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# MHD Natural Convection Nanofluid Flow between Two Vertical Flat Plates through Porous Medium Considering Effects of Viscous Dissipation, non-Darcy, and Heat Generation/Absorption

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**Received:** 22-August-2022

**Accepted:** 06-September-2022

**Published:** 01- October- 2023

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## ABSTRACT

The paper investigates the analytical and numerical solution of MHD natural convection of grade three of a non-Newtonian Nanofluid flow between two vertical flat plates through a porous medium under the influence of non-Darcy resistance force, viscous dissipation, and heat generation/absorption. Analytically the nonlinear partial differential equations describing the present problem are solved using Multi-step differential transform method (MDTM) one of the most successful approaches for calculating an approximate solution to a system's nonlinear differential equations [1]. Numerically paired linearized differential (momentum and energy) equations are transformed into a linear system of algebraic equations using the finite difference method (FDM). Graphs and tables are used to display the effects of different parameters on velocity and temperature. The comparisons between current results and available previous results are listed in the tables, which indicate that the current answers are very similar to the past answers. The study found that (MDTM) and (FDM) are powerful approaches for solving non-linear differential equations such as this problem.

**Keywords:** Natural convection Nanofluid, Darcy and non-Darcy medium, viscous dissipation, Heat Generation/Absorption.

# 1 INTRODUCTION

Because of their numerous industrial and technical uses, non-Newtonian fluids have gained more attention and importance in recent years than Newtonian fluids. When modelling non-Newtonian incompressible fluid flow, the differential equations that arise are very nonlinear and difficult [1]. The majority of fluid mechanics and heat transfer issues are nonlinear by definition. Ordinary or partial nonlinear differential equations can be used to simulate these issues and phenomena in order to determine their behavior in the environment. The majority of the physical and mechanical issues are defined by a set of paired nonlinear differential equations. Natural convection, for example, has a system of coupled nonlinear differential equations that arise in a variety of physical and technical settings such as "geothermal systems, chemical catalytic reactors, heat exchangers, and so on" [2-3].

(DTM) which is based on the Taylor series, an analytical solution, which has lately been widely employed in many issues, is one of the easy and dependable approaches for the answer to a system of nonlinear paired differential equations [3]. It generates a polynomial answer as an analytical solution. In fact, DTM differs from the usual high-order Taylor series approach, which needs the symbolic computation of the data functions' essential derivatives [11]. The (DTM) technique was initially used in the field of engineering by [12]. Recently, this method attracted many authors to solve the nonlinear equations [14-19].

When analytical answers are unattainable or very difficult, and we need to compare analytical and experimental approaches, numerical methods are very useful tools for solving highly nonlinear differential equations. Because there is no accurate analytic solution for all nonlinear equations, numerical methods have been widely employed to solve them. To obtain the needed precision, iterative approaches must be used to answer the linearized differential equations. Because of its simplicity, (FDM) is commonly used for solving differential equations, both "linear or nonlinear" [3] and [13].

Natural convection of non-Newtonian fluids has been documented in several studies [3-10]. Inside two parallel orthogonal flat plates, a non-Newtonian fluid was studied [20-21]. For the natural convective dissipative heat transfer of an incompressible third-grade non-Newtonian fluid moving through an infinite porous plate embedded in a Darcy–Forchheimer porous medium, an analytic approximate solution is presented [22]. Non-Darcy fully developed mixed convection in a porous medium channel with heat generation/absorption and hydromagnetic effects presented by [23]. In addition, other researchers have presented a variety of models and methodologies for studying convective flows of nanofluids [24-30].

The purpose of this work is to investigate the analytical and numerical solution of MHD natural convection of grade three of a non-Newtonian Nanofluid flow Inside two parallel orthogonal flat plates through a porous medium considering the effects of non-Darcy, viscous dissipation, and heat generation/absorption analytically and numerically using (MDTM) and (FDM). With different values of the parameters, (nanoparticle volume fraction, viscous dissipation viscosity parameter, porosity parameter, Hartman number, non-Darcy parameter, Eckert number, Prandtl number, and heat generation/absorption) the variance distribution of velocity and temperature was that govern the problem are presented. Finally, Comparisons to recently published works are made and demonstrated that the present results have high accuracy and are found to be in excellent agreement.

## 2 MATHEMATICAL FORMULATION

A diagram of the problem under investigation is depicted in Fig (1). It is made up of two vertically positioned flat plates. A non-Newtonian fluid is contained on two flat walls separated by  $2b$ . At  $x = +b$  and  $x = -b$ , the walls are kept at constant temperatures  $T_1$  and  $T_2$ , respectively, with  $T_1$  more than  $T_2$ . The fluid near the wall at  $x = -b$  rises, whereas the fluid near the wall at  $x = +b$  falls, due to the temperature differential [33]. Copper is included in the fluid, which is a water-based nanofluid. The thermal equilibrium between the base fluid and the nanoparticles is assumed, with no slide between them.

