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Fitting the Nuclear Binding Energy Coefficients for Liquid Drop Model and Applying a Mathematical Terms to the Closed Shell of Magic Nuclei

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ABSTRACT

The present study reconsiders the formula of the Liquid Drop Model (LDM) in addition to updating the terms of the energy parameters represented by the term of volume, surface, coulomb, asymmetry and pairing. This was performed using the least-squares method (LSM) by means of a computer program in the Fortran language to match the nuclear binding energy for more than 480 different nuclei, including the magic nuclei of the range ($2 \leq Z \leq 92$). A mathematical term represented by the closed shell term, in addition to the energy terms above, was derived once by the difference between the separation energy of protons and neutrons and again by the method of valence nucleons, which represents the highest energy level in any nucleus because it is the only energy that participates in spinning the nucleus. New energy parameters were obtained specifically for the Liquid Drop Model which enabled us to determine the theoretical nuclear binding energy in a good match with its experiment values for most of the nuclei used, especially the magical nuclei. The standard deviation (σ) was used as a statistical tool to determine the extent to which the model can be adopted to explain the behavior of the magic nuclei, in addition to the high accuracy in determining the theoretical nuclear binding energy. The value of the standard deviation (σ = 0.126) and (σ=0.144) for the two updated formulas of the model were the generalized liquid drop models (GLDM)$_1$ and (GLDM)$_2$, respectively.

1. INTRODUCTION

The fundamental understanding of nuclear properties, such as the explanation of fission and fusion processes, nuclear force saturation, and the existence of pairing, was paved by describing the nuclear mass by the (LDM) [1]. At the same time, we state that the (LDM) gives an approximation suitable for atomic masses and a variety of other phenomena, but it does not explain the appearance of magic numbers.

Cohen and *Swiatecki* [2] showed that the LDM presented by Bohr and Wheeler [3] is unable to explain why peaks in the binding energy curve occur at certain values for protons (Z) and neutrons (N). Nuclei in which the number of Z or the number of N or both is equal to the magic numbers (2,8,20,28,50,82,126) which are more stable than the others. Magic numbers have been interpreted as including closed shells of Z and N in nuclei, similar to filling electron shells in atoms. The experimental evidence that supports the existence of nuclear shells is the separation energies of Z or N, as the separation energies gradually increase with the number of Z and the number of N, except for certain numbers of Z and N (magic numbers), where sharp drops or sudden changes in the separation energy occur, which can be explained only based on the presence of nuclear shells [4]. The LDM's projected binding energies are lower than the actual binding energies of "magic nuclei." For example, the LDM predicts a binding energy of 477.7 MeV for nickel ($^{56}_{28}Ni$), but the measured value is 484.0 MeV, and the LDM predicts a binding energy of 1084 MeV for tin($^{132}_{50}Sn$) , while the actual value is

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1110 MeV. Therefore, it has become necessary to think seriously to investigate new terms that represent the term of closed shell, which can be positively reflected on the suitability of the nuclear binding energy theoretically when balancing it with its experimental values. This will lead to a deeper understanding and a more accurate perception of nuclear structures, especially the magic nuclei. This can be obtained through the difference between the separation energies of Z and N, as well as the valence nucleons that carry a specific spin that would contribute to the fit of the nuclear binding energy.

As we know, the binding energy in the LDM is represented as a function of mass number and atomic number using five energy coefficients (volume, surface, coulomb, asymmetry, and pairing), where numerical methods are used to produce a new coefficient of energy parameters for the Semi-Empirical mass formula (SEMF). Among these methods is the method of (LSM), which is one of the most common methods of fitting data. Other researchers [5] added two terms for closed shells when generalizing the LDM to clarify the effects of closed shells on calculating the alpha decay energy (Qα) of heavy nuclei. A previous [6] suggests that the separation energies are used to predict new vacuoles in the shell. Hirsch et al.[7] used the three formulas of the LDM to describe the nuclear masses. They also studied the coefficients of the three models and showed that the inclusion of shell effects allows a better fit. Cakirili et al.[8] studied the separation energies and their relationship to the closed shell. . In another study[9] an LDM has been developed to calculate nuclear binding energy for (Z=50), and then compare the results with the original model and show that the error rate does not exceed 1.64%. Ankita and Suthar[10] used different parameters of the LDM, calculating the experimental binding energies by SEMF and comparing the values with the experimental data. An earlier publication[11] revealed a second development of the LDM to define a new concept regarding nuclear stability and binding energy. Chanda[12] conducted a study on the nuclear binding energy of highly stable nuclei using the LDM, in addition to trying to explain the reason for the high binding energy at magic numbers. Other researcher [13] proposed a new model based on the LDM in calculating the rate of nuclear potential that mediates the relationship between the nuclear core and neutron (N) at the surface. Karthika et.al.[14] studied the "magic property" of light nuclei using nuclear separation energies. Based on the LDM and taking into account the correction of the shell, other scientists [15] proposed a formula for calculating the energy emitted by the radioactivity of the Z. Many researchers [16,17,18,19,20,21] use the LSM to obtain a new set of coefficients for an LDM.

The present research aims at adding a theoretical mathematical term to the LDM, which represents the term of the closed shells of the magic nuclei. This was conducted through a mathematical derivation based on the difference between the separation energies of the Z and N, which is a nuclear binding energy that is added to the magic nuclei to increase their binding energy in addition to another mathematical method based on the valence nucleon number of Z and N, which represent the highest energy level in the nucleus. This is in addition to a suitable work for the coefficients of volume, surface, coulomb repulsion, asymmetry and pairing in the LDM by the LSM. This was performed by designing a code in the Fortran 95 program for 480 different nuclei, including magic numberswithin the $(2 \leq Z \leq 92)$ range, in order to correct the values of nuclear binding energy, especially for magic nuclei, and balance them with experimental values.

