



Computing Lower Critical Mach Number by a Simple Method

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ABSTRACT

There are three well-known methods for computing lower critical Mach number of airfoils. The method of Prandtl-Glauert is an approximate method as it is based on a linearized approach. Two other methods based on a non-linearized approach are found in literature. They are methods of Karman-Tsein and Christianovitch. The latter is more accurate than the former, but it is of less accuracy.

This paper presents a simple method of determining lower critical Mach number of airfoils. The present method is as accurate as the method of Christianovitch. In the mean time, it is convenient to computational aerodynamics.

NOMENCLATURE

C_p pressure coefficient
 M Mach number
 SQR magnitude defined by eq. (3.b)
 γ ratio of specific heats
 A dimensionless velocity = v/v_{cr}

Subscripts

c compressible
 cr critical
 i incompressible
 min minimum
 ∞ free stream
 l lower value

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INTRODUCTION

Determining lower critical Mach number of two-dimensional airfoils is of important significance as it determines the upper limit of pure subsonic flow and the lower limit of transonic flow. That is why, there is a need to a simple rapid and accurate computational technique.

In literature, there are three well-known methods for computing lower critical Mach number of airfoils [1,2,3]. The first method is that of Prandtl-Glauert which is based on a linearized approach. That is the reason of its bad accuracy. The remaining two methods are based on a non-linearized approach in the hodograph plane. They are the method of Karman-Tsien and that of Christianovitch. The former is less accurate than the latter because it includes two approximations [3]. One approximation is the tangent-gas approximation and the other one is approximating of flow density as a function of Mach number by Maclaurin expansion then taking first two terms only. The method of Christianovitch is more accurate as it has one approximation only; that is the second one. But this method is not convenient for computational aerodynamics. The first two methods could be applied in the area of computational aerodynamics but with bad accuracy.

The present method is based on a non-linearized in the hodograph plane and it has one approximation only. It is the same approximation as the method of Christianovitch. The former could be considered as an active modification to the latter. Moreover, it is more simpler than the latter. Consequently, it is more convenient for computational aerodynamics.

MATHEMATICAL DEVELOPMENT

Basic Assumptions

- a) Two-dimensional fluid flow.
- b) Steady compressible fluid flow.
- c) Pure subsonic flow (no mixed flow).
- d) $\gamma = 1,4$ for the air. The solution is carried out in the hodograph plane.

Mathematical Formulation

Deducing the formula, of interest, that computes the lower critical Mach number of airfoils starts by a relation between compressible and incompressible dimensionless velocities M_c and M_i respectively. This relation is presented in a detailed form in reference 4. The relation of interest has the following form [4].

$$(M_{\infty c}) = 1,095 \sqrt{1 - \sqrt{1 - 1,68 M_{\infty i}^2}} \quad (1)$$

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The equation of pressure coefficient in case of incompressible fluid flow in terms of λ has the form [3,4]:

$$C_{pi} = 1 - (\lambda_i / \lambda_{\infty i})^2$$

$$\lambda_i = \lambda_{\infty i} \sqrt{1 - C_{pi}}$$

As the Mach number M_{∞} reaches critical Mach number $M_{\infty cr1}$, hence $C_{pi} = C_{pi min}$ and $\lambda_c = 1$ and in this case:

$$\lambda_i = 0,7576 \quad [3, 4]$$

$$\text{Hence, } 0,7576 = \lambda_{\infty i} \sqrt{1 - C_{pi min}} \quad (2)$$

Finding out $\lambda_{\infty i}$ from eq. (2) and substituting its magnitude into eq. (1), the expression of $\lambda_{\infty c}$ at $M_{\infty} = M_{\infty cr1}$ has the form:

$$(\lambda_{\infty c})_{M_{\infty cr1}} = 1,095 \sqrt{1 - SQR} \quad (3.a)$$

$$\text{where: } SQR = \sqrt{(3,575 \cdot 10^{-2} - C_{pi min}) / (1 - C_{pi min})} \quad (3.b)$$

Applying the basic relation of gas dynamics between M_{∞} and λ_{∞} in the critical condition: $M_{\infty} = M_{\infty cr1}$ and $\lambda_{\infty} = \lambda_{\infty cr}$

$$M_{\infty cr1} = (\lambda_{\infty})_{M_{\infty cr1}} / \sqrt{(1,2 - 0,2 \lambda_{\infty cr})} \quad (4)$$

Substituting eq.(3.a) into eq.(4) and rearranging the expression, the formula of $M_{\infty cr1}$ could be written in the form:

$$M_{\infty cr1} = 2,236 / \sqrt{(\{5/(1 - SQR)\} - 1)} \quad (5)$$

Where SQR is determined by eq. (3.b). Eq.(5) determines the numerical value of lower critical Mach number in terms of the minimum of pressure coefficient of the airfoil in the incompressible fluid flow.

