AERODYNAMIC CHARACTERISTICS OF QUASI-SLENDER WING - BODY COMBINATIONS AT SUPERSONIC VELOCITIES

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ABSTRACT

A procedure for determining the lift and pitching moment derivatives of not so slender wing-body combination in supersonic flow is presented. It is based on an introduced extension of the approximate slender body theory by retaining additional terms that have been neglected by virtue of body slenderness.The extension included the dependence of the solution on body geometry and flow Mach number. An application is given for the case of thin flat triangular wing mounted on conical pointed body. Computed results are compared with the original theory and other published theoritical and experimental data where good agreement is obtained.

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1. INTRODUCTION

Slender and quasi-slender wing-body combinations are currently appearing in the aerodynamic design of modern high speed airplanes and missiles. Among analytical techniques for determining their aerodynamic characteristics is the Slender Body theory originated by M.Munk[1] and R.T.Jones [2]. The theory solves approximately the flow around slender wings and bodies elongated in direction of flight at small angles of attack. In view of body elongation in direction of flight , the theory assumes that the flow pattern near the body at any transverse section is the same as in two dimensional incompressible flow. The theory proved to be applicable throughout the whole flight speeds range, from M=0 to supersonic velocities. Being simple and useful, it was extended to deal with different wing-body combinations [3]&[7] and with unsteady flows[6].

Quasi-slender bodies have larger relative lateral dimensions than slender bodies. Application of Slender Body theory for determination of their aerodynamic characteristics leads to very approximate results. Several extensions were derived for modifying the theory to deal with quasi-slender bodies [7]& [11]. The extension given in [7] uses a complicated procedure based on solving a system of integral equations ,and is applicable to wings only. Extension given in [11] has its validity limited to bodies of revolution only. The presented work is an extension of the Slender Body theory for solving the aerodynamic characteristics of quasi-slender wing-body combinations having arbitrary cross-sectional shapes and wing planforms. It introduces a more accurate solution of the linearized potential flow equation upon which the original theory is based.

2. INTRODUCED EXTENSION OF THE SLENDER BODY THEORY

The linearized supersonic potential flow around bodies is given by the solution of the partial differential equation

$$(M_\infty^2-1)\frac{\partial^2\phi}{\partial x^2}-\frac{\partial^2\phi}{\partial y^2}-\frac{\partial^2\phi}{\partial z^2}-2M_\infty\frac{\partial^2\phi}{\partial x\partial t}-M_\infty^2\frac{\partial^2\phi}{\partial t^2}=0 \qquad (2.1)$$

where $\phi\,(x,y,z,t)$ is the nondimensional perturbation velocity potential, and M_∞ is the flight Mach number. The solution of Eqn.(2.1) is subjected to the basic boundary conditions of flow around bodies :

i-Zero normal velocity component at the body surface

$$\left.\frac{\partial\phi}{\partial\nu}\right|_c=\frac{D\nu}{Dt}=\frac{\partial\nu}{\partial t}+\frac{\partial\nu}{\partial x} \qquad (2.2a)$$

where ν is the direction of the local normal to body surface C , and

ii-Vanishing perturbation potential and perturbation velocity on and outside the generated Mach cone.

$$\phi(x,y,z,t)=\frac{\partial}{\partial x}\phi(x,y,z,t)=0 \qquad y\geqslant z\geqslant\frac{x}{\sqrt{M_\infty^2-1}}\ ,\quad 0\leqslant x\leqslant\infty \qquad (2.2b)$$

Equation(2.1)and its boundary conditions are derived in the moving system of coordinates shown in Fig.2.1, where x-axis coinsides with the direction of undisturbed velocity U_∞ . All linear dimensions are nondimensionalized with respect to body length ℓ and the time with respect to the quantity $\ell\,/\,U_\infty$, which can be interpretted as the time taken by the body to travel its characteristic length ℓ .

