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ENERGY SAVING VIA OPTIMIZATION OF AIRCRAFT FLIGHT PATH

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ABSTRACT

The continuous increasing cost of aircraft operations, which is greatly related to the cost of fuel, motivated the concerned services to examine all possible solutions that could limit that highly increasing cost. Short term solutions for the actually in service aircrafts can be obtained by optimizing the flight trajectory. This problem has been formulated as an optimal control problem. Most of the available optimization works are based on off-line calculations without any feed back effect. However a real time implementation may offers a highly desirable adaptation to possibly existing non-nominal conditions, which in turns is reflected through a more gain in economy of fuel. The realization of such a real time simulation could be achieved through the use of a Flight Management computer system FMCS. such a system should in addition to other possible tasks, generate, in a real time, the optimal flight profile in the vertical plane. The methodology used to solve this optimal control problem is that of the singular perturbation theory, with which the original problem is decomposed to sub problems of lower dimensions and then matched together to get the composed solution. The detailed analysis and application to different aircrafts yield to the following results:-

- Cruise phase : The optimal cruise is an ascending one. A deterioration in the specific range of the order of 3-4% is noted when the flight altitude is below the optimal one of 4000 ft.
- Climbing phase : The gain is of the order of 5-7% when compared with the conventional profile.
- Descente phase : For a fixed time of arrival option, a substantial economy of fuel is obtained.

The D.O.C gain is of the order of 4% for long range trajectories (those including cruise phase) and 12% for short range tra jectories.

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I. INTRODUCTION

The continuous increasing cost of aircraft operations, which is greatly related to the cost of fuel, forced the concerned compan- ies , navigation services, and aircraft industeries to investi- gate all possible solutions that could limit that highly increasing cost. Proposed solutions can be classified into two categorie s:

(1) Long term solutions that imply the design of more economic aircraft by improvement of aerodynamic and structure, use of new materials, more efficient propulsive system, etc...

(2) Short and medium term solutions related to the optimization of aircraft trajectory and handling of airtraffic system.

Concerning the aircraft trajectory, many improvements could be achieved for the different successive encountered phases :-

a- Take off and climb phase : reduction of the maximum take off thrust in function of the aircraft weight, and adoption of optimized profiles for climbing.

b- Cruise phase : changing the Mach number accord- ing to flight level, weight, external conditions, and choice whenever possible, of an adapted flight level.

c- Descent phase : optimization procedure and dece- lerated approach. The above strategy could be achieved in terms of an optimization criterion

II. PROBLEM FORMULATION

The trajectory optimization procedure, which consists of determin- ing the"best" admissible trajectory satisfying the constraints of a given mission, necessitates to precise the following :-

i) The aircraft dynamic model, specially to define the state and the control variables.

ii) The initial and final conditions.

iii) The constraints on the controls and the states.

iv) The optimization criterion.

II.1 The Aircrfft Model

For a subsonic commercial aircraft, We can consider only the following longitudinal equations of motion;

$$\begin{aligned} \dot{x} &= v\cos\gamma \\ \dot{h} &= v\sin\gamma \\ m\dot{v} &= T - D - mg\sin\gamma \\ mv\dot{\gamma} &= L - mg\cos\gamma \end{aligned} \qquad (1)$$

Where x, h, v and γ are the state variables representing respec- tively, the horizontal distance, the altitude, the velocity and the flight path angle. The controls are T , the thrust and L, the lift.

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II.2. Initial and Final Conditions

To avoid airtraffic constraint on the velocity below 10000 feet ,we consider the following boundary conditions;

$$\begin{array}{ll} x(t_o) = x_o & x(t_f) = \text{Range } R \\ h(t_o) = h(t_f) = 10000 \text{ feet} & \\ v(t_o) \;,\; \gamma(t_o) \quad \text{and} & v(t_f) \;,\; \gamma(t_f) \end{array} \tag{2}$$

are either free or imposed; the final time t_f will be considered firstly free and then fixed.

