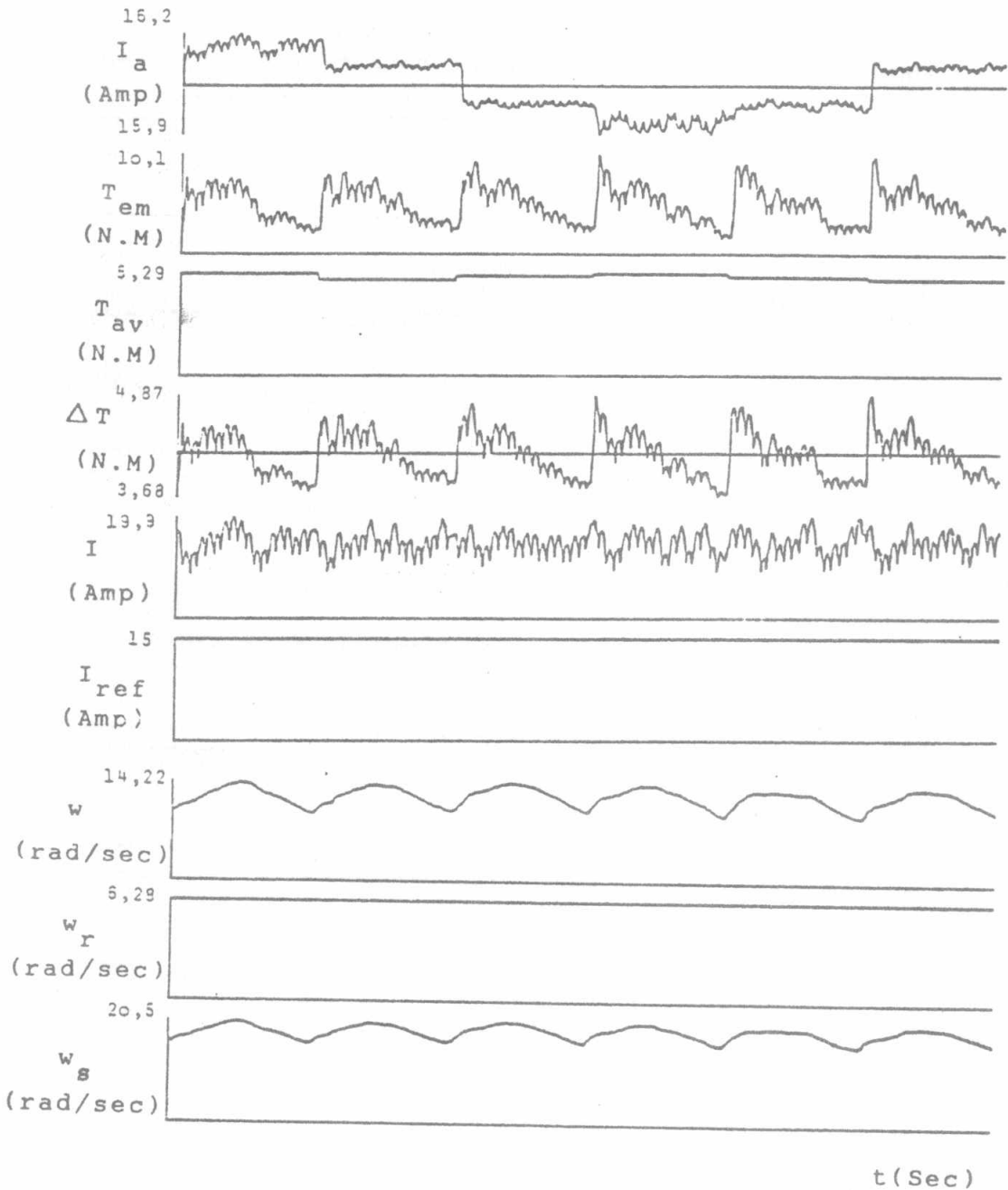


Fig.7. Machine fed by square wave current.