**Table (1) lists the nanofluid's thermo-physical properties [31-32].**

Material	Density $(\rho)$ (kg/m <sup>3</sup> )	$C_p$ (J/Kg.k)	K (w/m.k)	$\beta \times 10^5$ (k <sup>-1</sup> )
Pure water	997.1	4179	0.613	21
Copper	8933	385	401	1.67

The effective density- $\rho_{nf}$ , the effective dynamic viscosity- $\mu_{nf}$ , the heat capacitance- $(\rho C_p)_{nf}$  and the thermal conductivity- $\kappa_{nf}$  of the Nanofluid can be expressed by the solid volume fraction  $\varphi$  as:

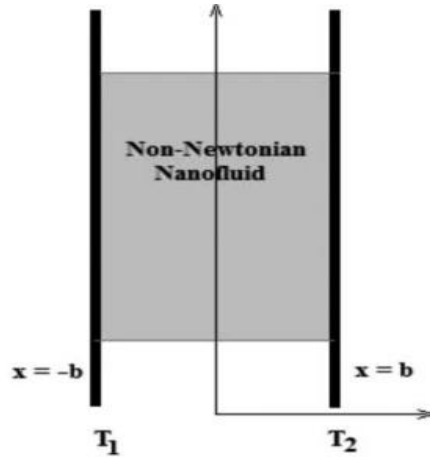
$$\rho_{nf} = \rho_f (1 - \varphi) + \rho_s \varphi \quad (1)$$

$$\mu_{nf} = \frac{\mu_f}{(1 - \varphi)^{2.5}} \quad (2)$$

$$(\rho C_p)_{nf} = (\rho C_p)_f (1 - \varphi) + (\rho C_p)_s \varphi \quad (3)$$

$$\frac{\kappa_{nf}}{\kappa_f} = \frac{\kappa_s + 2\kappa_f - 2\varphi(\kappa_f - \kappa_s)}{\kappa_s + 2\kappa_f + \varphi(\kappa_f - \kappa_s)} \quad (4)$$

$$\frac{\sigma_{nf}}{\sigma_f} = 1 + \frac{3\left(\frac{\sigma_s}{\sigma_f} - 1\right)\varphi}{\left(\frac{\sigma_s}{\sigma_f} + 2\right) - \left(\frac{\sigma_s}{\sigma_f} - 1\right)} \quad (5)$$



**Figure 1.** shows the motion of nanofluid between two vertical flat walls

The governing "momentum-energy" equations are developed, accordingly, based on the assumptions, as [1], [3], and [14]:

$$\mu_{nf} \frac{d^2 u}{dx^2} + 6\beta_3 \left(\frac{du}{dx}\right)^2 \frac{d^2 u}{dx^2} + \rho_0 \gamma (T - T_m) g - \frac{\mu_{nf}}{K_{nf}} u - \frac{\sigma_{nf} \beta_0^2}{\rho_{nf}} u - \frac{\rho_0 B}{K_{nf}} u^2 = 0, \quad (6)$$

$$K_{nf} \frac{d^2 T}{dx^2} + 2\beta_3 \left(\frac{du}{dx}\right)^4 + \mu_{nf} \left(\frac{du}{dx}\right)^2 + Q_0 (T - T_m) = 0, \quad (7)$$

Rajagopal [7] has demonstrated that by using the similarity variables:

$$v = \frac{u}{u_0}, \eta = \frac{x}{b} \text{ and } \theta = \frac{T - T_m}{T_1 - T_2}, \quad (8)$$

After substituting the above parameters, Eqs. (6) and (7) can be simplified to shown below:

$$\frac{d^2 v}{d\eta^2} + 6\delta(1 - \varphi)^{2.5} \left(\frac{dv}{d\eta}\right)^2 \frac{d^2 v}{d\eta^2} + \theta - \frac{P}{B} v - A H_a^2 (1 - \varphi)^{2.5} v - \frac{F_s}{B} v^2 = 0 \quad (9)$$

$$\frac{d^2 \theta}{d\eta^2} + 2\delta E_c P_r \left(\frac{dv}{d\eta}\right)^4 + \left(\frac{E_c P_r}{B}\right) (1 - \varphi)^{-2.5} \left(\frac{dv}{d\eta}\right)^2 + \frac{\alpha}{B} \theta = 0, \quad (10)$$

Where  $A = \frac{\sigma_{nf}}{\sigma_f}$ ,  $B = \frac{\kappa_{nf}}{\kappa_f}$ ,  $\mu_{nf} = \frac{\mu_f}{(1 - \varphi)^{2.5}}$ ,  $v_0 = \frac{\rho_0 \gamma g b^2 (T_1 - T_2)}{\mu_{nf}}$ ,  $\delta = \frac{\beta_3 u_0^2}{\mu_f b^2}$  is dimensionless non-Newtonian viscosity,  $P = \frac{b^2}{k_f}$  is porosity parameter,  $H_a^2 = \frac{\sigma_f \beta_0^2 b^2}{\mu_f}$  is Hartman number,  $F_s = \frac{\rho_0 B u_0 b^2}{\mu_f k_f}$  non-Darcy,  $E_c = \frac{u_0^2}{c_f (T_1 - T_2)}$  is Eckert number,  $P_r = \frac{\mu_f c_f}{k_f}$  is Prandtl number, and  $\alpha = \frac{Q_0 b^2}{k_f}$  is dimensionless Heat generation (source)/heat absorption (sink) parameter.