II.3. Constraints

The constraints are related eitherto the aircraft cababilities or the airtraffic limitations. Weconsider only the first types given by;

$$\begin{array}{ccccc} V_{min} \leq & V & \leq & V_{max} \\ & h & \leq & h_{max} \\ T_{min} \leq & T & \leq & T_{max} \end{array} \tag{3}$$

II.4. Criterion

For a commercial aircraft , the objective is to minimize the Direct Operating Cost (DOC) defined by;

$$J_1 = \int_{t_o}^{t_f} (C_f \dot{f} + C_t)\, dt \tag{4}$$

where $\dot{f}$ is the fuel flow rate , c_f and c_t are the unit cost of fuel and time respectively . The minimization of J_1 depends on the ratio C_f / C_t . A Performance index frequently used in the literature which results directly from (4) is given by:-

$$J_2 = \int_{t_o}^{t_f} [\sigma \dot{f} + (1-\sigma)]\, dt \tag{5}$$

where ;

$$\sigma = \frac{C_f / C_t}{[1 + C_f/C_t]}$$

Knowing well C_f and C_t , one value of σ will be quite sufficient. On the other hand, the knoweldge of the entire curve sketched in Fig(1) gives atrade off between fuel and time. Typically there is a knee in the curve. Before the fuel crisis, the airlines operated to the left of minimum DOC since faster routes meant more passangers and higher profits. Now they operate close to minimum DOC.

Defining the elements in II-1,2,3 and4,the optimal control problem consists in calculating the optimal controls T and L that transfer the aircraft from the initial to the final conditions, satisfying the constraints and minimizing the criterion,

III. METHODOLOGY

III.1 Historical background

Aircraft trajectory optimization problem has been treated by many authors [1] but most of the previous work was based on a reduced model. They introduced the total energy E defined by;

$$E = h + V^2/2g \qquad (6)$$

to replace h and V. They also considered that γ as a fast state variable and neglected its dynamics. Furthermore, x was treated as independent variable, leaving only the following state equation,

$$\frac{dE}{dx} = \frac{(T - D)}{mg} \qquad (6\text{-}a)$$

III.2 Solution by Singular Perturbation Theory "SPT".

The SPT methodology has been recently used to solve optimal control problems exhibiting fast and slow dynamics [2],[3]. In opposit to the model reduction technique, the SPT offers the advantages that the fast dynamics neglected with a low order model will be further taken into consideration through boundary layers and analyzed with respect to a stretched time scale.

The first step in solving a problem by the SPT technique consists in putting the model (1) in the standard form of a singularly perturbed system (SPS). That requires the identification of fast, slow variables, and the perturbation parameter ε .
For a non linear system, there is noclassical method allowing the realization of such transformation up till now. However, transformation to the SPS form is almost based on the idea we have about the variable behaviour, or by using some emperical formulas [4].
In fact aircraft trajectory comprises essentially two phases;

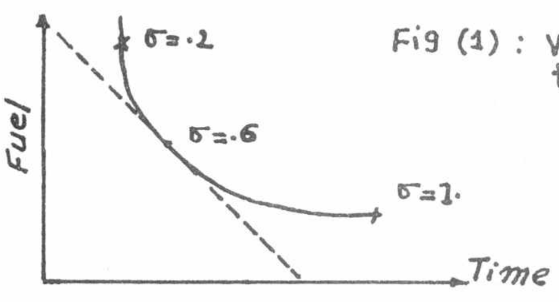

Fig (1) : variation of consumption and flight time in function of the parameter σ for a given mission i- Cruise phase where the altitude h, the velocity v, and the flight path angle γ remain almost constant while only the horizontal distance x varies.