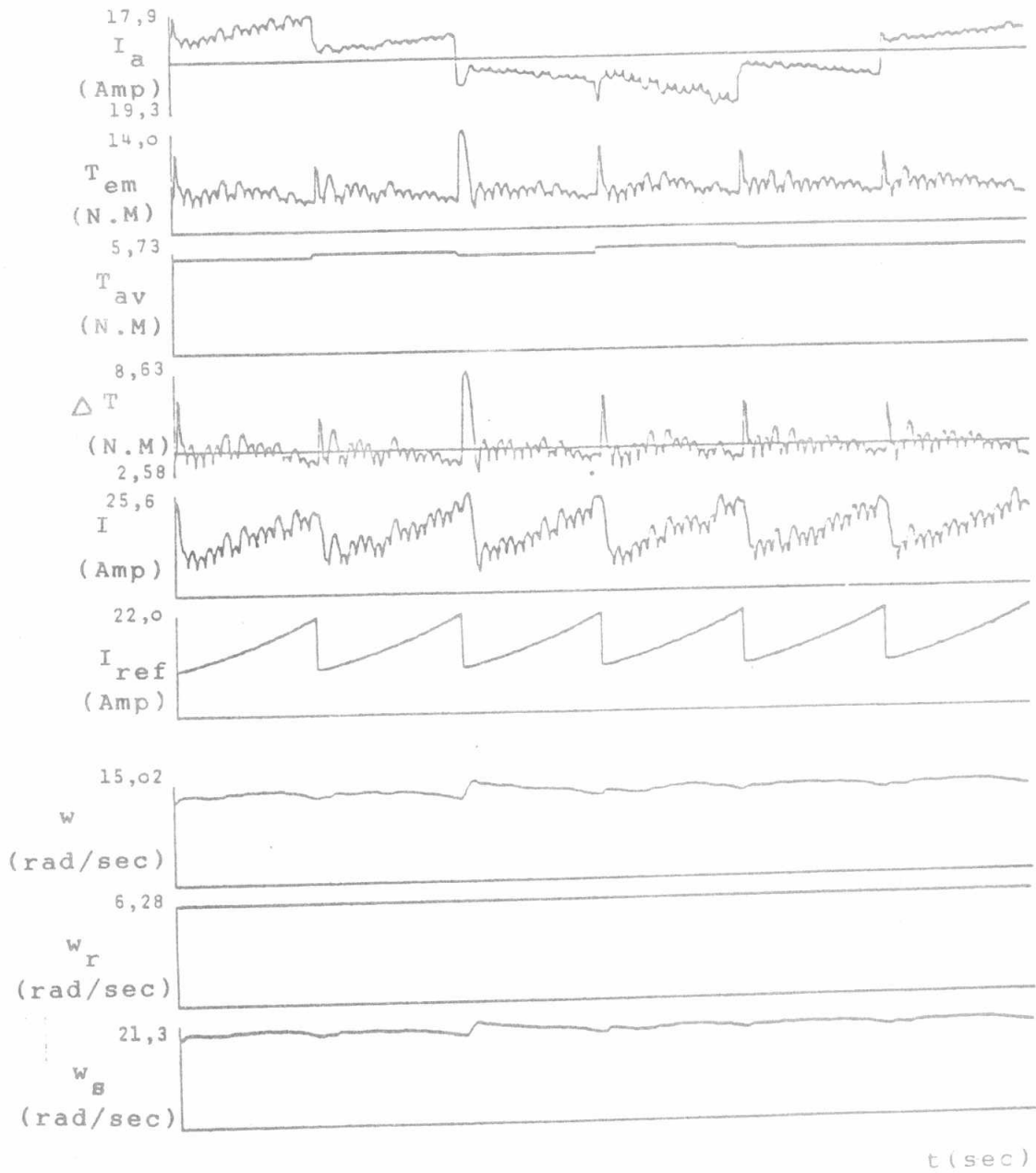


Fig.8. Machine fed by optimal current.