The relevant boundary conditions are as follows:

$$v(-1) = 0, \theta(-1) = \frac{1}{2}, \tag{11}$$

$$v(1) = 0, \theta(1) = -\frac{1}{2}. \tag{12}$$

In Eq. (10), heat generation is indicated by  $\alpha > 0$  whereas heat absorption is shown by  $\alpha < 0$ . Also,  $B_r = E_c P_r = \frac{\mu_f u_0^2}{k_f(T_1 - T_2)}$  [3], this is known as the Brinkman number, and there are significant correlations between Brinkman number and viscous dissipation, where the Brinkman number  $\left(B_r = \frac{\text{viscous dissipation}}{\text{conduction}}\right)$  is the ratio of viscous dissipation to conduction.

### 3 ANALYTICAL METHOD FOR THE SOLUTION

It is known that when using (DTM) in systems of strong nonlinear differential equations or with an infinite domain, the results we get are diverginse. Furthermore, power series are ineffective when the independent variable has large values. To address this problem, the (MDTM) has been created for the analytical solution of differential equations, and it is discussed in this section.

By Appling differential transformation theorems on Eqs. (9) and (10), can be obtained in the following recursive relations:

$$(k + 1)(k + 2)V(k + 2) + 6\delta (1 - \varphi)^{2.5} \sum_{r_2=0}^k \sum_{r_1=0}^{r_2} (r_1 + 1)(r_2 - r_1 + 1)(k - r_2 + 1)(k - r_2 + 2)V(r_1 + 1)V(r_2 - r_1 + 1)V(k - r_2 + 2) + \theta(k) - \frac{P}{B}V(k) - AH_a^2(1 - \varphi)^{2.5}V(k) - \frac{F_s}{B} \sum_{r=0}^k v(k)v(k - r) = 0, \tag{13}$$

$$(k + 1)(k + 2)\theta(k + 2) + 2\delta E_c P_r \sum_{r_3=0}^k \sum_{r_2=0}^{r_3} \sum_{r_1=0}^{r_2} (r_1 + 1)(r_2 - r_1 + 1)(r_3 - r_2 + 1)(k - r_3 + 1)V(r_1 + 1)V(r_2 - r_1 + 1)V(r_3 - r_2 + 1)V(k - r_3 + 1) + \frac{E_c P_r}{B}(1 - \varphi)^{-2.5} \sum_{r=0}^k (r + 1)(k - r + 1)V(r + 1)V(k - r + 1) + \frac{\alpha}{B}\theta(k) = 0, \tag{14}$$

The differential transforms of  $u(\eta)$  and  $\theta(\eta)$  are  $V(k)$  and  $\theta(k)$ , respectively.

The boundary conditions' differential transform (11-12) is as follows:

$$V(0) = 0, \theta(0) = \frac{1}{2}, \quad (15) \quad \sum_{k=0}^i v(k)2^k = 0,$$

$$\sum_{k=0}^i \theta(k)2^k = -\frac{1}{2}, \quad (16)$$

We can consider the following boundary conditions (11–12):

$$v(-1) = 0, \theta(-1) = \frac{1}{2}, \quad (17)$$

$$v'(-1) = \lambda, \theta'(-1) = \omega, \quad (18)$$

Then, differential transform of (17-18) are given by:

$$V(0) = 0, \theta(0) = \frac{1}{2}, \quad (19)$$

$$V(1) = \lambda, \theta(0) = \omega, \quad (20)$$

Moreover, by substituting equations (19) and (20) into equations (13) and (14) and by the recursive method we can calculate other values of  $V(k)$  and  $\Theta(K)$ .

#### 4 A NUMERICAL METHOD FOR THE SOLUTION

Both, non-linear coupled ordinary differential equations (9-10) and boundary conditions (11-12) are answered for the flow velocity and temperature using (FDM). Because nonlinearity of this system, the following linearized form should be used:

$$\frac{d^2 v}{d\eta^2} \left( \left( 1 + 6\delta(1-\varphi)^{2.5} \left( \frac{d\bar{v}}{d\eta} \right)^2 \right) \right) + \theta - \left( \left( AH_a^2 (1-\varphi)^{2.5} + \frac{P}{B} \right) - \frac{F_s}{B} \bar{v} \right) v = 0, \quad (21)$$

$$\frac{d^2 \theta}{d\eta^2} + E_c P_r \frac{dv}{d\eta} \left( 2\delta \left( \frac{d\bar{v}}{d\eta} \right)^3 + \frac{(1-\varphi)^{-2.5}}{B} \frac{d\bar{v}}{d\eta} \right)^3 + \frac{\alpha}{B} \theta = 0, \quad (22)$$

Where bar notation denotes the iterated terms that convert Eqs. (9-10) to a linearized one.

By applying Taylor's expansions of the dependent variables about the central point for Eqs. (21 -22) to obtain a system of algebraic equations [3].

$$\frac{dv_i}{d\eta} = \frac{v_{i+1} - v_{i-1}}{\Delta} + o(\Delta^2) \quad (23)$$

$$\frac{d^2 v_i}{d\eta^2} = \frac{v_{i+1} - 2v_i + v_{i-1}}{\Delta^2} + o(\Delta^2) \quad (24)$$

$$\frac{d^2 \theta_i}{d\eta^2} = \frac{\theta_{i+1} - 2\theta_i + \theta_{i-1}}{\Delta^2} + o(\Delta^2) \quad (25)$$

Where  $i = 1, 2, 3, \dots, m+1$  and  $m$  the number of subintervals of the finite domain of solution ( $-1 < \eta < 1$ ).