For steady and quasi-steady flows the time derivatives of the perturbation potential are zero or negligibly small respectively,and Eqn(2.1) reduces to the known Prandtl Glauert equation

$$(M_\infty^2-1)\frac{\partial^2\phi}{\partial x^2}-\frac{\partial^2\phi}{\partial y^2}-\frac{\partial^2\phi}{\partial z^2}=0 \qquad (2.3)$$

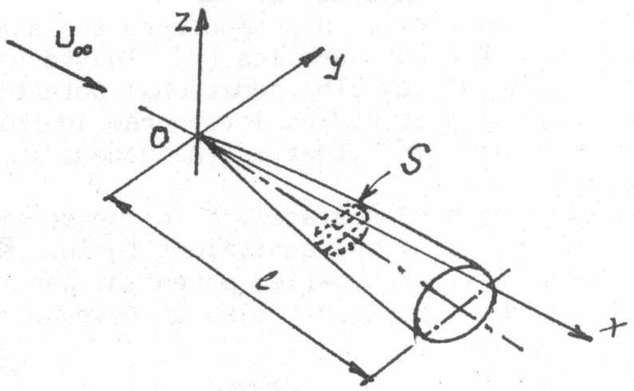

Fig (2.1) Body and Coordinate System

In terms of cylindrical coordinates x , r and φ, Eqn.(2.3) changes to

$$\frac{\partial^2\phi}{\partial r^2}+\frac{1}{r}\frac{\partial\phi}{\partial r}+\frac{1}{r^2}\frac{\partial^2\phi}{\partial\varphi^2}=m^2\frac{\partial^2\phi}{\partial x^2} \qquad (2.4)$$

where $m=\sqrt{M_\infty^2-1}$

The solution of Eqn.(4.2) which converges far from the body is obtained by applying Laplace transform and the boundary conditions (2.2)

$$\bar{\phi}(p,y,z,t)=\sum K_n(mpr)\quad A_n(p,t)\cos n\varphi+B_n(p,t)\sin n\varphi \qquad (2.5)$$

where $\bar{\phi}$ is the Laplace transform of ϕ , p is the transformation variable and $K_n(mpr)$ is the modified Bessel function of the second kind.
The expansions of $K_n(mpr)$ for small values of their arguments are :

$$K_0(mpr)\doteq-\left[\ln\frac{mpr}{2}+\gamma\right]\left[1+O(r^2)\right] \qquad (2.6a)$$

$$K_1(mpr)\doteq\frac{1}{mpr}+\frac{mpr}{2}\left[\ln\frac{mpr}{2}+\gamma-\frac{1}{2}\right]+\frac{1}{mpr}\,O(r^2) \qquad (2.6b)$$

$$K_n(mpr)=\left[\frac{(n-1)!}{2}\left(\frac{2}{mpr}\right)^n\right]\left[1+O(r^2)\right] \qquad (2.6c)$$

where γ is the Euler constant (γ = 0.57...)
Substituting the above relations in (2.5) and performing the inverse Laplace transform ,the general solution of (2.4) is given by the real part of the complex potential

$$\phi(x,y,z,t)=Rl(\chi)=Rl\ a_0(x,t)\ln w+b_0(x,t)+\sum\frac{a_n(x,t)}{w^n}+\Delta\chi \qquad (2.7a)$$

$$\Delta \chi = -\frac{m^2}{4}\left[\frac{\partial^2 \bar{a}_1(x,t)}{\partial x^2} + \int_0^{x-mr} \frac{\partial^2 \bar{a}_1(x_0,t)/\partial x_0^2}{\sqrt{(x-x_0)^2-(mr)^2}}\right] w \tag{2.7b}$$

where w complex variable

a_0, b_0 a_n coefficients to be determined

$\bar{a}_1$ complex conjugate of a_1

Equation (2.7a),without the last term,corresponds to the slender body solution of Adams and Sears [7],Ward[8]and Miles [9]. They considered only the first terms in the expansions of K_n .The additional potential $\Delta\chi$ represents the extension for quasi-slender bodies,introduced in this work by retaining the terms up to order r^2 ,that is the second term in (2.6b) in addition to the first one.

For the flow solution near the body surface , r in the square roots under the integral sign,is substituted by the equivalent radius R_e given in terms of x only.The additional perturbation potential can be then interpretted as the real part of the complex potential corresponding to additional perturbation cross flow velocity

$$\Delta v = -\frac{m^2}{4}\left[\frac{\partial^2 a_1(x,t)}{\partial x^2} + \int_0^{x-mR_e} \frac{\partial^2 a_1(x_0,t)/\partial x_0^2}{\sqrt{(x-x_0)^2-m^2R_e^2(x)}}\right] \tag{2.8}$$

The complex transverse force

$$F(x,t) = Y(x,t) + i\,Z(x,t) \tag{2.9}$$

acting on the body between O and x is given by[9],

$$\frac{\partial F}{\partial x} = \rho_0 U^2 \ell^2 \frac{DQ}{Dt} \tag{2.10}$$

where Q is the nondimensional virtual momentum given by

$$Q = 2\pi a_1 + \frac{D}{D_t}(w_g S) \tag{2.11}$$

and S area of local cross-section.

w_g position of section centroid.

