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## JOUKOWSKI AIRFOIL IN SUBSONIC FLOW

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## ABSTRACT

The thin airfoil theory for the case of steady, compressible, inviscid, and uniform flow past a Joukowski airfoil located along the horizontal axis is considered. Velocity potential is extended to a second-order approximation.

Flow quantities, at the body surface, such as speed, pressure coefficient, and drag coefficient are obtained, up to a secondorder approximation, for various values of Mach number "M" less than unity.

Approximate value of the critical Mach number in the related compressible flow about the thin airfoil when its section is contracted with, the Prandtl-Glauert correction factor, in the stream direction, is calculated.

The results are plotted and difficulties in calculations are discussed.

[^0]The problem to he considered is thn carn of steady, compressible, inviccid, and uniform flow past a symmetrical Joukowski thin airfoil as shown in Fig. 1


Fig. 1 Uniform flow past a symmetrical Joukowski airfoil

According to Van Dyke [1], it is convenient to work with the velocity notential, because the connection between the stream function and the velocity is complicated by variations of density.
For nlane flow of a perfect gas the full potential equation given by Oswatitsch 2 is

$$
\begin{aligned}
\frac{\partial^{2} \phi}{\partial x^{2}}+\frac{\partial^{2} \phi}{\partial y^{2}} & =m^{2}\left\{\left(\frac{\partial \phi}{\partial x}\right)^{2} \frac{\partial^{2} \phi}{\partial x^{2}}+2 \frac{\partial \phi}{\partial x} \frac{\partial \phi}{\partial y} \cdot \frac{\partial^{2} \phi}{\partial x \partial y}+\left(\frac{\partial \phi}{\partial y}\right)^{2} \frac{\partial^{2} \phi}{\partial y^{2}}\right. \\
& \left.+\left(\frac{\gamma-1}{2}\right)\left[\left(\frac{\partial \phi}{\partial x}\right)^{2}+\left(\frac{\partial \phi}{\partial y}\right)^{2}-1\right]\left[\frac{\partial^{2} \phi}{\partial x^{2}}+\frac{\partial^{2} \phi}{\partial y^{2}}\right]\right\} ;(1.1)
\end{aligned}
$$

where $M$ is the free-stream Mach number and $\gamma$ is the adiabatic index.
The dimensionless velocity components are

$$
\begin{equation*}
u=\frac{\partial \phi}{\partial x} \quad, \quad v=\frac{\partial \phi}{\partial y} \tag{1.2}
\end{equation*}
$$

Let the velocity of the flow at infinity along the body axis be $U=1$.
Let the thickness function be $T(x)=(1-x) \sqrt{1-x^{2}}$, describes a symmetrical Joukowski airfoil. The thickness ratio is $\varepsilon$ at midchord, and $1.30 \mathcal{E}$ at the thickest point, $x=-0.5$, as shown in Fig. 1
Soundary conditions considered are:
(i) A uniform stream at infinity is given by

$$
\begin{equation*}
\phi(x, r) \rightarrow x \quad \text { as } \quad x^{2}+y^{2} \longrightarrow \infty \tag{1.3.1}
\end{equation*}
$$

where $r^{2}=x^{2}+y^{2}$.

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(ii) The flow tangent condition to each fixed surface may be written as

$$
\begin{equation*}
\frac{v}{u}=\frac{\frac{\partial \phi}{\partial y}}{\frac{\partial \phi}{\partial x}}=\frac{d y}{d x}=\mp \varepsilon \frac{1+x-2 x^{2}}{\sqrt{1-x^{2}}} \text { at } y= \pm \varepsilon(1-x) \sqrt{1-x^{2}} \text {. } \tag{1.3.2}
\end{equation*}
$$

where the upper sign for the upper surface and the lower one for the lower surface.