2. THEORETICAL FRAMEWORK

2.1 Liquid Drop Model (LDM)

The LDM was developed by Von Weizsäcker [22] according to the essential assumption that the nucleus may be thought of as a drop of incompressible material which arises because the internal density is approximately equal. The interaction between nucleons is strong, and the binding energy of the nucleus - according to the LDM is linearly dependent on its mass number (the volume of the nucleus). Moreover, Nucleons on the nucleus' surface can react with nucleons deep within the nucleus. Therefore, their binding energy is lower than the rest. Furthermore, the Z has a coulomb repulsion force among themselves that would reduce the binding energy of the nucleus in general. Increasing the number of N to the number of Z would create higher energy levels, which would negatively affect their binding energy. Therefore, most stable arrangement contains equal numbers of Z and N. In addition, the pairing term raises the binding energy of the nucleus when the nucleons are paired in the form of pairs, meaning that the nucleus is even-even, unlike the odd-odd nucleus.

The nuclear binding energy can be written as a function of the mass number A and the atomic number Z based on the LDM as follows[4]:

$$B(A,Z) = a_v A - a_s A^{\frac{2}{3}} - a_c \frac{Z^2}{A^{\frac{1}{3}}} - a_a \frac{(\frac{A}{2}-Z)^2}{A} + a_p A^{-\frac{1}{2}} \quad \mathbf{(1)}$$

Where $(a_v, a_s, a_c, a_a, a_\rho)$ represents the term of volume, surface, coulomb, asymmetry, and pairing term,[16] respectively, and their values are $(14.8, 16.8, 0.703, 28.8, 11.2)$, respectively.

2.2 Least-Squares Method (LSM)

The LSM is based on minimizing the amount of error $(\mathcal{E})$ in calculating the coefficients of volume, surface, coulomb, asymmetry, and pairing in the LDM especially when taking different types of nuclei as an extension of super-mass nuclei or when adding newly discovered nuclei. The quantity $(\mathcal{E})$ is represented by the following equation:

$$\mathcal{E} = \sum_i \left(y_i - B(Z_i, A_i)\right)^2 = \sum_i \left(y_i - Bi(a_v, a_s, a_c, a_a, a_\rho)\right)^2 \quad \mathbf{(2)}$$

Where (y_i) is empirical value for the binding energy of the nucleus, and (B_i) is the theoretical binding energy obtained from the equation (1). In other words, there is a possibility to obtain the coefficients of volume, surface, coulomb, asymmetry, and pairing by minimizing the function $(\mathcal{E})$. In other words, its first derivative must equal to zero.

$$\frac{\partial \mathcal{E}}{\partial a_v} = 0, \frac{\partial \mathcal{E}}{\partial a_s} = 0, \frac{\partial \mathcal{E}}{\partial a_c} = 0, \frac{\partial \mathcal{E}}{\partial a_a} = 0, \frac{\partial \mathcal{E}}{\partial a_\rho} = 0 \quad \mathbf{(3)}$$

Through equation (3), we get the matrix equation (4)

$$\begin{bmatrix}
+\sum_i A_i^2 & -\sum_i A_i^{\frac{5}{3}} & -\sum_i {Z_i}^2 A_i^{\frac{2}{3}} & -\sum_i \left(\frac{Ai}{2} - Z_i\right)^2 & +\sum_i \boldsymbol{\delta} i \mathrm{A} i^{1/2} \\
+\sum_i A_i^{\frac{5}{3}} & -\sum_i A_i^{\frac{4}{3}} & -\sum_i {Z_i}^2 A_i^{\frac{1}{3}} & -\sum_i \frac{\left(\frac{Ai}{2} - Z_i\right)^2}{A_i^{\frac{1}{3}}} & +\sum_i \boldsymbol{\delta} i \mathrm{A} i^{1/6} \\
+\sum_i {Z_i}^2 A_i^{\frac{2}{3}} & -\sum_i {Z_i}^2 A_i^{\frac{1}{3}} & -\sum_i \frac{Z_i^4}{A_i^{\frac{2}{3}}} & -\sum_i \frac{{Z_i}^2\left(\frac{Ai}{2} - Z_i\right)^2}{A_i^{\frac{4}{3}}} & +\sum_i \frac{\boldsymbol{\delta} i {Z_i}^2}{A_i^{\frac{5}{6}}} \\
+\sum_i \left(\frac{Ai}{2} - Z_i\right)^2 & -\sum_i \frac{\left(\frac{Ai}{2} - Z_i\right)^2}{A_i^{\frac{1}{3}}} & -\sum_i \frac{{Z_i}^2\left(\frac{Ai}{2} - Z_i\right)^2}{A_i^{\frac{4}{3}}} & -\sum_i \frac{\left(\frac{Ai}{2} - Z_i\right)^4}{A_i^2} & +\sum_i \frac{\boldsymbol{\delta} i\left(\frac{Ai}{2} - Z_i\right)^2}{A_i^{\frac{3}{2}}} \\
+\sum_i \boldsymbol{\delta} i \mathrm{A} i^{1/2} & -\sum_i \boldsymbol{\delta} i \mathrm{A} i^{1/6} & -\sum_i \frac{\boldsymbol{\delta} i {Z_i}^2}{A_i^{\frac{5}{6}}} & -\sum_i \frac{\boldsymbol{\delta} i\left(\frac{A_i}{2} - Z_i\right)^2}{A_i^{\frac{3}{2}}} & +\sum_i \frac{\boldsymbol{\delta} i^2}{A_i}
\end{bmatrix}$$

$$\begin{bmatrix} av \\ a_s \\ a_c \\ a_a \\ a_p \end{bmatrix} = \begin{bmatrix} \sum_i y_i A_i \\ \sum_i y_i A_i^{\frac{2}{3}} \\ \sum_i y_i \frac{{Z_i}^2}{A_i^{\frac{1}{3}}} \\ \sum_i y_i \frac{\left(\frac{Ai}{2} - Z_i\right)^2}{A_i} \\ \sum_i \frac{\boldsymbol{\delta} i y_i}{A_i^{\frac{1}{2}}} \end{bmatrix} \quad \mathbf{(4)}$$