In the slender body solution [9],the coefficient a_1 is determined using the conformal mapping of the exterior of the body cross-sections S to the exterior of circles with radii c keeping the flow at infinity undistorted.

$$a_1 = \bar{v}\,Res_f + v c^2 - a_0 w_g \tag{2.12}$$

where Res_f is the residue of the mapping function.Substituting the above relation in (2.11)we get

$$Q(x,t) = (2\pi c^2 - S)v + 2\pi\, Res_f\, \bar{v} \tag{2.13}$$

where $v = Dw_g/D_t$ is the cross flow velocity component and $\bar{v}$ is its complex conjugate.For motions in the vertical plane $v = -\bar{v}$,and equation (2.13) becomes

$$Q(x,t) = [2\pi(c^2 - Res_f) - S]\,v \tag{2.14}$$

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For circular cross-sections, the quantity in square brackets equals the section area. Accordingly, for cross-sections of general form, this quantity can be interpretted as the area of an equivalent circle of raduis R_e given by

$$S_e = [2\pi(c^2 - Res)_f - S] = \pi R_e^2 \tag{2.15}$$

This circular cross-section has the same virtual momentum as that of the given body section of general form.
By virtue of (2.14) and (2.10), the body formed by these equivalent cross-sections has the same aerodynamic loading (excluding viscous effects) as the original body. Using this argument, the integral in the expression for the additional potential of the presented extention is evaluated. For calculating this potential near the body surface at small deflections, the equivalent radius R_e, function of x only, substitutes r in these integrals, enabling their easy solution.
Aerodynamic forces and moments are determined within this extension using an iterational procedure with the first step given by the original Slender Body theory. The coefficient a_1 is firstly calculated from relation (2.12) using the cross flow velocity component $v = -Dw_g/Dt$. The additional transverse velocity given by (2.8) is then evaluated and added to the cross flow velocity Dw_g/D_t. The resulting value of v is used for calculating the virtual momentum Q of the quasi-slender body from relation (2.14) .

3. THE APPLICATION TO WING-BODY COMBINATIONS

The presented extended theory is applied for calculating the slopes of lift and moment curves as function of angle of attack of quasi-slender wing body combinations in steady supersonic flow. The chosen configuration is a low aspect ratio wing of local span $2b(x)$, mounted on a pointed body of revolution of radius $a(x)$, see Fig.(3.1).

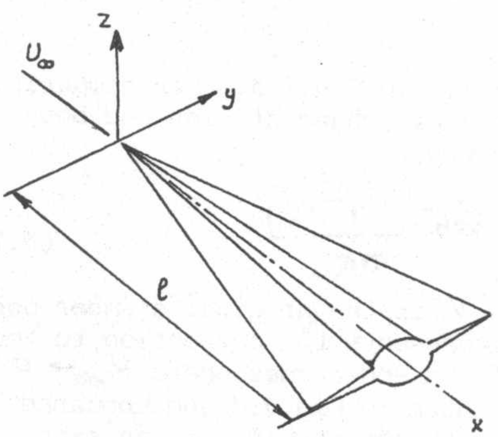

Fig.(3.1) Selected wing-body combination

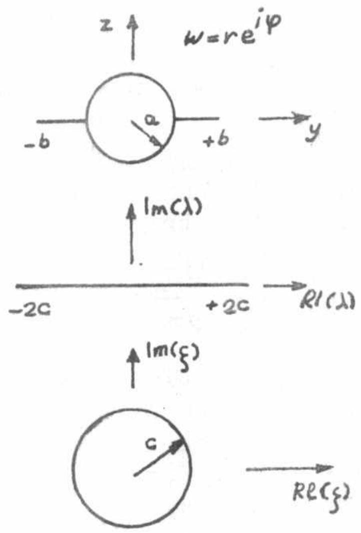

Fig.(3.2) Conformal mapping of body cross-sections

The double Joukovsky transformation

$$w + \frac{a^2}{w} = \lambda = \zeta + \frac{c^2}{\zeta} \tag{3.1}$$

where

$$c(x) = \frac{1}{2} b(x) \left[1 + \frac{a^2(x)}{b^2(x)} \right] \tag{3.2}$$

mapps the domain exterior to the winged body cross-sections in the w plane to the domain exterior to the flat plate $-2c < \lambda < 2c$ in the λ plane and hence to the exterior of the circle $|\zeta| = c$, as shown in Fig.3.2. Considering the case of rigid cross-sections, the equivalent cross-sectional areas of the wing -body combination are