ANALYSIS
From the asymptotic condition (1.3.1), it is possible to write velocity potential in the form

$$
\phi=x+\Phi
$$

such that the perturbation potential $\Phi$ vanishes at infinity. Hence, from (2.1), we get the following

$$
\begin{align*}
& \frac{\partial \Phi}{\partial x}=1+\frac{\partial \Phi}{\partial x} \quad, \quad \frac{\partial \Phi}{\partial y}=\frac{\partial \Phi}{\partial y} \\
& \frac{\partial^{2} \phi}{\partial x^{2}}=\frac{\partial^{2} \Phi}{\partial x^{2}}, \frac{\partial^{2} \Phi}{\partial x \partial y}=\frac{\partial^{2} \Phi}{\partial x \partial y}, \frac{\partial^{2} \phi}{\partial y^{2}}=\frac{\partial^{2} \Phi}{\partial y^{2}} \tag{2.2}
\end{align*}
$$

Upon employing (2.1) and (2.2) into (1.1), we get

$$
\begin{align*}
& \frac{\partial^{2} \Phi}{\partial y^{2}}+\beta^{2} \frac{\partial^{2} \Phi}{\partial x^{2}}=M^{2}\left\{\left[\left(\frac{\partial \Phi}{\partial x}\right)^{2}+2 \frac{\partial \Phi}{\partial x}\right] \frac{\partial^{2} \Phi}{\partial x^{2}}+2\left(\frac{\partial \Phi}{\partial x}+1\right) \frac{\partial \Phi}{\partial y} \frac{\partial^{2} \Phi}{\partial x \partial y}\right. \\
& \left.+\left(\frac{\partial \Phi}{\partial y}\right)^{2} \frac{\partial^{2} \Phi}{\partial y^{2}}+\left(\frac{\partial-1}{2}\right)\left[\left(\frac{\partial \Phi}{\partial x}+1\right)^{2}+\left(\frac{\partial \Phi}{\partial y}\right)^{2}-1\right]\left[\frac{\partial^{2} \Phi}{\partial x^{2}}+\frac{\partial^{2} \Phi}{\partial y^{2}}\right]\right\}
\end{align*}
$$

where $\beta^{2}=1-m^{2}$
Applying the Prandtl-
ital scale), we write

$$
\begin{equation*}
\tilde{x}=x \quad, \quad \tilde{y}=\beta_{Y} \tag{2.5}
\end{equation*}
$$

Hence we get

$$
\left.\begin{array}{cc}
\frac{\partial}{\partial x} \equiv \frac{\partial}{\partial \tilde{x}} & , \frac{\partial}{\partial y} \equiv \frac{\partial}{\partial \tilde{y}} \beta \\
\frac{\partial^{2}}{\partial x^{2}} \equiv \frac{\partial^{2}}{\partial \tilde{x}^{2}} \quad, \quad \frac{\partial^{2}}{\partial x \partial y} \equiv \beta \frac{\partial^{2}}{\partial \tilde{x} \partial \tilde{y}}, \frac{\partial^{2}}{\partial y^{2}} \equiv \beta^{2} \frac{\partial^{2}}{\partial \tilde{y}^{2}} \tag{2.6}
\end{array}\right\}
$$

Substituting (2.6) in (2.3), we get

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$$
\begin{align*}
\frac{\partial^{2} \Phi}{\partial \tilde{y}^{2}}+\frac{\partial^{2} \Phi}{\partial \tilde{x}^{2}} & =\left(\frac{M}{\beta}\right)^{2}\left\{\left[\left(\frac{\partial \Phi}{\partial \tilde{x}}\right)^{2}+2 \frac{\partial \Phi}{\partial \tilde{x}}\right] \frac{\partial^{2} \Phi}{\partial \tilde{x}^{2}}+2 \beta^{2}\left(\frac{\partial \Phi}{\partial \tilde{x}}+1\right) \frac{\partial \Phi}{\partial \tilde{y}} \frac{\partial^{2} \Phi}{\partial \tilde{x} \partial \tilde{y}}\right. \\
& +\beta^{4}\left(\frac{\partial \Phi}{\partial \tilde{y}}\right)^{2} \frac{\partial^{2} \Phi}{\partial \tilde{y}^{2}}+\left(\frac{\gamma-1}{2}\right)\left[\left(\frac{\partial \Phi}{\partial \tilde{x}}+1\right)^{2}+\beta^{2}\left(\frac{\partial \Phi}{\partial \tilde{y}}\right)^{2}\right. \\
& \left.-1]\left[\frac{\partial^{2} \Phi}{\partial \tilde{x}^{2}}+\beta^{2} \frac{\partial^{2} \Phi}{\partial \tilde{y}^{2}}\right]\right\} \tag{2.7}
\end{align*}
$$