The matrix equation above represents a set of the linear equation of five variables, which were solved by the Gauss's method using a computer program written in Fortran 95 which fits several coefficients as terms of the LDM down to seven coefficients when adding a new term represented by the shell term. The values of these coefficients were tabulated for the terms of volume, surface, coulomb, asymmetry and pairing in the new formula of the LDM. The results we obtained from this method are explained by equation (5), and the equation of the LDM becomes as follows:

$$B(A,Z) = 15.81\,A - 18.55\,A^{\frac{2}{3}} - 0.715\frac{Z^2}{A^{1/3}} - 23.59\frac{\left(\frac{A}{2}-Z\right)^2}{A} \pm 14.7A^{-\frac{1}{2}} \qquad (\mathbf{5})$$

Due to the terms of the new energy coefficients obtained from the above equation, it was found out that they differ from their counterparts in the equation (1), and that this difference depends on the number of the nuclei used to fit the coefficients of the LDM.

2.3 Derivation of Shell Term for Magic Nuclei

Until now, the LDM in its present form is an incomplete structure, despite the discovery of the new nuclei. Although many improvements have been made over the years, this formula does not give an explanation for magic nuclei, whether in the number of Z or N. Most of the nuclear properties show differences near certain values of the number of Z and N. Experimental facts indicate that nuclei are more stable at those numbers, which form closed shells for Z and N. And the stability of the magic nuclei is greater than the stability of the neighboring nuclei. The experimental evidence that supports the existence of the nuclear shells separation energies of Z and N measured in the form of sharp deviations from the expected values so that the effects of closed shells appear more clearly [23]. According to the difference between the energies separating Z and N, as [24]:

$$S_p = B(A,Z) - B(A-1,Z-1) \qquad (\mathbf{6})$$

$$S_n = B(A,Z) - B(A-1,Z,N-1) \qquad (\mathbf{7})$$

Where $S_p, S_n\, are$ the separation energy of Z and N, respectively. According to the LDM as in equation (1), the difference between the energy of separation of Z and N is as follows:

$$S_p - S_n = -a_v(A-1) + a_s(A-1)^{\frac{2}{3}} + a_c\frac{(Z-1)^2}{(A-1)^{\frac{1}{3}}} + a_a\frac{\left(\frac{A-1}{2}-(Z-1)\right)^2}{A-1} - a_p(A-1)^{-\frac{1}{2}} + a_v(A-1)$$

$$-a_s(A-1)^{\frac{2}{3}} - a_c\frac{(Z)^2}{(A-1)^{\frac{1}{3}}} - a_a\frac{(\frac{A-1}{2}-Z)^2}{A-1} + a_p(A-1)^{-\frac{1}{2}} \qquad (\mathbf{8})$$

$$S_p - S_n = -a_c(2Z-1)(A-1)^{-\frac{1}{3}} + a_a(A-2Z)(A-1)^{-1} \qquad (\mathbf{9})$$

For stable nuclei, we use the following relationship [24]:

$Z \approx \frac{A}{2}(1 - \frac{a_c}{a_a}A^{\frac{2}{3}})$

$$S_p - S_n = \frac{a_c}{A-1}[A^{\frac{5}{3}} - (A-1)^{\frac{5}{3}} + \frac{a_c}{a_a}A^{\frac{5}{3}}(A-1)^{\frac{5}{3}}]$$

$$S_p - S_n \approx \frac{a_c}{A-1}[\frac{a_c}{a_a}A^{\frac{5}{3}}(A-1)^{\frac{2}{3}} + A^{\frac{5}{3}} - (A-1)^{\frac{2}{3}}(A-1)] \qquad (\mathbf{10})$$

When the mass number is more than one, the following approximation can be used:

$A \gg 1, A-1 = A$

$a_c = 0{,}715\,MeV, a_a = 23.59\,MeV$

$$\therefore\ S_p - S_n = 0.0217\,A^{\frac{4}{3}}\,MeV$$

$$= a_{sh}A^{\frac{4}{3}}\,MeV \qquad (\mathbf{11})$$

Where $a_{sh}$is the shell constant.