## 5 RESULTS AND DISCUSSION

In this paper, the (FDM) and (MDTM) is applied successfully to the present problem. Tables and graphics of the results are really helpful to demonstrate the efficiency and accuracy of (FDM) and (MDTM) for the problem stated in this work. In order to ensure that the current results are accurate, we compared these results with the previously published work. Figures (2–8) (a) and (b) indicates the effects of  $\delta$ ,  $P$ ,  $Ha$ ,  $F_s$ ,  $B_r$ ,  $\alpha$  and  $\phi$  on  $V(\eta)$  and  $\theta(\eta)$  profiles.

Figures 2 (a) and 2 (b), are shown the effect of viscoelastic parameter  $\delta$  on the velocity and temperature profiles, when other parameters are constants. The dimensionless non-Newtonian viscosity shows how important the inertia impact is in comparison to the viscous effect. From figures 2 (a) and 2 (b), It has been noticed that as the viscoelastic parameter  $\delta$  is increased,  $V(\eta)$  and  $\theta(\eta)$  decrease.

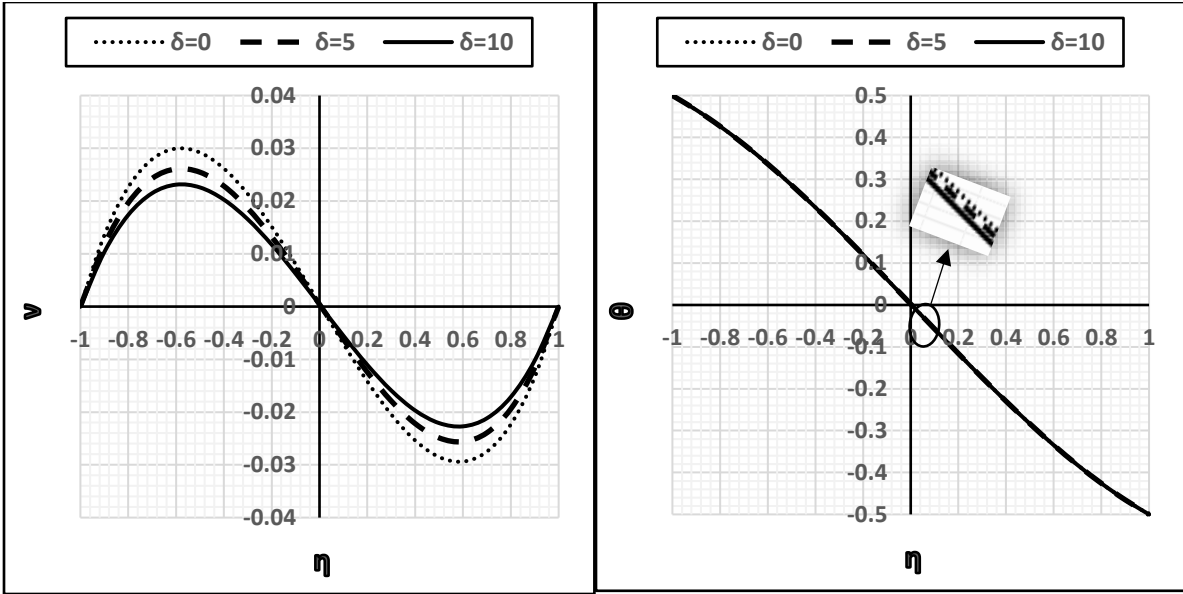
It is also found that  $\delta$  has a greater impact on  $V(\eta)$  than on  $\theta(\eta)$ .

It can be seen that increasing in  $P$ ,  $Ha$ , and  $F_s$  leads to a decrease in  $V(\eta)$  Figures (3–5) (a) because of its resistance to motion and the rate of shear increases and causes a decrease in the boundary layer thickness.

The effect of Brinkman number  $B_r$ , Heat generation (source)/heat absorption (sink) parameter  $\alpha$  and the nanoparticle concentration  $\phi$  is presented in the Figures (6–8) (a) As it is shown, the velocity distribution  $V(\eta)$  increases slightly by increasing values of  $B_r$ ,  $\alpha$  and  $\phi$  due to the improvement in the energy exchange rate and increased the heat dissipation.

In addition, the effect of Brinkman number  $B_r$ , Heat generation (source)/heat absorption (sink) parameter  $\alpha$ , and the nanoparticle concentration  $\phi$  is presented in the Figures (6–8) (b) as it is shown, the temperature distribution  $\theta(\eta)$  increases by increasing values of  $B_r$ ,  $\alpha$  and  $\phi$  due to of dissipation.