Upon employing (2.1),(2.5) and (2.6) into boundary conditions (1.3.1) and (1.3.2) we get
(i)
and
(ii)

$$
\begin{aligned}
& \Phi \longrightarrow 0(1) \quad \text { as } \beta^{2} \tilde{x}^{2}+\tilde{y}^{2} \longrightarrow \infty \\
& \frac{\beta \frac{\partial \Phi}{\partial \tilde{y}}}{1+\frac{\partial \Phi}{\partial \tilde{x}}}=\mp \varepsilon\left[\frac{1+\tilde{x}-2 \tilde{x}^{2}}{\sqrt{1-\tilde{x}^{2}}}\right] \text { at } \tilde{y}= \pm \varepsilon \beta(1-\tilde{x}) \sqrt{1-\tilde{x}^{2}}
\end{aligned}
$$

We seek the asymptotic expansion of the solution as the thinkness parameter $\mathcal{E} \rightarrow 0$. In the limit, the Joukowski airfoil degenerates to a line which causes no disturbance of the free stream, so the basic solution is the uniform parallel flow.
We tentatively assume that the asymptotic series for the perturbation potential $\Phi$ has, for a given thickness function $T(x)$, the form

$$
\begin{equation*}
\Phi(\tilde{x}, \tilde{y} ; \varepsilon) \sim \varepsilon \Phi_{1}(\tilde{x}, \tilde{y})+\varepsilon^{2} \Phi_{2}(\tilde{x}, \tilde{y})+\varepsilon^{3} \Phi_{3}(\tilde{x}, \tilde{y})+\ldots \tag{2.9}
\end{equation*}
$$

Substituting (2.9) in (2.7) and equating like powers of $\mathcal{E}$. we get

$$
\begin{align*}
& \frac{\partial^{2} \Phi_{1}}{\partial \tilde{y}^{2}}+\frac{\partial^{2} \Phi_{1}}{\partial \tilde{x}^{2}}=0 \\
& \frac{\partial^{2} \Phi_{2}}{\partial \tilde{y}^{2}}+\frac{\partial^{2} \Phi_{2}}{\partial \tilde{x}^{2}}=\left(\frac{M}{\beta}\right)^{2} \quad(\gamma+1) \frac{\partial \Phi_{1}}{\partial \tilde{x}} \frac{\partial^{2} \Phi_{1}}{\partial \tilde{x}^{2}} \\
&+2 \beta^{2} \frac{\partial \Phi_{1}}{\partial \tilde{y}} \frac{\partial^{2} \Phi_{1}}{\partial \tilde{x} \partial \tilde{y}}+(\gamma-1) \beta^{2} \frac{\partial \Phi_{1}}{\partial \tilde{x}} \frac{\partial^{2} \Phi_{1}}{\partial \tilde{y}^{2}} \tag{2.10.2}
\end{align*}
$$

Substituting (2.9) in (2.8.1) and equating like powers of $\mathcal{E}$. we get


$$
\begin{equation*}
\text { as } \beta^{2} \tilde{x}^{2}+\tilde{y}^{2} \longrightarrow \infty \tag{2.11.1}
\end{equation*}
$$

$$
\begin{equation*}
\text { as } \beta^{2} \tilde{x}^{2}+\tilde{y}^{2} \rightarrow \infty \tag{2.11.2}
\end{equation*}
$$

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In order to substitute the expansion (2.9) in the tangency condition (2.8.2) we must transfer (2.8.2) to the axis $\tilde{y}=0$. This can be done by using Taylor series expansion on the $\tilde{x}$ axis and equating like powers of $\mathcal{E}$, we get

$$
\begin{equation*}
\frac{\partial \Phi_{1}}{\partial \tilde{y}}(\tilde{x}, O \pm)=\mp\left(\frac{1+\tilde{x}-2 \tilde{x}^{2}}{\beta \sqrt{1-\tilde{x}^{2}}}\right) \tag{2.12.1}
\end{equation*}
$$

$\frac{\partial \Phi_{2}}{\partial \tilde{y}}(\tilde{x}, O \pm)=\mp\left(\frac{1+\tilde{x}-2 \tilde{x}^{2}}{\beta \sqrt{1-\tilde{x}^{2}}}\right) \frac{\partial \Phi_{1}}{\partial \tilde{x}}(\tilde{x}, 0)$

$$
\begin{equation*}
-\beta(1-\tilde{x}) \sqrt{1-\tilde{x}^{2}} \frac{\partial^{2} \Phi_{1}}{\partial \tilde{y}^{2}}(\tilde{x}, u) \tag{2.12.2}
\end{equation*}
$$

Here $\tilde{y}=O \pm$ refers to the top and bottom sides of the slit to which théaairfoil degenerates in the limit as $\mathcal{E} \rightarrow 0$, and across which $\frac{\partial \Phi}{\partial \tilde{Y}}$ is discontinuous.