$$S_e(x) = \pi b^2(x) \left[1 - \frac{a^2(x)}{b^2(x)} + \frac{a^4(x)}{b^4(x)} \right] \tag{3.3}$$

Solving for simplicity, the case of delta wing mounted on a conical body, where $a(x)/b(x) = k$ and $b(x) = k_w x$ (k and k_w are constants), the additional cross flow velocity derivative with respect to angle of attack is found to be independent of x

$$\Delta v_z^{\alpha} = - \left[\frac{m k_w (1+k^2)}{2} \right]^2 \left[1 + \cosh^{-1} \frac{1}{m k_w (1-k^2+k^4)} \right] \tag{3.4}$$

The nondimensional lift curve and moment curve slopes with respect to angle of attack related to wing planform area $A = \ell^2 k_w$, reference length ℓ and the dynamic pressure $\frac{1}{2} \rho_\infty U_\infty^2$ are then

$$\frac{1}{k_w} \left. \frac{\partial C_z}{\partial \alpha} \right|_{wb} = 2\pi (1-k^2+k^4) \left[1 - \frac{1}{4} m^2 k_w^2 (1+k^2)^2 \left\{ 1 + \cosh^{-1} \frac{1}{m k_w (1-k^2+k^4)} \right\} \right] \tag{3.5}$$

and

$$\left. \frac{\partial C_m}{\partial \alpha} \right|_{wb} = \left. \frac{\partial C_z}{\partial \alpha} \right|_{wb} \left(x_c - \frac{2}{3} \right) \tag{3.5}$$

Results of Eqn.3.5 are presented graphically in Fig.3.3. For comparison, the presented solution is applied for the limit cases of wing and body alone. For the wing case, $a = k_b = 0$ .Hence

$$\frac{1}{2k_w} \left. \frac{\partial C_z}{\partial \alpha} \right|_{w} = \pi \left[1 - \frac{1}{4} m^2 k_w^2 \left(1 + \cosh^{-1} \frac{1}{m k_w} \right) \right] \tag{3.7}$$

In the previous relation ,the first term π is the original slender body theory value of $C_z^{\alpha}/2k_w$. The second term represents the correction to the not so slender bodies and tends to zero for slender ones, where $k_w \to 0$. Results of Eqn.(3.7) are graphically presented in Fig.3.4 and compared with experimental data from [14]together with the results of the exact theory[13] and the extended slender body theory[7] , where good agreement is observed. Moreover ,the presented solution is much easier than the extension given in [7] which necessitates the solution of a system of integral equations.The moment curve results pridicted linear variation with lift coefficient and a position of aerodynamic center at two thirds of wing root chord, which agrees with the known value for similar wing-body combinations.

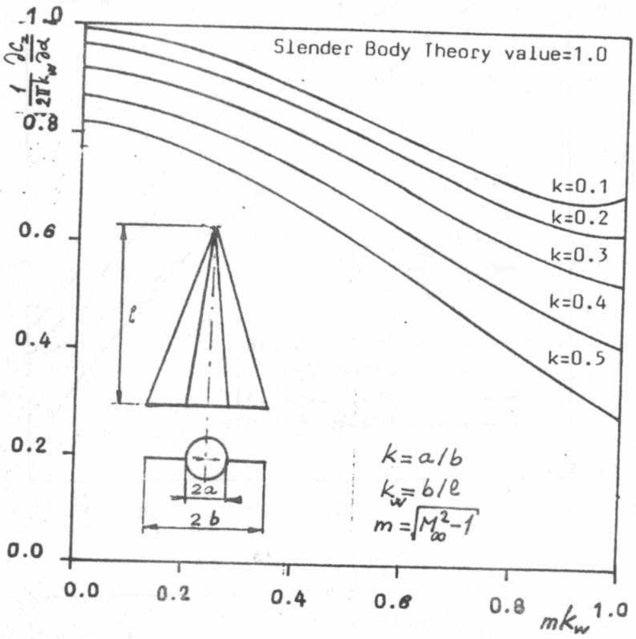

Fig 3.3 Lift force derivative of wing - body combination according to presented extension of S.B.theory.