ii- climb or descent phase, where the aircraft changes its h, v and γ between their boundary and cruise values over a relatively short travelling distance. This allows to put the system given by equations (1) in the following form of a "Forced " signgularly perturbed system "FSPS" [5];

$$\dot{x} = v \cos\gamma \qquad (7\text{-a})$$
$$\varepsilon \dot{h} = v \sin\gamma \qquad (7\text{-b})$$
$$\varepsilon m \dot{v} = T - D - mg\sin\gamma \qquad (7\text{-c})$$
$$\varepsilon m v \dot{\gamma} = L - mg\cos\gamma \qquad (7\text{-d})$$

To avoid solution of Two point Boundary value problem "TPBVP", we try to find an inner separation between h, v and γ which allows to put the previous system of equations in a multitime scale.

To realize such separation, the equations (7-b, C and d) are linearized about the cruise values. The corresponding linearized model is;

$$\frac{d}{dt}\begin{bmatrix}\delta h\\ \delta v\\ \delta\gamma\end{bmatrix} = \begin{bmatrix} 0 & \sin\gamma & v\cos\gamma \\ \frac{\partial}{\partial h}\frac{(T-D)}{m} & \frac{\partial}{\partial v}\frac{(T-D)}{m} & -g\cos\gamma \\ \frac{1}{mv}\frac{\partial L}{\partial h} & \frac{1}{mv^2}\left[v\frac{\partial L}{\partial v}-(L-mg\cos\gamma)\right] & \frac{g\sin\gamma}{v}\end{bmatrix}\begin{bmatrix}\delta h\\ \delta v\\ \delta\gamma\end{bmatrix} \qquad (8)$$

For this linearized model, the well known methods for dynamic separation [6] can be applied. Numerical application relative to the model of the Airbus shows that γ is the fastest state variable and V is faster than h. Thus the system of equations (7) can be rewreitten in the following multitime scale form;

$$\begin{aligned}\dot{x} &= v\cos\gamma\\ \varepsilon\dot{h} &= v\sin\gamma\\ \varepsilon^2 m\dot{v} &= T - D - mg\sin\gamma\\ \varepsilon^3 m v\dot{\gamma} &= L - mg\cos\gamma\end{aligned} \qquad (9)$$

III.3 Zero Order approximation by SPT

a- Global system

The application of the maximal principle to the problem formulated in (II) yields to the following Two point Boundary value problem (TPBVP),

$$\begin{aligned}\dot{x} &= V\cos\gamma\\ \varepsilon\dot{h} &= V\sin\gamma\\ \varepsilon^2 m\dot{V} &= T - D - mg\sin\gamma\\ \varepsilon^3 mV\dot{\gamma} &= L - mg\cos\gamma\\ \dot{\lambda}_x &= -\partial H/\partial x\\ \varepsilon\dot{\lambda}_h &= -\partial H/\partial h\\ \varepsilon^2\dot{\lambda}_v &= -\partial H/\partial v\\ \varepsilon^3\dot{\lambda}_\gamma &= -\partial H/\partial\gamma\end{aligned} \qquad (10)$$

Where His the Hamlotonian function, given by;

$$H = C_p\dot{P} + C_t + \lambda_x v\cos\gamma + \lambda_h v\sin\gamma + \frac{\lambda_v}{m}(T - D - mg\sin\gamma) + \frac{\lambda_\gamma}{mv}(L - mg\cos\gamma)$$

together with the boundary conditions given in (2) and the following terminal conditions;

$$\lambda_x(t_f) = 0 \qquad (11)$$
$$\lambda_h(t_f) = 0$$

b- Reduced system

It is obtained by substituting $\varepsilon = 0$ in the global system we notice that the original control $\bar{L}$ and $\bar{T}$ are constrained and replaced by the pseudo - controls $\bar{h}$ and $\bar{V}$ determined by;

$$\bar{h}, \bar{V} = \arg\min \frac{C_p\bar{\dot{P}} + C_t}{\bar{V}} \Bigg|_{\substack{\bar{\gamma} = 0 \\ \bar{T} = \bar{D} \\ \bar{L} = mg}} \qquad (12)$$