## SOLUTION OF THE FIRST-ORDER PROBLEM

Solve

$$
\begin{equation*}
\frac{\partial^{2} \Phi_{1}}{\partial \tilde{x}^{2}}+\frac{\partial^{2} \Phi_{1}}{\partial \tilde{y}^{2}}=0 \tag{3.1}
\end{equation*}
$$

Subject to
(i)

$$
\begin{align*}
& \text { (i) } \Phi_{1} \longrightarrow o(1) \quad \text { as } \beta^{2} \tilde{x}^{2}+\tilde{y}^{2}  \tag{3.2.1}\\
& \text { (ii) } \frac{\partial \Phi_{1}}{\partial \tilde{y}}\left(\tilde{x}, O_{ \pm}\right)=\mp\left(\frac{1-\tilde{x}-2 \tilde{x}^{2}}{\sqrt{1-\tilde{x}^{2}}}\right) \tag{3.2.2}
\end{align*}
$$

which is equivalent to an incompressible problem.
Solution of the first-order problem (given by Ching and Roth [8]) is:

$$
\begin{align*}
& \frac{\partial \Phi_{1}}{\partial \tilde{x}}(\tilde{x}, \tilde{y})=  \tag{3.3}\\
& \frac{1}{\pi} \int_{-1}^{1} \frac{(\tilde{x}-s) \frac{\partial \Phi_{1}}{\partial \tilde{y}}(s, 0+)}{(\tilde{x}-s)^{2}+\tilde{y}^{2}} d s \\
& \frac{\partial \Phi_{1}}{\partial \tilde{y}}(\tilde{x}, \tilde{y})=\frac{1}{\pi} \int_{-1}^{1} \frac{\tilde{y} \frac{\partial \Phi_{1}}{\partial \tilde{y}}(s, 0+)}{(\tilde{x}-s)^{2}+\tilde{y}^{2}} d s
\end{align*}
$$

which is obtained by replacing the body by a distribution sources and sinks (due to the symmetry of the airfoil) and with equal numbers (due to the closed shape of the body).Also, solution can be verified by direct substitution.

Upon employing the condition (3.2.2) in (3.3) and (3.4) and along the $\tilde{x}$-axis, using Bois [3], we get:

$$
\begin{align*}
\frac{\partial \Phi_{1}}{\partial \tilde{x}}(\tilde{x}, 0) & =-\frac{1}{\pi \beta} \int^{1} \frac{1+s-2 s^{2}}{(\tilde{x}-s) \sqrt{1-s^{2}}} d s \\
& =\frac{1}{\beta}(1-2 \tilde{x}) \tag{3.5}
\end{align*}
$$

$\frac{\partial \Phi_{1}}{\partial \tilde{y}}(\tilde{x}, 0)=0$
$\begin{aligned} \frac{\partial^{2} \Phi_{1}}{\partial x^{2}}(\tilde{x}, 0) & =\frac{1}{\pi \beta} \int_{-1}^{1} \frac{1+s-2 s^{2}}{(\tilde{x}-s)^{2} \sqrt{1-s^{2}}} d s \\ & =\frac{-2}{\beta}\end{aligned}$

$$
\begin{equation*}
=\frac{-2}{\beta}^{-1} \tag{3.7}
\end{equation*}
$$

$\frac{\partial^{2} \Phi_{1}}{\partial \tilde{y}^{2}}(\tilde{x}, 0)=\frac{2}{\beta}$
SOLUTION OF THE SECOND-ORDER PROBLEM

$$
\begin{align*}
\frac{\partial^{2} \Phi_{2}}{\partial \tilde{x}^{2}}+\frac{\partial^{2} \Phi_{2}}{\partial \tilde{y}^{2}} & =\left(\frac{M}{\beta}\right)^{2}\left[(\gamma+1) \frac{\partial \Phi_{1}}{\partial \tilde{x}} \frac{\partial^{2} \Phi_{1}}{\partial \tilde{x}^{2}}+2 \beta^{2} \frac{\partial \Phi_{1}}{\partial \tilde{y}} \frac{\partial^{2} \Phi_{1}}{\partial \tilde{x} \partial \tilde{y}}\right. \\
& \left.+(\gamma-1) \beta^{2} \frac{\partial \Phi_{1}}{\partial \tilde{x}} \frac{\partial^{2} \Phi_{1}}{\partial \tilde{y}^{2}}\right] \tag{4.1}
\end{align*}
$$