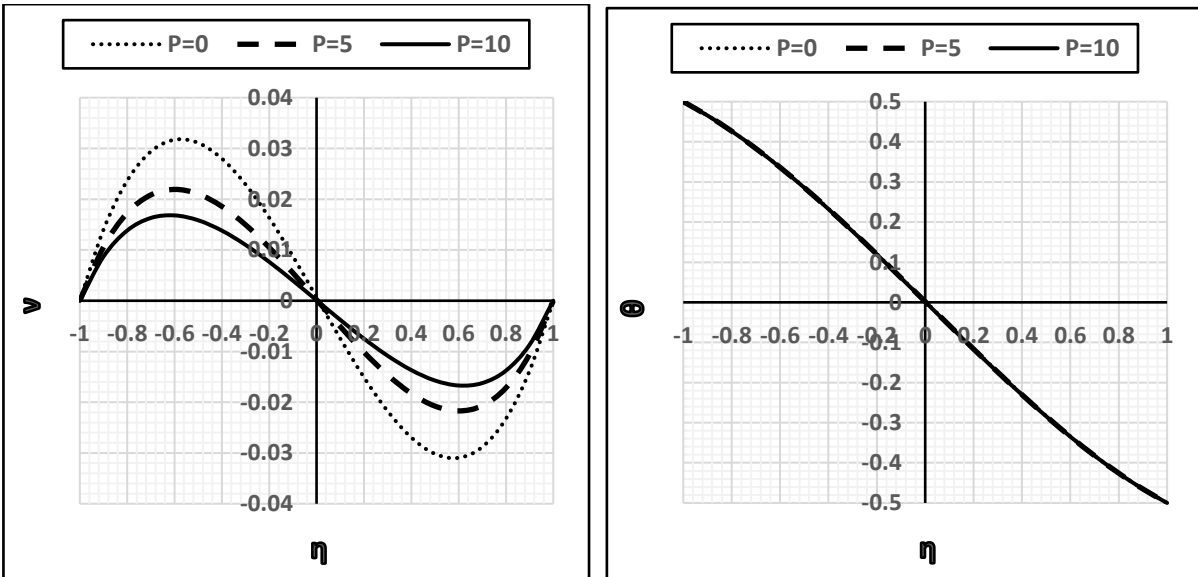
Also, it can be seen that the effect of  $P$ ,  $Ha$  and  $F_s$  on  $\theta(\eta)$  is very little almost non-existent Figures (3–5) (b), because  $P$ ,  $Ha$  and  $F_s$  does not explicitly occur in the energy equation. Therefore, it can be concluded that  $P$ ,  $Ha$ , and  $F_s$  have a negligible impact on the flow of  $\theta(\eta)$ .



(a)

(b)

Figure 2: Variation of  $V(\eta)$  and  $\theta(\eta)$  at different values of  $\delta$  when  $P=1$ ,  $Ha=1$ ,  $Fs=1$ ,  $Ec=1$ ,  $Pr=1$ ,  $\phi=0$  and  $\alpha=1$ .



(a)

(b)

Figure 3: Variation of  $V(\eta)$  and  $\theta(\eta)$  at different values of  $P$  when  $\delta=1$ ,  $Ha=1$ ,  $Fs=1$ ,  $Ec=1$ ,  $Pr=1$ ,  $\phi=0$  and  $\alpha=1$ .



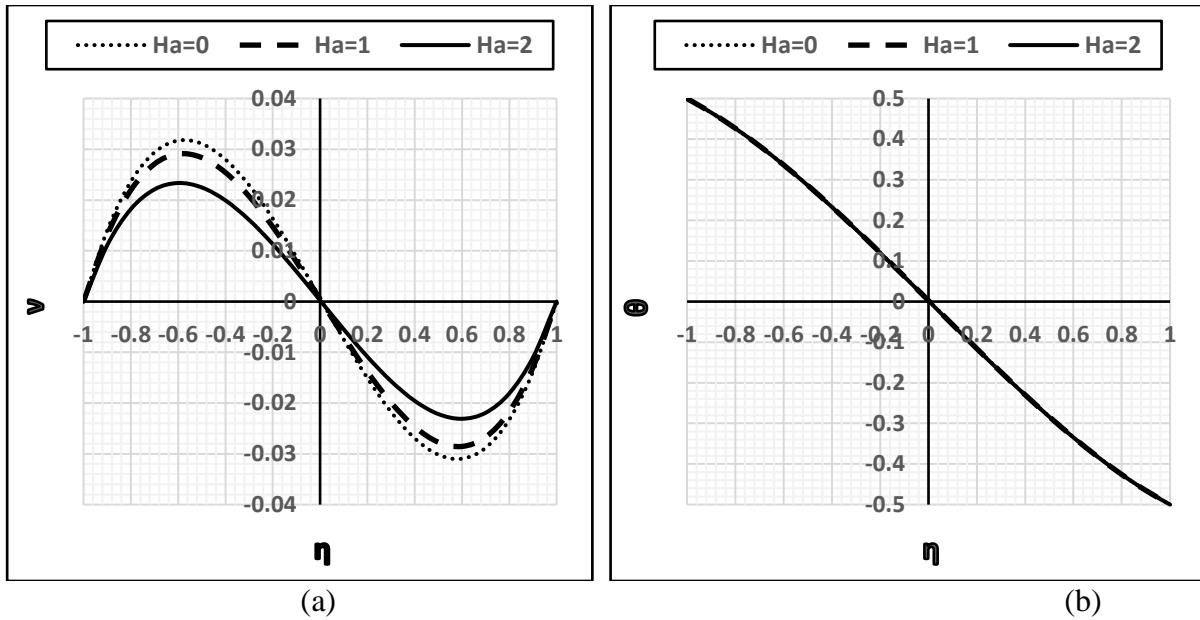


Figure 4: Variation of  $V(\eta)$  and  $\theta(\eta)$  at different values of  $Ha$  when  $\delta = 1, P = 1, Fs = 1, Ec = 1, Pr = 1, \phi = 0$  and  $\alpha = 1$ .