The symbol (-) is used to indicate that the variables are calculated in the reduced system.

c- First boundary layers

The equations in the first Initial Boundary Layer(1 IBL) are determined with respect to the stretched time scale;

$$\tau_1 = \frac{t - t_0}{\varepsilon} \qquad (13)$$

For Zero order approximation ($\varepsilon = 0$ in the resulting equations), the controls $\hat{L}_1$ and $\hat{T}_1$ are always constrained and replaced by the pseud-controls $\hat{v}_1$ and $\hat{\gamma}_1$ determined by

$$\hat{v}_1, \hat{\gamma}_1 = \arg\min \frac{C_p\hat{\dot{P}}_1 + C_t + \bar{\lambda}_x \hat{v}_1\cos\hat{\gamma}_1}{\hat{v}_1\sin\hat{\gamma}_1} \Bigg|_{\substack{\hat{T}_1 = \hat{D}_1 + mg\sin\hat{\gamma}_1 \\ \hat{L}_1 = mg\cos\hat{\gamma}_1 \\ \hat{h}_1 = \text{constant}}} \qquad (14)$$

The equations in the Terminal Boundary Lager (TBL) are obtained with respect to the time scale;

$$\sigma_1 = \frac{t_f - t}{\varepsilon} \qquad (15)$$

d- Second boundary layers.

The equations in the 2 IBL are obtained with respect to the time scale,

$$\tau_2 = \frac{t - t_0}{\varepsilon^2} \qquad (16)$$

for $\varepsilon = 0$, the lift is always constrained, the thrust $\hat{T}_2$ and the flight path angle $\hat{\gamma}_2$ are determined by;

$$\hat{T}_2 , \hat{\gamma}_2 = \arg\min \left. \frac{C_p \hat{f}_2 + C_t + \bar{\lambda}_x \hat{V}_2 \cos\hat{\gamma}_2 + \hat{\lambda}_{h_1} \hat{V}_2 \sin\hat{\gamma}_2}{(\hat{T}_2 - \hat{D}_2 - mg\sin\hat{\gamma}_2)/m} \right|_{\hat{L}_2 = mg\cos\hat{\gamma}_2} \tag{17}$$

e - Third boundary layer

The equations in the 3IBL are obtained with respect to the time scale;

$$\tau_3 = \frac{t - t_o}{\varepsilon^3} \tag{18}$$

and the original controls $\hat{L}_3$ and $\hat{T}_3$ are obtained by;

$$\hat{L}_3 , \hat{T}_3 = \arg\min \frac{C_p \hat{f}_3 + C_t + \bar{\lambda}_x \hat{V}_3 \cos\hat{\gamma}_3 + \hat{\lambda}_{h_1} \hat{V}_3 \sin\hat{\gamma}_3 + \lambda_{V_2}(\hat{T}_3 - \hat{D}_3 - mg\sin\hat{\gamma}_3)/m}{\hat{L}_3 - mg\cdot\cos\hat{\gamma}_3} \tag{19}$$

In a similar way, the equations in the TBL are obtained with respect to the time scale,

$$\sigma_3 = \frac{t_f - t}{\varepsilon^3}$$

IV - RESULTS AND SYNTHESES

IV.1 Cruise Phase

With the decrease of the mass, the optimal altitude increases while the velocity decreases. For a given mass the optimal altitude is nearly the same for the two extreme cases of minimum time or minimum consumption while the optimal velocity is greater in the first case than in the second. For the case of minimum consumption it has been noted that some parameters remain nearly constant. These parameters are; M the optimal Mach number; m/δ , mass divided by the atmospheric pressure ratio and RF the Range Factor defined as the travelled distance by one kg of fuel multiplied by the mass of the aircraft . The variation of the mass can be taken into consideration in either of the two ways:

a) Either by dividing the equation $\dot{\bar{x}} = \bar{V}$ by the equation $\dot{m} = -\dot{\bar{f}}$ which gives ;

$$\frac{d\bar{x}}{dm} = -\frac{\bar{V}}{\bar{f}} = -\frac{\overline{RF}}{m} \tag{20}$$

The integration of the previous equation gives;

$$m = m_i \, e^{-\frac{\bar{x}}{\overline{RF}}} \tag{21}$$

At each Δx , the optimal cruise parameters are updating for the new mass.

b) Or in the general case , by updating the trajectory systematically for the new mass at each Δt.