Subject to
(i) $\Phi_{2}-0$ as $\beta^{2} \tilde{x}^{2}+\tilde{y}^{2} \longrightarrow \infty$
(ii) $\frac{\partial \Phi_{2}}{\partial \tilde{y}}(\tilde{x}, 0 \pm)=\mp\left(\frac{1+\tilde{x}-2 \tilde{x}^{2}}{\beta \sqrt{1-\tilde{x}^{2}}}\right) \frac{\partial \Phi_{1}}{\partial \tilde{x}}(\tilde{x}, 0)$

$$
\begin{equation*}
-\beta(1-\tilde{x}) \sqrt{1-\tilde{x}^{2}} \frac{\partial^{2} \Phi_{1}}{\partial \tilde{y}^{2}}(\tilde{x}, 0) \tag{4.2.2}
\end{equation*}
$$

Substituting (3.5) and (3.8) in (4.2.2) we get

$$
\begin{equation*}
\frac{\partial \Phi_{2}}{\partial \tilde{y}}(\tilde{x}, O \pm)=\mp \frac{(1-2 \tilde{x})\left(1+\tilde{x}-2 \tilde{x}^{2}\right)}{\beta^{2} \sqrt{1-\tilde{x}^{2}}}-2(1-\tilde{x}) \sqrt{1-\tilde{x}^{2}} \tag{4.2.3}
\end{equation*}
$$

In order to calculate surface values, as we will see in the next section, we need only $\frac{\partial \Phi_{2}}{\partial \tilde{x}}(\tilde{x}, 0)$ from the second-order problem. Therefore, it is now appropriate to apply the Hillert transformation, for $\Phi_{2}$ more details see Muskhelishvili $[4]$.


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where P.V. refers to the Cauchy principal value of the divergent integral when $-1 \leq \tilde{x} \leq 1$.

Upon employing (4.2.3) into (4.3) for $\tilde{y}=0+$ and carrying out the integrations, using Bois [3], we get

$$
\begin{equation*}
\frac{\partial \Phi_{2}}{\partial \tilde{x}}(\tilde{x}, 0)=-\frac{1}{2 \beta^{2}}\left(\frac{1-\tilde{x}}{1+\tilde{x}}\right)(1+2 \tilde{x})^{2} \tag{4.4}
\end{equation*}
$$

## FLOW OUANTITIES AT THE BODY SURFACE

Flow quantities, at the body surface, such as the speed $q_{\text {. }}$, the pressure coefficient $C_{p}$, and the drag coefficient $C_{d}$ can be expressed as power series in the thickness parameter ${ }^{\mathcal{E}}$ e by relating them, again through Taylor series expansion, to the velocity components on the $\tilde{x}$-axis.
Thus the surface speed $g_{S}$ is found to be

$$
\begin{align*}
q_{s}^{2} & =1+\varepsilon\left[2 \frac{\partial \Phi_{1}}{\partial \tilde{x}}(\tilde{x}, 0)\right]+\varepsilon^{2}\left[\left(\frac{\partial \Phi_{1}}{\partial \tilde{x}}\right)^{2}(\tilde{x}, 0)+2 \frac{\partial \Phi_{2}}{\partial \tilde{x}}(\tilde{x}, 0)\right. \\
& \left.+\beta^{2}\left(\frac{\partial \Phi_{1}}{\partial \tilde{y}}\right)^{2}(\tilde{x}, 0)\right]+\ldots \tag{5,1}
\end{align*}
$$

Using $(3.5),(3,6),(2.5)$, and $(4.4)$ we get

$$
\begin{equation*}
q_{s}^{2}=1+\varepsilon\left[\frac{2(1-2 \tilde{x})}{\beta}\right]+\varepsilon^{2}\left[\frac{2 \tilde{x}\left(4 \tilde{x}^{2}-3\right)}{\beta^{2}(1+\tilde{x})}\right]+\ldots \tag{5.2}
\end{equation*}
$$