When equation (11) is added to equation (5), we get a new update of the (LDM) that deals with magic nuclei along with all other nuclei.

$$B(A,Z) = 15.81\,A - 18.55\,A^{\frac{2}{3}} - 0.715\frac{Z^2}{A^{\frac{1}{3}}} - 23.59\,\frac{(\frac{A}{2}-Z)^2}{A} \pm 14.7\,A^{-\frac{1}{2}} + 0.0217\,A^{\frac{4}{3}} \qquad (\mathbf{12})$$

When applying the above equation to all the nuclei under study, it was shown that the theoretical values are close to the experimental values with an acceptable deviation rate. It is worth noting that the above equation is symbolized by the generalization LDM (GLDM)1

2.4 Adding a Correction Term to the Shell

The effects of magic numbers have long been the focus of the nuclear microscopic theory. Several methods for dealing with these effects have been proposed in the literature, but there is no known general dependence for Z and N, which could constitute an additional term in the Semi-Empirical mass formula (SEMF). The corrective term that will be adopted in the present study is the valence nucleon coefficient [25, 26], which is largely unrelated to other SEMF terms.

$$B_{shell}(N_n, N_P) = a_{sh1}P + a_{sh2}P^2 \qquad (\mathbf{13})$$

where $P = (N_n N_p)/(N_n + N_p)$, and(N_n , N_p), represent N and Z valence, which are located in the last energy levels and in turn participate in spinning the nucleus. When these two terms are added to the SEMF, they become as follows:

$$B(A,Z) = a_v A - a_s A^{\frac{2}{3}} - a_c \frac{Z^2}{A^{\frac{1}{3}}} - a_a \frac{(\frac{A}{2} - Z)^2}{A} \pm a_p A^{-\frac{1}{2}} - a_{sh1}P + a_{sh2}P^2 \qquad (\mathbf{14})$$

By fitting the coefficients of this formula using the LSM for only 261 magic nuclei, a new form of the equation could be obtained(14)

$$B(A,Z) = 14.2\,A - 15.3\,A^{\frac{2}{3}} - 0.57\,\frac{Z^2}{A^{\frac{1}{3}}} - 19.4\,\frac{(\frac{A}{2} - Z)^2}{A} \pm 12\,A^{-\frac{1}{2}} - 0.63\,P + 1.74\,P^2 \qquad (\mathbf{15})$$

When applying the above equation to all the nuclei under study, the theoretical values approached the experimental with a very acceptable deviation rate. The equation shall be symbolized by the generalized LDM $(GLDM)_2$.

2. 5 Determining the Standard Deviation of the Proposed Models

In order to determine the accuracy of the two equation (12),(15), and compare them with the experimental results, the standard deviation was calculated [27].

$$\sigma = \sum_{i=1}^{N} \frac{|BEexp - BEtheo|}{N} \qquad (\mathbf{16})$$

BE_{exp} : representing experimental values.

BE_{theo}: representing theoretical values.

3. RESULTS AND DISCUSSION

Table (1) shows the values of the coefficients obtained when fitting equations (1), (14), according to $(GLDM)_1$, $(GLDM)_2$, respectively

Table (1): The values of the coefficients energy from fitting equations (1), (14), according to $(GLDM)_1$, $(GLDM)_2$, respectively

Models	a_v	a_s	a_c	a_a	a_p	a_{sh1}	a_{sh2}
$(GLDM)_1$	15.8136	18.54563	0.71462	23.59992	14.7231	__	___
$(GLDM)_2$	14.1910	15.29089	0.57864	19.42074	12.0301	0.63938	**1.7499**

Table (2): The comparison of the coefficients that we obtained by the LSM for both models $(GLDM)_1$, $(GLDM)_2$, with a set of different values of the coefficients that were calculated in the previous works in a similar way

Coefficients (*MeV*)	a_v	a_s	a_c	a_a	a_p	a_{sh1}	a_{sh2}
Ref. [16]	15.78	18.34	0.71	23.21	12.00	__	__
Ref. [19]	15.52	17.48	0.67	24.58	__	__	__
Ref. [20]	14.64	14.08	0.64	21.07	11.54	__	__
Ref. [21]	15.55	16.96	0.70	23.03	_	__	__
Present Work$(GLDM)_1$	15.81	18.55	0.71	23.59	14.7	__	__
Present Work$(GLDM)_2$	14.19	15.29	0.58	19.42	12.03	0.64	1.74

It is shown that the range of the accuracy finiteness for both methods used in the research when matching the values of the coefficients to the LDM is shown in the Table above when comparing the obtained values with the values of previous works. As for the coefficients that were reached for the model $(GLDM)_2$, they can be relied upon, especially in obtaining a standard deviation of no more than (0.144) as in Table (3), when the theoretical values of the binding energy of magic nuclei after adding two valence nucleons are compared to the experimental values

Figure (1) shows a comparison between the average experimental binding energy and the average theoretical binding energy obtained from equation (5) for all the studied nuclei

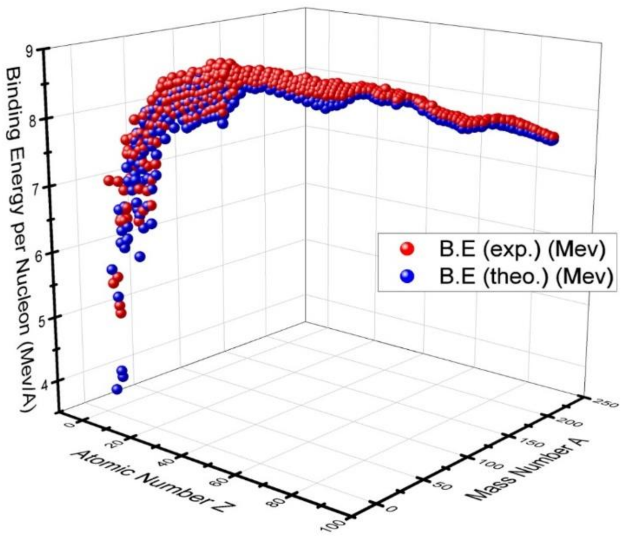

Fig. (1): A comparison between the average experimental binding energy and the average theoretical binding energy obtained from equation (5) for all the studied nuclei.