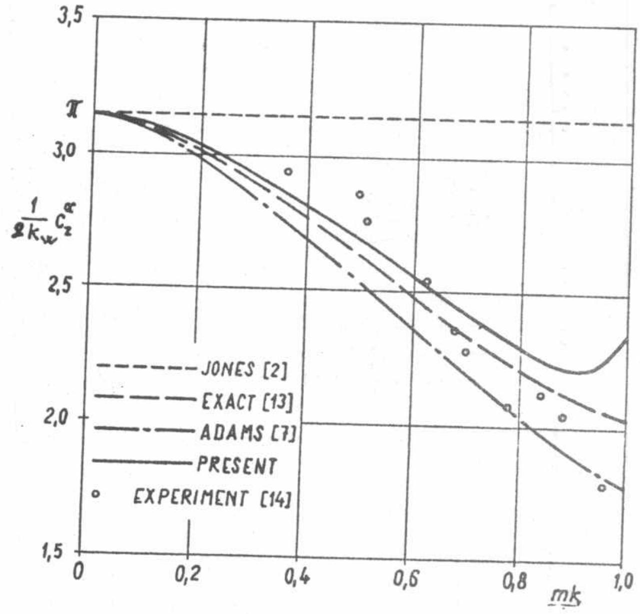

Fig.3.4 Comparison of different theories and experiment for the lift force deriv-derivative of delta wing.

For the case of body alone(cone in this case), $k=1$ & $k_w=k_b$.The lift curve slope related to cone base area $S=\pi\ell^2k_b^2$ is thus given by

$$\frac{\partial C_z}{\partial \alpha} = 2\left[1-(m\,k_b)^2\left(1+\cosh^{-1}\frac{1}{m\,k_b}\right)\right] \qquad (3.8)$$

Results of Eqn.3.8 are plotted as function of cone semi-vertex angle δ_c for a moderate range of supersonic Mach numbers together with the results of linearized solution of [15] , emperical formula $c_z^\alpha=2\cos 2\delta_c$ [16] , and the exact shock expansion theory [13],noting that S.B. theory value of this quantity is 2.0. . Good agreement is observed .

A practically useful result for estimating the effect of body on total lift of wing-body arrangment is obtained by deviding Eqn. 3.5 by Eqn. 3.7

$$\frac{L_{wb}}{L_w} = (1-k^2+k^4)\,\frac{\left[1-\frac{1}{4}m^2k_w^2(1+k_b^2)\left\{1+\cosh^{-1}\frac{1}{mk_w(1-k^2+k^4)}\right\}\right]}{\left[1-\frac{1}{4}m^2k_w^2\left\{1+\cosh^{-1}\frac{1}{mk_w}\right\}\right]} \qquad (3.9)$$

where

$$\left.\frac{L_{wb}}{L_w}\right|_{wb} = (1-k^2+k^4) \qquad (3.10)$$

is the Slender Body theory value of this ratio [12] and which can be derived within the previous analysis by setting $k_w=0$ in Eqn.3.9. Results of Eqn.3.9 are numerically presented in Tab.3.1 where it shows larger decriment of lift of the wing-body combination due to increasing the relative lateral dimensions of the body with respect to the wing.

Conformal mapping of wing-body combination cross-sections can be solved either either computationally using the method of trigonometric interpolation [17] or experimentally using electrohydrodynamic analogy[18].