COMPARISON & ANALYSIS

The present method is compared to the three-well-known methods: Prandtl-Glauert, Karman-Tsein and Christianovitch, in the following way;

- Pressure distribution of the incompressible fluid flow C_{pi} on a classical airfoil NACA 4412 is measured at 40 m/s in the closed circuit low-speed wind tunnel in the laboratory of aerodynamics. Then, the lower Critical Mach number is computed by four methods. Table 1 presents a comparison of numerical results obtained.
- Figure 1 presents the dependence of $M_{\infty cr1}$ versus $C_{pi min}$ by the four methods of interest.

It is evident that the present method is as accurate as the method of Christianovitch because it is based on the same non linearized approach . In the mean time, it is more accurate than other two methods: Karman-Tsein and Prandtl-Glauret. Moreover, it is more convenient than that of Christianovitch for computational aerodynamics. This means that the method presented in this paper has removed weak points of methods of Karman-Tsein and Christianovitch with comparable accuracy.

CONCLUSION

The method presented in this paper is simple and rapid to determine lower critical Mach number of airfoils. It is as accurate as the method of Christianovitch. Hence, it is convenient for the area of computational aerodynamics.

REFERENCES

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Table 1. Lower Critical Mach Number of NACA 4412

| Method | Values of $M_{\infty cr1}$ |
|-----------------------|----------------------------|
| Prandtl-Glauert (P-G) | 0,672 |
| Karman-Tsein (K-T) | 0,647 |
| Christianovitch (Ch) | 0,603 |
| Present Method (Pr) | 0,60315 |

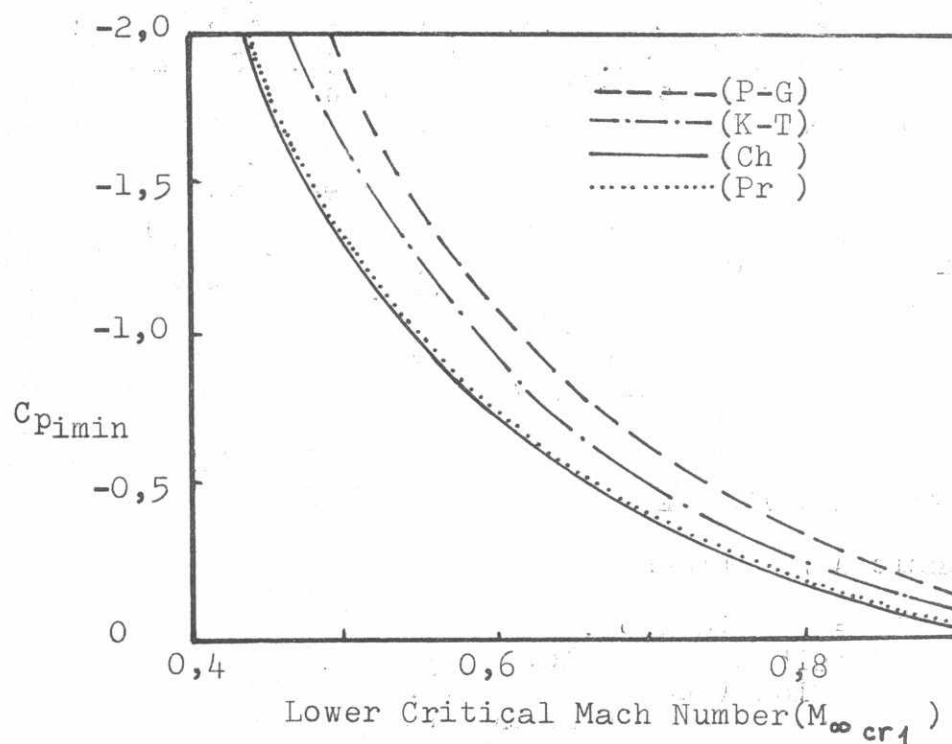


Fig.1 Comparison Between Four Methods