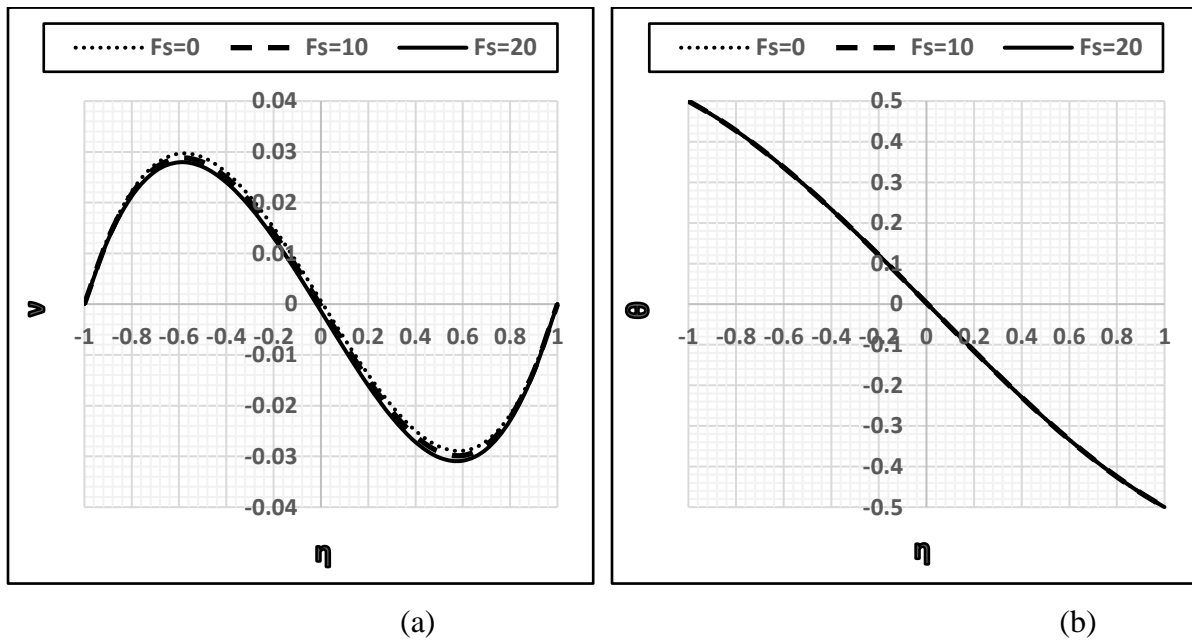


Figure 5: Variation of  $V(\eta)$  and  $\theta(\eta)$  at different values of  $Fs$  when  $\delta = 1, P = 1, Ha = 1, Ec = 1, Pr = 1, \phi = 0$  and  $\alpha = 1$ .

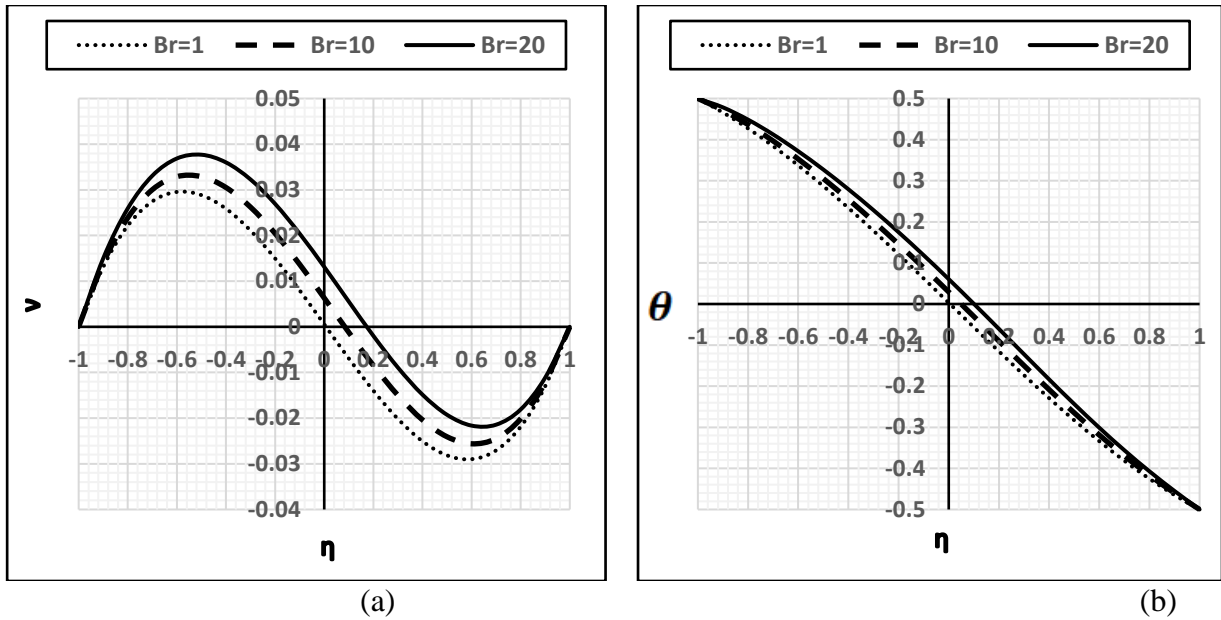


Figure 6: Variation of  $V(\eta)$  and  $\theta(\eta)$  at different values of  $Br$  when  $\delta=1$ ,  $P=1$ ,  $Ha=1$ ,  $Fs=1$ ,  $Pr=1$ ,  $\phi=0$  and  $\alpha=1$ .