From Figure (1), it is shown that the theoretical binding energy obtained from equation (5) and using the coefficients calculated using the (LSM) is very compatible with the experimental values, especially at medium and heavy nuclei (except for some heights at (A=100,140,200), due to the presence of magic numbers in these regions, as equation (5) does not take into account the effect of the shell. So, when adding the shell term to the equation of the (LDM), these differences disappear. As for the $(A \leq 20\ )$ region, there is an acceptable discrepancy with the experimental values. Figures (2 and 3) show the the difference between the experimental binding energy and the theoretical binding energy in the presence of the shell-term with the mass number A and atomic number Z of a model $(GLDM)_1$ for the studied nuclei within the range $(2 \leq Z \leq 92)$ and a model $(GLDM)_2$, for magic nuclei only, respectively.

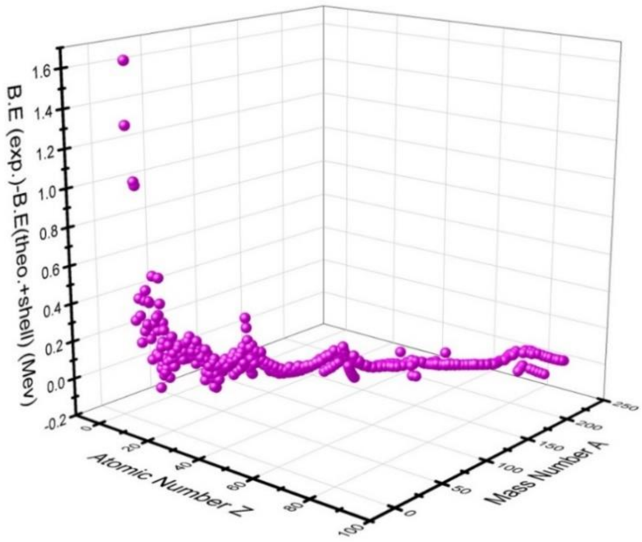

Fig. (2): The difference between the experimental and theoretical binding energy in the presence of the shell term with the mass number and atomic number according to the model $(GLDM)_1$ for all studied nuclei

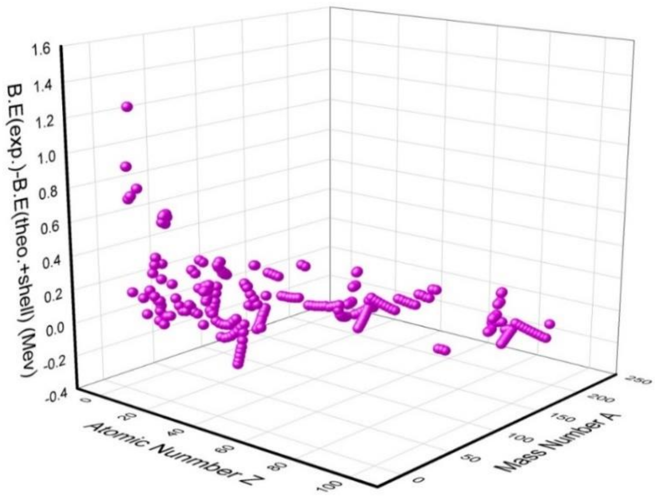

Fig. (3): The difference between the experimental and theoretical binding energy in the presence of the shell-term with the mass number and atomic number according to the model $(GLDM)_2$, for all studied magic nuclei.

It should be noted that the more the difference between the theoretical and experimental values is close to zero, the closer the model is to its adoption. It is clear from Figures (2 and 3), and for both models that the difference between the experimental and theoretical nuclear binding energy in the presence of the shell-term and the valence nucleon term, respectively, is of high value for light nuclei of magic numbers, whether in the number of Z or the number of N in addition to the neighboring nuclei. While this difference decreases significantly and is centered around zero in the medium and heavy nuclei, this leads to an acceptable agreement with the experimental values. This indicates the possibility of adding the shell term represented by ($S_p - S_n = a_{sh}A^{\frac{4}{3}}\,MeV$) and ($a_{sh1}P + a_{sh2}P^2$) to the LDM. The standard deviation included in Table (3) and for both models with the values (0.126),(0.144), respectively proves the validity of the assumption on which the mathematical derivation is built when adding that term, and thus the two models become complementary, especially in the values of the coefficients that were found through the fit process using the LSM. Looking back at the term shell in the model $(GLDM)_1$ it is noted that it depends on the hypothesis of the difference between the separation energy of Z and N, where Z and N are equal in light nuclei and it takes equal energy to separate a Z and N in light nuclei. While in stable heavy nuclei, the number of N is greater than the number of Z, so it needed a higher energy of separation for Z in equilibrium with N. All of these facts are based on the assumption that the nuclear force is approximately equal for each nucleon pair [23]. As for the shell term in the model $(GLDM)_2$, it depends on the valence nucleons of the closed shells of the magic nuclei.