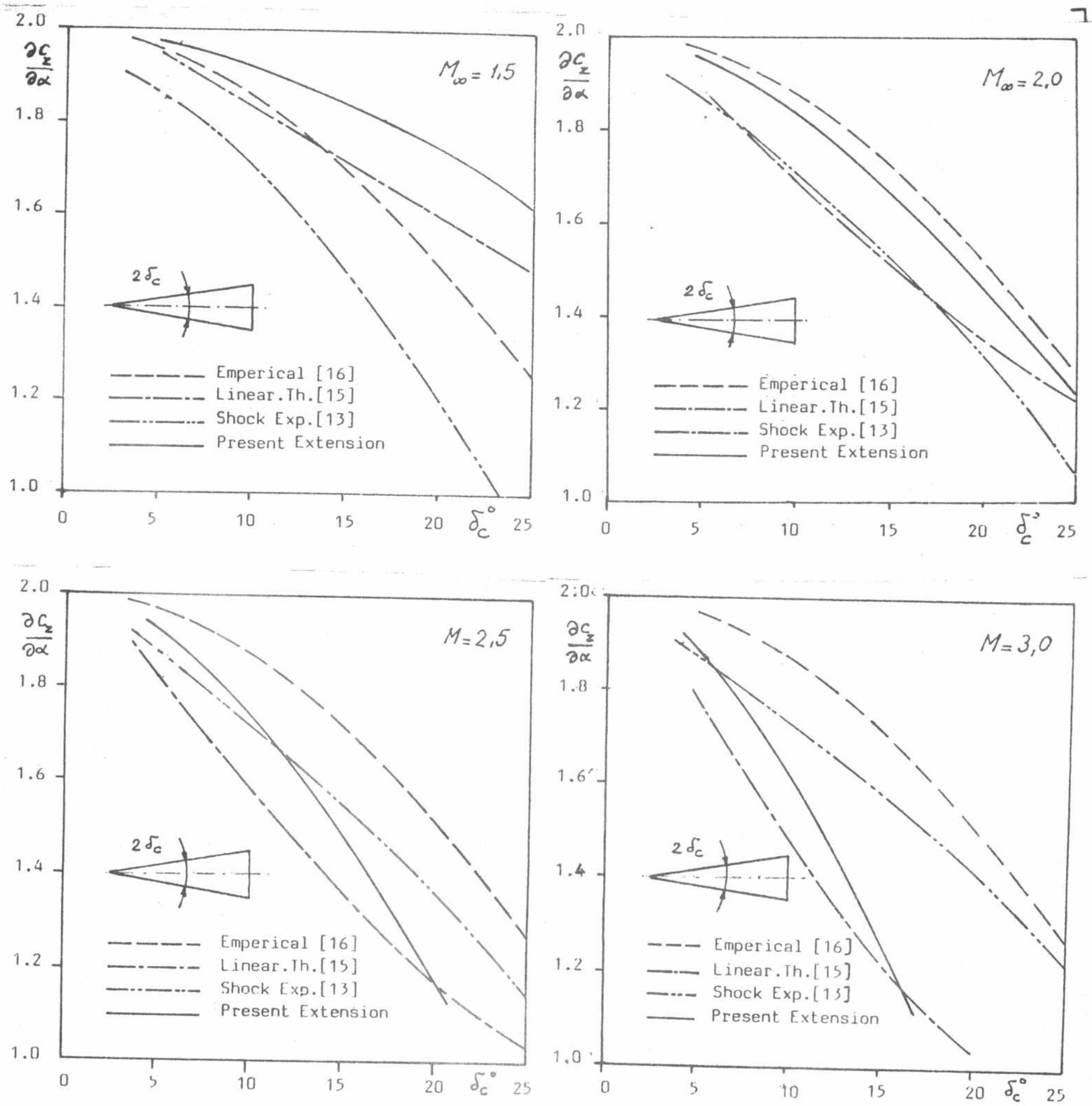

Fig.3.5 Comparison of several theories for the lift force derivative of quasi-slender cones for different Mach numbers.

mk_w	k = 0.2	0.3	0.4	0.5
0.2	0.9967	0.9925	0.9862	0.9774
0.4	0.9886	0.9736	0.9511	0.9201
0.6	0.9752	0.9424	0.8935	0.8261
0.8	0.9547	0.8953	0.8074	0.6871

Tab.3.1 The ratio L_{wb} / L_w for different wing and wing-body combinations slenderness ratios.

4. CONCLUSIONS

The determination of aerodynamic characteristics of quasi-slender wing-body combination can be performed satifactorily using the presented extension of the Slender Body Theory. Obtained results are in good agreement with available theoritical and experimental data. The presented extension allowed the introduction of effects of body shape and flow Mach number which were not included in the original theory solution . Moreover ,the applied procedure and the form of final results are much simpler than other used theoritical techniques.

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