The surface pressure coefficient $c_{p_{s}}$, following Curle and
Davies [5], is qiven by

$$
\begin{equation*}
c_{p_{s}}=\frac{p-p_{o}}{\frac{1}{2} \rho_{0} U^{2}}=1-q_{S}^{2} \tag{5.3}
\end{equation*}
$$

Hence, upon employing (5.2) into (5.3), we get

$$
\begin{equation*}
c_{p_{s}}=-\varepsilon\left[\frac{2}{\beta}(1-2 \tilde{x})\right]-\varepsilon^{2}\left[\frac{2 \tilde{x}\left(4 \tilde{x}^{2}-3\right)}{\beta^{2}(1+\tilde{x})}\right]-\ldots \tag{5.4}
\end{equation*}
$$

The drag coefficient $C_{d}$ can be calculated by integrating the pressure over the surfaç of the airfoil according to

$$
\begin{equation*}
c_{d}=\frac{1}{\frac{1}{2} p_{0} U^{2}} \int p_{s} d y \quad \text { on } \quad y=\varepsilon(1-x) \sqrt{1-x^{2}} \tag{5.5}
\end{equation*}
$$

Using (5.3), we get

$$
\begin{equation*}
\frac{p_{S}}{\frac{1}{2} \rho_{0} U^{2}}=c_{p_{s}}+\frac{p_{0}}{\frac{1}{2} \rho_{0} U^{2}} \tag{5.6}
\end{equation*}
$$

Hence (5.5) becomes


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$$
\begin{align*}
c_{d_{s}} & =-\varepsilon \int\left\{-\varepsilon\left[\frac{2(1-2 x)}{\beta}\right]-\varepsilon^{2}\left[\frac{2 x\left(4 x^{2}-3\right)}{\beta^{2}(1+x)}\right]\right. \\
& \left.-\cdots+2 \frac{p_{0}}{\rho_{0}}\right\}\left\{\frac{1+x-2 x^{2}}{\sqrt{1-x^{2}}}\right\} d x \tag{5.7}
\end{align*}
$$

Carrying out the integrations in (5.7),keeping only firstand second-order terms gives

$$
\begin{equation*}
c_{d_{s}} \sim-\frac{2 \pi \varepsilon^{2}}{\beta} \tag{5.8}
\end{equation*}
$$

This result is obviously incorrect since it contradicts d'Alembert's principle. This is due to the rounded leading edge.
Jones [6] (see Van Dyke [1], pp.55) has shown that this leadingedge drag can be recovered by calculating the drag not from surface pressures but with a momentum contour that avoids the region of invalidity near the leading edge.

## CONCLUSION AND DISCUSSIONS

The reqion of invalidity is within a distance from the leading edge of the order of the nose radius which is $4 \varepsilon^{2}$ 。 In that vicinity the airfoil can be approximated by parabola having the same nose radius.


Fiq. 2 Osculating parabola for the leading edge
The first and second approximations for $q_{S}$ and $C_{p}$ versus $-1<x<1$ are plotted in Fig. 3 and Fig. 4 , where the divergence of the series near the singular points $x=-1$ is evident. From equations (5.2) and (5.4) we find:

$$
\begin{align*}
& q_{s 2}=-\frac{1}{2 \beta^{2}}\left(\frac{1-x}{1+x}\right)(1+2 x)^{2}  \tag{6.1}\\
& c_{p_{s 2}}=-\frac{2 x}{\beta^{2}}\left(\frac{4 x^{2}-3}{1+x}\right) \tag{6.2}
\end{align*}
$$

Hence, we find that as the Mach number $M$ of the uniform stream increases so will the maximum surface sneed until it becomes equal to the local speed of sound $c$. When this occurs, the free


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stream Mach number $M$ is said to have reached a critical value for the flow and we write $M=M_{c r}$.
Using the second order approximation for the maximum surface speed and following the method suggested by curle and Davies [7], we get

$$
\begin{equation*}
\left(1-M_{c r}\right)\left(1-M_{c r}^{2}\right) \sim-1.85 M_{c r} \varepsilon^{2} \tag{6.3}
\end{equation*}
$$

from which More may be calculated. If $\varepsilon \ll 1$, ( $1-M_{c r}$ ) must be
small and wet

$$
\begin{equation*}
M_{c r} \sim 1-0.15 \varepsilon^{2} \tag{6.4}
\end{equation*}
$$

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