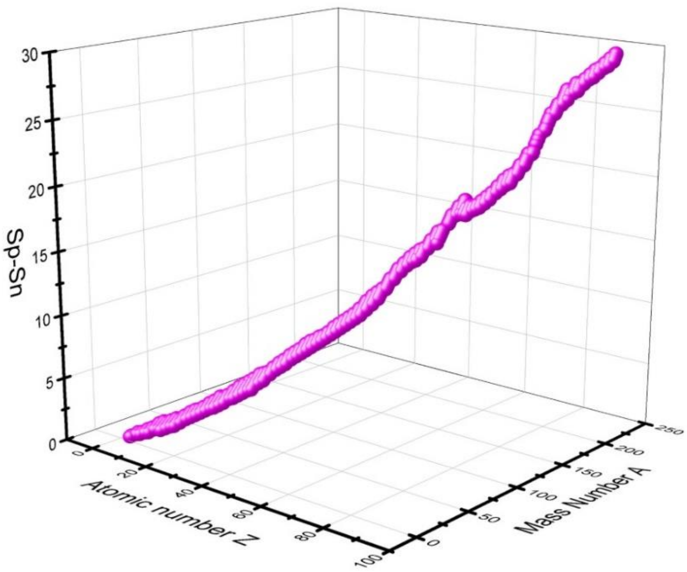

Fig. (4): The relationship between the shell term (Sp − Sn) with the mass number A and atomic number Z of all studied nuclei ($2 \leq Z \leq 92$) of the model $(GLDM)_1$

The result represented in the above Figure shows that the difference between the separation energies of Z and N ($Sp - Sn$) is proportional to the increase in the mass number of all nuclei in general. Separation energies are proven to reveal a wealth of information about the nuclear structure. They show the main shell closures at $P = P\,(magic)$ or $N = N\,magic$, which are represented by severe discontinuities in S_p, S_{2p}, S_n and S_{2n}, as a function of Z and N. The development of nuclear collectivity is reflected in a smooth variation of separation energy as a function of N and Z [28].

The proton subshell closures are reflected in the behavior of separation energies due to their nature (proton-neutron interaction). If the major proton's spherical shell closure does not influence the two neutron separation energies, the proton subshell closures are reflected in the behavior separation energies due to their nature (proton-neutron interaction). The numbers of Z and N are often equal in light, In stable heavy nuclei, the number of N is bigger than the number of Z. As a result, the energy required to remove a Z or N in light nuclei is roughly identical. The energy required to remove a Z from a heavy nucleus, on the other hand, is more than that required to remove an N, and this energy rises as the mass number rises. The Sn is the amount of energy required to break the nuclear bonds that hold the N in the nucleus. $Sp = Sv - Sc$ is made up of two parts. The nuclear part (Sv) is equivalent to neutron separation and represents the energy required to break nuclear bonds; the coulomb part Sc represents the additional effect of electrostatic repulsion between the Z and the remnant nucleus after the bonds have been broken. In heavy nuclei the following relation is obtained[29]:

$$\left.\begin{matrix} \mathrm{Sv} > \mathrm{S} \\ \mathrm{Sv} - \mathrm{Sn} \cong \mathrm{Sc} \end{matrix}\right\} \mathrm{A} > 50, \qquad (\mathbf{17})$$

This relationship (17) indicates that breaking a proton's nuclear connection requires more energy than separating an N from a heavy nucleus (A > 50). Sc is almost equal to the difference. To compensate for the electric repulsion energy, protons are more firmly bound by nuclear forces. Hence, proton emission (in comparison to neutron emission) is inhibited in heavier nuclei [29].

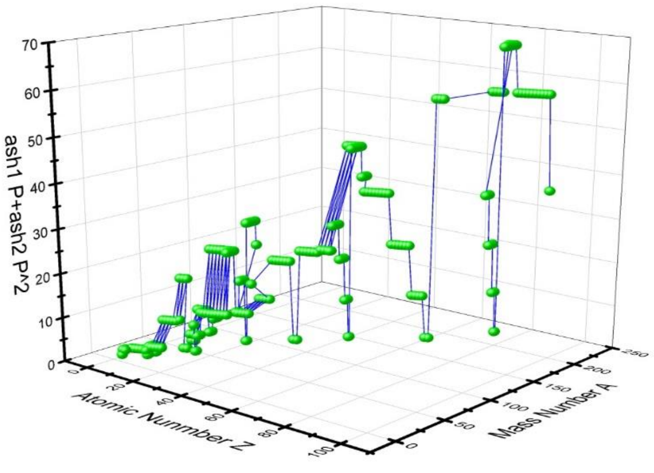

Fig. (5): The relationship between the shell term, represented in equation (13) and the mass number with atomic number , with the magic number of all magic nuclei understudy for the model (GLDM)₂

The above Figure shows how important it is to add the shell term in a model $(GLDM)_2$ to the LDM, as a result of the added increase in the nuclear binding energy of magic nuclei, whether in the number of Z or number of N or both. This is reflected in the approach of the results with the experimental values of the binding energy.

The valence nucleons that represent high energy levels will add another energy that will reflect positively on the magic nucleus binding energy, being the only one that participates in spinning the nucleus. For example, nuclei with atomic numbers (8,20,50) will increase their binding energy as the number of N in those nuclei increases, considering that they are isotopes belonging to a nucleus. This means that the increase in the number of N will be an additional energy gain for the nucleus. It should also be noted that the oxygen ($Z = 8, N = 7$) and carbon ($Z = 7, N = 8$) nuclei give very close values to the shell term, which is estimated at ($2.21903 MeV$), and the same is repeated with two nuclei calcium (Z=20, N=21) and scandium nucleus (Z=21, N=20) with values (10.773 MeV) because both of those nuclei contain magic numbers, either the number of N or the number of Z. It was found that the valence of the nucleon was able to improve the standard deviation from (0.323) to (0.144). This is due to the increase in the theoretical binding energy of the magic nuclei and getting it close to the experimental values, especially at the medium and heavy nuclei. Although this term is largely unrelated to the terms of the other formulation of SEMF, its relative contribution is very impressive.

The standard deviation values indicate the possibility of adopting the two models in interpreting magic numbers. The results can be considered very acceptable due to the improvement shown by the first model by (33%) and by (55%) for the second model. The more accurate is the fit of the coefficients, the better are the results.