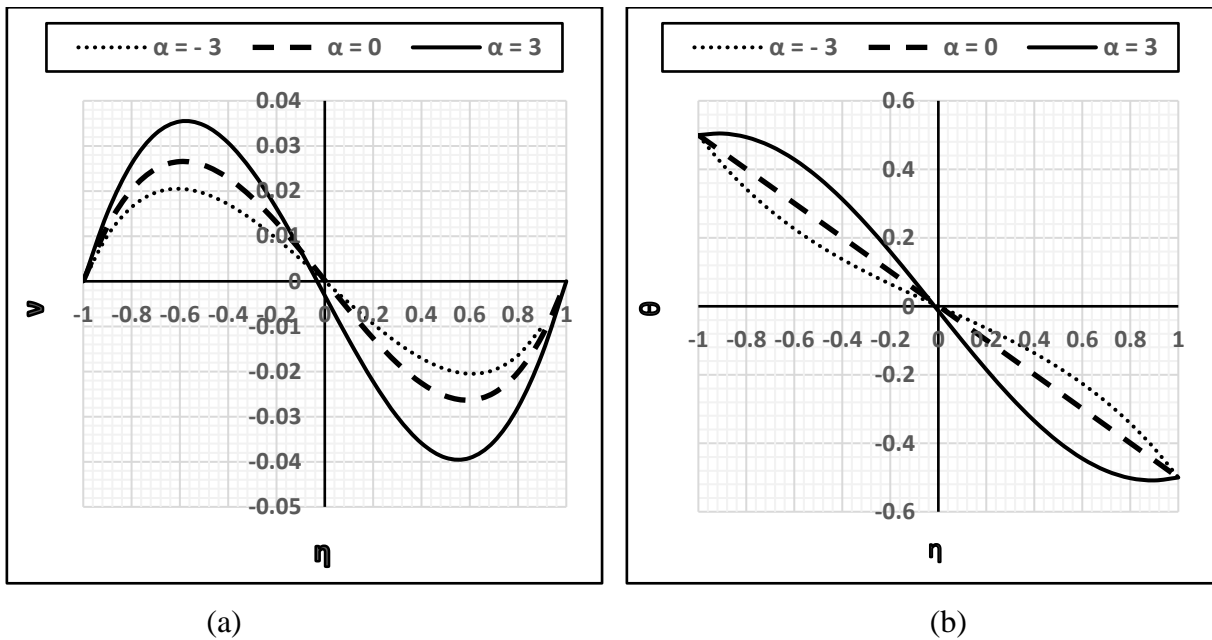
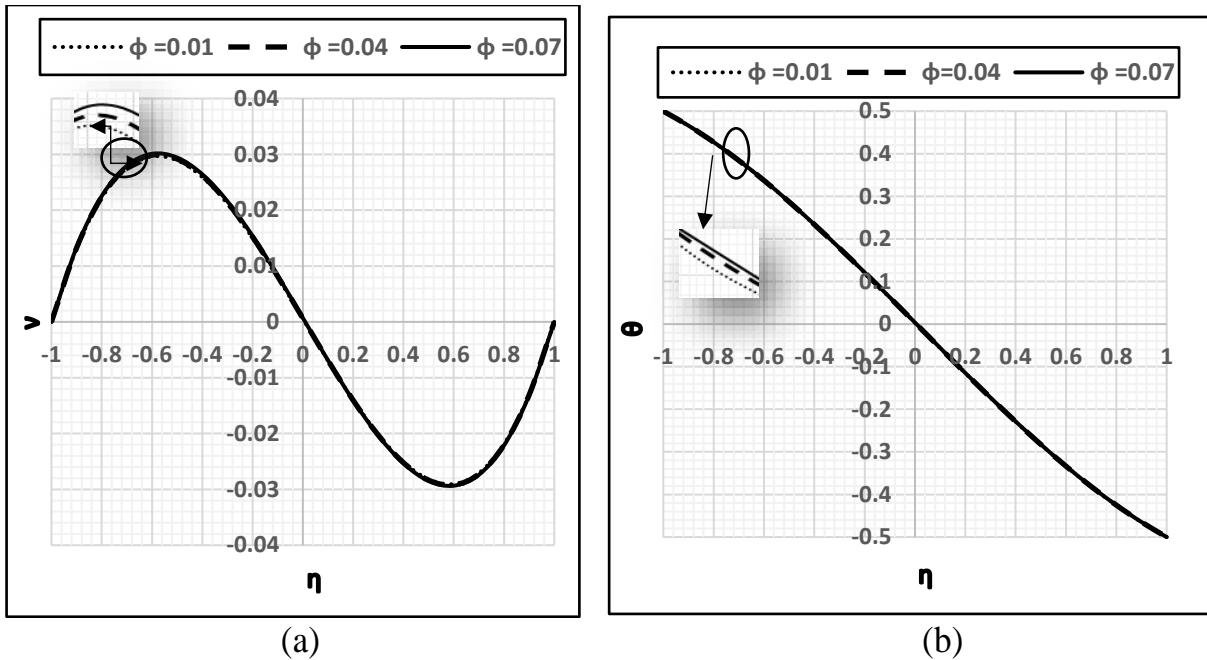


Figure 7: Variation of  $V(\eta)$  and  $\theta(\eta)$  at different values of  $\alpha$  when  $\delta=1$ ,  $P=1$ ,  $Ha=1$ ,  $Fs=1$ ,  $Ec=1$ ,  $\phi=0$  and  $Pr=1$ .