If the flight level is imposed due to airtraffic regulations, the optimalvelocity can be obtained by integrating equation (12) for a constant $\bar{h}$. However this constraint deteriorates substantially the crieterion. This deterioration is as much important as the imposed flight level is lower than the optimal altitude.

IV.2 Climb Phase

The optimal mach number (or velocity) and flight path angle are determined by the equation (14) at each altitude between the initial and cruise levels. For a given mass it has been noticed that the optimal mach number increases progressively with the altitude till the appearance of compressibility phenomenon where it decreases before it increases again to tend asymptotically to the cruise value. The optimal flight path angle decreases progressively with the altitude and becomes zero at the optimal cruise altitude. The profile for the case of minimum consumption is slower than those corresponding to the minimum DOC. The optimal trajectory for minimum consumption or DOC reachs the cruise altitude asymptotically, hence the travelled distance during this phase is more important than that corresponding to the actual conventional profiles. Conventional profile consists in following one or two segments with constant Indicated Air Speed (IAS) followed by a segment with constant mach number. All these segments are carried out at maximum climb thrust.

For the same travelled distance, the realized gain on the 6 DOC in following the optimal trajectory is of the order of 5 to 7% with respect to the conventional profile. It has also been noted that parametric profile, which consists in following a classical procedure of constant IAS/M with optimizing the thrust and taking M as the optimal cruise Mach number, represents an economy on DOC nearly of the same order of magnitude but with important facility of pilotage.

IV.3 Descent phase with fixed time of arrival . This phase is similar to the previous one. During the cruise and climb phases the final time t_f was assumed free. If this assumption appears logical for these two phases it will not seems appropriate for the descent phase where the time of arrival will be imposed by the airtraffic. In this case the objective will be to minimize the consumed fuel. Amathematical artifice was made to convert the pervious criterion-minimum consumption-to the usual one of minimum DOC by adpoting an appropriate value for the time cost c_t. Up till now this coefficient has been taken positive for clear physical reasons. It will be possible t0"absorb" time with negative values for Ct. The problem is to know the maximum delay which can be achieved and the corresponding limiting value of c_t. For this, we allow that an inferior limit for the aerodynamic velocity in cruise corresponds to the value $V_{\dot{f}_{min}}$ (Maximum Endurance velocity).
That arises the following question: what is the smallest value of ct such that the resulting optimal cruise velocity will be equal to $V_{\dot{f}_{min}}$?

It has been demonstrated in[7] that the limit value of ct is givien by,

$$c_{t_{min}} = - c_f \, \dot{f}_{min} \tag{22}$$

The objective will be to find the value of the coefficient ct which will be appropriate to the assigned "time of" arrival. Two situations can be considered:

- If the assigned value of the time of arrival "t_{fa}" is less than the estimated landing time "to" calculated with the profil of minimum consumption, then a positive value of ct will be inserted - on the contrary if "t_{fa}" is greater than "to" a negative value of C_t has to be adopted.

A substantial economy of fuel is realized with the introduction of the previous option in the program. As an example we considered an aircraft at a distance of 150 NM (Nautical Mile) from its arrival point (at 10000 feet), the pilot is informed that he is authorized to land after 30 minutes. we noticed a global ecomomy of fuel of 450 kg by adopting the previous technique of fixed time of arrival over the DOC trajectory with the assumption that the waiting time is absorbed by holding at the cruise altitude.

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