Figures (6,7) show the acceptable agreement of the average experimental nuclear binding energy, the theoretical values of the original LDM and the theoretical values of the proposed models.

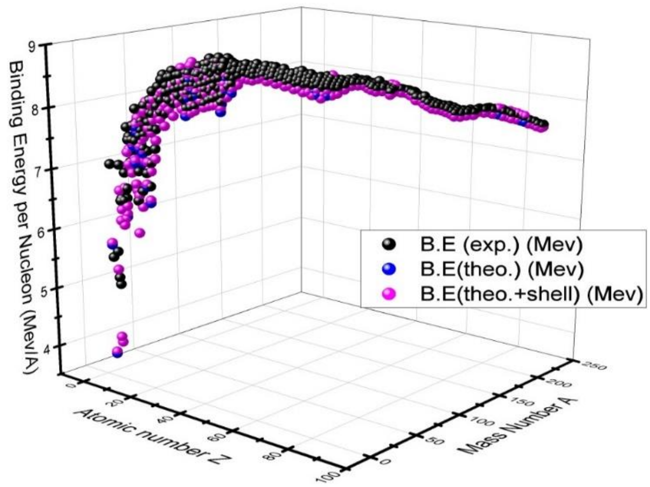

Fig. (6): The average nuclear binding energy with mass number A and atomic number Z for experimental and theoretical values of the original LDM and theoretical values of the models (GLDM)₁ which contains magic and non-magic nuclei

Table (3): The standard deviation values of the two models used before and after adding the shell term, along with the percentage of improvement that occurred in the two models

The model	Standard deviation(*σ)* *without shell*	The model	Standard deviation(*σ)* *with shell*	Improvement rate
LDM_1	0.188	$GLDM_1$	0.126	**33%**
LDM_2	0.323	$GLDM_2$	0.144	**55%**

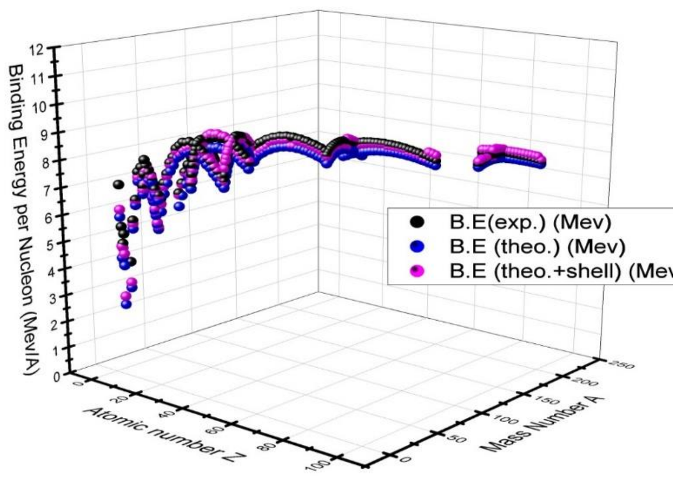

Fig. (7): The average nuclear binding energy with mass number A and atomic number Z for experimental and theoretical values of the original LDM and theoretical values of the model represented by magic nuclei only $(GLDM)_2$

Figure (7) shows, at first glance, that the rate of the nuclear binding energy does not show a sequential order as in the model $(GLDM)_1$, due to the fact that the model $(GLDM)_2$ deals the with magic nuclei only. It can be said in general, that there is an acceptable agreement for the rate of empirical nuclear binding energy with calculated theoretical values through the two models $(GLDM)_1$, $(GLDM)_2$. It could be noticed that the theoretical values, after adding the shell term, are close to the experimental values of two models $(GLDM)_1$,$(GLDM)_2$. This confirms the possibility of adopting the proposed shell-terms in the interpretation of the magic numbers for all the studied nuclei and a wide range, especially for the medium and heavy nuclei. It is observed that the actual increases in the binding energy of the magic numbers, led to the theoretical nuclear binding energy being remarkably close to the experimental values in addition to the values of the standard deviation and improvement percentage. This is shown in Table (3) for the two proposed models from the original model, which indicates the possibility of adopting the two models.

4. CONCLUSION

There is no doubt that the effect of the shell term on the LDM is very important. From the results, the following can be concluded:

- The updates of the values of the coefficients of the LDM obtained through the LSM show their suitability with previous similar works.
- The results obtained through the two proposed models in the interpretation of the magic numbers show that there are acceptable discrepancies between the experimental values and the theoretical values calculated in the current work.
- The statistical relationships of the standard deviation showed the possibility of adopting the two models in the interpretation of magic numbers.
- The two models $(GLDM)_1$, and $(GLDM)_2$, can be adopted because of the broad range of magic and non-magic nuclei that exceeded 480 nuclei.
- The results show that as the mass number A of all nuclei increases, the shell term obtained in equation (11) increases as well.

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6. REFERENCES

[1] Bohr, A. N., & Mottelson, B. R. (1998). *Nuclear Structure (In 2 Volumes)*. World Scientific PublishingCompany.https://doi.org/10.1142/3530

[2] Cohen, S., & Swiatecki, W. J. (1963). The deformation energy of a charged drop: Part v: Results of electronic computer studies. *Annals of Physics*, *22*(3), 406–437.

[3] Bohr, N., & Wheeler, J. A. (1939). The mechanism of nuclear fission. Physical Review, 56(5), 426. https://doi .org/10 .1103/PhysRev.56.426.

[4] Basdevant, J.-L., & Rich, J. (2005). Fundamentals in nuclear physics: From nuclear structure to cosmology. Springer Science & Business Media.