**Figure 8: Variation of  $V(\eta)$  and  $\theta(\eta)$  at different values of  $\phi$  when  $\alpha = 1$ ,  $\delta = 1$ ,  $P = 1$ ,  $Ha = 1$ ,  $Fs = 1$ ,  $Ec = 1$  and  $Pr = 1$ .**

In addition, tables (2–3) shows comparison between (MDTM) and (FDM) with ((LSM) and (GM) [10]). As can be seen, this approximate analytical and numerical solution agree with the solutions that are relevant.

**Table 2: Comparison solution by (MDTM) and(FDM) with ((LSM) and (GM) [10]) for  $V(\eta)$  when  $\delta=1, P=1, Ha=1, Fs=0, Ec=1, Pr=1, \phi=0$  and  $\alpha=1$ .**

$\eta$	Present $V(\eta)$		$v(\eta)$	
	MDTM	FDM	LSM	GM
-	0	0	0	0
-	0.013146724396251194		0.01296	0.01301
-	0.022204193479333386		0.021828	0.02191
-	0.02755244209846112		0.027059	0.02716
-	0.029639149959951303		0.029106	0.02921
-0.5	0.028947913723925744		0.028426	0.02853
-	0.025971060940111916		0.025473	0.02556
-	0.021192239867421657		0.0207	0.02077
-	0.01507875939700477		0.014563	0.01461
-	0.008081105071870518		0.007517	0.00753
0	0.0006368290320354		1.57E-05	6.53E-
0.1	-0.00682327533902678	-	-0.00749	-
0.2	-0.01386806518847978	-	-0.01453	-0.0146
0.3	-0.02005894400597227	-	-0.02067	-
0.4	-0.02494368906027614	-	-0.02545	-
0.5	-0.02805253285788286	-	-0.0284	-
0.6	-0.02889854812818517	-	-0.02909	-
0.7	-0.02698511095476234	-	-0.02704	-
0.8	-0.02182304241995679	-	-0.02182	-
0.9	-0.01295756242140307	-	-0.01295	-
1	$5.39770700205 \times 10^{-8}$	0	0	0

**Table 3: Comparison solution by (MDTM) and(FDM) with ((LSM) and (GM) [10]) for  $\theta(\eta)$  when  $\delta=1, P=1, Ha=1, Fs=0, Ec=1, Pr=1, \phi=0$  and  $\alpha=1$ .**

$\eta$	Present $\theta(\eta)$		$\theta(\eta)$	
	MDTM	FDM	LSM	GM
-	0.5	0.5	0.5	0.5
-	0.4660129153558711	0.46564572059348	0.4651	0.465708161
-	0.4272408396641567	0.42672020151194	0.4255	0.426457054
-	0.3841456096762759	0.38358185751875	0.3816	0.382797641
-	0.3371967627858102	0.33663535990197	0.3340	0.335280886
-	0.2868770365169094	0.28632737459272	0.2832	0.28445775
-	0.2336865386068267	0.23314192399689	0.2298	0.230879199
-	0.1781451881225290	0.17759541263291	0.1742	0.175096193

-	0.1207933267826234	0.12023136111228	0.1170	0.117659697
-	0.0621905976823350	0.06161489692170	0.0588	0.059120673
0	0.0029132687030886	0.00232705384206	8.35E-	3.01E-05
0	-0.056449821953170	-	-	-
0	-0.115302559964060	-	-	-
0	-0.173048004801702	-	-	-
0	-0.229095968876270	-	-	-
0	-0.282871753769532	-	-	-
0	-0.333826111811522	-	-	-
0	-0.381446504288412	-	-	-
0	-0.425269654243197	-	-	-
0	-0.464895204894587	-	-	-
1	-0.5	-0.5	-0.5	-0.5

## 6 CONCLUSION

We used (MDTM) and (FDM) to calculate non-Darcy MHD natural convection of grade three of non-Newtonian Nanofluid flow between two infinite parallel vertical plates via a porous medium with effects of viscous dissipation and heat generation/absorption in the current research. Figures illustrate effectiveness of  $\delta$ ,  $P$ ,  $H_a$ ,  $F_s$ ,  $P_r$ ,  $E_C$ , and  $\alpha$  on  $V(\eta)$  and  $\theta(\eta)$ . Furthermore, the findings of this study show that nanoparticle volume fraction and viscous dissipation have an impact on velocity and temperature. Comparisons with available previously published works are performed and showed that the present methods for solutions and results have high accuracy and are found to be in excellent agreement as shown in the tables. The tables' findings are extremely beneficial in various engineering life applications.

### Declarations

All authors: analyzed and interpreted data; writing the article or critically editing it for intellectual substance and approved the final version.

This work has not been submitted to and is not currently being reviewed by any other journal or publishing venue.

The authors have no financial or other ties to any of the organizations mentioned in the manuscript.

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