[5] Dong, J., Zuo, W., Gu, J., Wang, Y., & Peng, B. (2010). α-decay half-lives and Q α values of superheavy nuclei. *Physical Review C*, *81*(6), 64309. 10.1103/PhysRevC.81.064309

[6] Radha, C. A., Ramasubramanian, V., & Samuel, E. J. J. (2011). Shell Closures and Structural Information from Nucleon Separation Energies. *Electronic Journal of Theoretical Physics*, *8*(25), 327–342.

[7] Hirsch, J. G., Barbero, C. A., & Mariano, A. E. (2011). Nuclear masses, deformations and shell effects. *Journal of Physics: Conference Series*, *322*(1), 12017. doi:10.1088/1742-6596/322/ 1/012017

[8] Cakirli, R. B., & Casten, R. F. (2013). Nuclear binding and nuclear structure. *International Journal of Mass Spectrometry*, *349*, 187–191. https://doi.org/10.1016/j.ijms.

[9] UVS, S., & Lakshminarayana, S. (2015). On the role of strong interaction in understanding nuclear beta stability line and nuclear binding energy. *Proceedings of the DAE-BRNS Symp. on Nucl. Phys*, *60*, 118.

[10] Ankita, & Suthar, B. (2016). Nuclear binding energy using semi empirical mass formula. *AIP Conference Proceedings*, *1728*(1),20024. http://dx.doi.org/10.1063/1.4946074

[11] UVS, S., & Lakshminarayana, S. (2017). A new approach to understand nuclear stability and binding energy. *Proceedings of the DAE Symp. on Nucl. Phys*, *62*, 106.

[12] Chanda, D. (2018). A study of nuclear binding energy of magic number nuclei and energy splitting considering Independent particle shell model. *International Journal of Advanced Scientific Research and Management*, *3*(2).

[13] Sirma, K. K., Chelimo, L. S., & Khanna, K. M. (2020). A modified nuclear model for binding energy of nuclei. *World Scientific News*, *143*, 203–223.

[14] Karthika, C., Kokila, C., & Balasubramaniam, M. (2021). Appearance/Disappearance of Magic Number in Light Nuclei. Journal of Nuclear Physics, Material Sciences, Radiation and Applications,9(1),109–115. https://jnp.chitkara.edu.in/

[15] Pan, X., Zou, Y.-T., Liu, H.-M., He, B., Li, X.-H., & Xiang, D. (2021). Released energy formula for proton radioactivity based on the liquid-drop model. *Communications in Theoretical Physics*, *73*(7), 75302. http://doi.org/10.1088/1572-9494/abf822.

[16] Chowdhury, P. R., Samanta, C., & Basu, D. N. (2005). Modified Bethe–Weizsäcker mass formula with isotonic shift and new driplines. *Modern Physics Letters A*, *20*(21),1605–1618. https://doi.org/10.1016/0003-4916(63)90385-3.

[17] Royer, G., & Gautier, C. (2006). Coefficients and terms of the Liquid Drop Model and mass formula. *Physical Review C*, *73*(6), 67302. http://hal.in2p3.fr/in2p3-00089830

[18] Kim, B., & Cha, D. (2010). On the mass number dependence of the semi-empirical mass formula. *Journal of the Korean Physical Society*, *56*(5), 1546–1549.

[19] Vahid, M. M. A., Mohsen, M., & Bagher, A. M. (2017). Estimation of Semi-Empirical Mass Formula Coefficients. *Nuclear Science*, *2*(1), 11–15. doi:10.11648/j.ns.20170201.13

[20] Benzaid, D., Bentridi, S., Kerraci, A., & Amrani, N. (2020). Bethe–Weizsäcker semiempirical mass formula coefficients 2019 update based on AME2016. *Nuclear Science and Techniques*, *31*(1), 1–6. https://doi.org/10.1007/s41365-019-0718-8

[21] Mohammed-Azizi, B., & Mouloudj, H. (2021). Least-squares fitting applied to nuclear mass formulas. Solution by the Gauss–Seidel method. *International Journal of Modern Physics C*, 2250076. http://doi.org/10.1142/s0129183122500760

[22] Von Weizsäcker, C. F. (1935) The Theory of Nuclear Masses. Magazine for Physics, 96(7-8) 431-458 . https://doi.org/10.1007/BF01337700

[23] Ali, F. M., & Hadi, H. A. (2019). Improving of phenomenological formula for nuclear energy decay (Qα-value) of heavy and super heavy nuclei. *International Journal of Nuclear Energy Science and Technology*, *13*(2), 163–179.

[24] Krane, K. S. (1991). *Introductory nuclear physics*. John Wiley & Sons.

[25] Angeli, I. (1991). Effect of valence nucleons on nuclear binding energy. *EPL (Europhysics Letters)*, *16*(5), 429.

[26] Kirson, M. W. (2008). Mutual influence of terms in a semi-empirical mass formula. *Nuclear Physics*

A,*798*(1–2),29–60. http://doi.org/10.1016/j.nuclphysa.2007.10.011

[27] Jian-Min, D., Hong-Fei, Z., Wei, Z., & Jun-Qing, L. (2008). Half-lives of superheavy nuclei in Z= 113 alpha decay chain. *Chinese Physics Letters*, *25*(12), 4230.

[28] Kanungo, R. (2007). Shell closures in the N and Z= 40–60 region for neutron and proton-rich nuclei. *Physics Letters B*, *649*(1), 31–34.

[29] Blatt, J. M., & Weisskopf, V. F. (1952). Theoretical Phys. 21 (1953) 124. *Nuclear Physics (John Wiley & Sons, Inc., 15) Annual Progress Reports of ONR Generator New York*.