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2 AN APPROACH TO THE OPTIMUM DESIGN OF THE ROTOR SYSTEM OF SINGLE-ROTOR HELICOPTERS

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4 ABSTRACT

This paper discusses the various aerodynamic and dynamic considerations involved in the design of the rotor system of single-rotor helicopters. Design parameters of a helicopter rotor and their influence on the rotor thrust efficiency, stall behaviour, helicopter flying qualities, vibration and noise at different flight modes are well defined.

Some of these parameters have contradictory effects on the rotor flight characteristics. Others are either strictly related together, or cannot be arbitrarily varied because of structural, mechanical and aerodynamic limitations. Necessary compromises should then be made in order to come up with the optimum design of the rotor system. A definite procedure for optimizing values of these parameters to give the best flight characteristics of the helicopter concerning performance, stability, control, vibration and noise at different flight modes and user's requirements is clearly submitted by the author. This will be of great value to helicopter designers, both in modifying existing helicopters and designing new ones.

INTRODUCTION

The purpose of a helicopter is to transport persons or any other payloads from place to place quickly, safely, comfortably and economically. Military combat helicopters should have, in addition, enough manoeuvreability and agility. To perform its mission quickly and economically the helicopter should have a rotor system with a high thrust efficiency and acceleration capability. For safety requirements the rotor system should be far from stall, aeroelastic instabilities and excessive control sensitivity. To provide comfort the rotor system should have adequate angular-rate-damping, low noise and vibration. Manoeuvreability of a

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helicopter increases with higher control power and reduced angular-rate-damping of its rotor system. Values of the rotor damping and control power depend upon the allowable levels of the helicopter stability and control sensitivity.

OPTIMUM DESIGN APPROACH

Design parameters that do affect the aerodynamic and dynamic characteristics of a helicopter rotor can be summarized into its diameter, tip speed, rotational speed, solidity, number of blades; blade aerofoil, twist, taper, aspect ratio, Lock's number, hub design and shaft tilt. These parameters have different influences and sometimes contradictory effects on the rotor performance, and its contribution into helicopter stability, control, vibration and noise. Some of them are strictly related together, or limited by structural, mechanical and aerodynamic limitations. It is then required to make the necessary compromises to get the optimum design of the rotor system taking into consideration the customer's requirements. The aim of this work is to give the way in which this may be achieved. This process is, inevitably, an iterative one as many of the parameters are linked together.

Rotor Diameter

Principal considerations in selecting the rotor diameter are the rotor thrust efficiency and the overall size of the helicopter as limited by such requirements as air transportability, carrier operations and nap-of-the-earth (NOE) flight capability. If there are no physical size constraints, the rotor diameter can be optimized such that it gives the maximum thrust efficiency, T/P, at a fundamental flight mode. Chosen flight mode for optimizing the rotor diameter is the low speed maximum manoeuvring, i.e. with thrust requirement in-and-around the hover and a thrust margin specified by the acceleration requirement from hovering or nearly hovering condition.

The total power required by a hovering rotor [1-2] to produce a specified thrust, T, is given by

$$P = P_i + P_p \tag{1}$$

where $P_i = kT^{3/2} / \sqrt{2 \rho A}$ (2)

and $P_p = \rho s A \bar{C}_D V_t^3/8$ (3)

Expressions (2) and (3) show that the induced power decreases and on the contrary the profile power increases as the rotor disc area is increased. The disc area can then be optimized as to give the minimum total power required to produce a specified thrust or to maximize the rotor thrust efficiency T/P, and in this case

$$\frac{\partial}{\partial A} (T/P) = 0 \tag{4}$$

6 The optimum disc area for a given thrust, T, is therefore

$$A_{opt.} = T\left[k/s\,\bar{C}_D\right]^{2/3} \Big/ (1/2)\rho V_t^{\,2} \tag{5}$$

This indicates that a smaller size is possible with higher tip speeds. The maximum possible tip speeds are given afterwards. The thrust capability of both the main and tail rotors must satisfy the user requirements in symmetric and assymmetric manoeuvres at the extremes of climatic conditions. For a main rotor, T=nW, where n is the maximum specified load factor from hover. In case of a tail rotor design, $T=T_Q+\Delta T$, where $T_Q=P_{MR}/X.\Omega_{MR}$ is the tail rotor thrust required to balance the main rotor torque, and $\Delta T=I_{zz}\ddot{\psi}/X$ is the thrust margin required for a specified yaw acceleration at hover. Typically, n=2-3 and $\ddot{\psi}$=0.5-0.8 rad/sec^2 for helicopters in current use.

The induced power factor, k, is the ratio between the real and ideal induced powers [2], i.e.

$$k=P_i/P_{io} = \int_0^1 x\,v_i^{\,3}\,dx \Big/ \sqrt{2}\left[\int_0^1 x\,v_i^{\,2}\,dx\right]^{3/2} \tag{6}$$

Using the combined blade-element-momentum theory, the induced velocity distribution along the blade of a hovering rotor is given by

$$v_i = \frac{asV_t}{16}\left[-1+\sqrt{1+\frac{32\,\theta x}{as}}\right] \tag{7}$$

The local pitch angle of a blade section is related to the local lift coefficient by the relation

$$\theta = \alpha + \phi = (C_L/a) + (v_i/V_t x) \tag{8}$$

To optimize the rotor diameter as to give the maximum T/P, the optimum solidity given later by equation(12) should be used. Equations (5), (6), (7) and (8) together with the relation of optimum solidity given later show that it is not possible to obtain an explicit formula for the rotor diameter. These equations should then be solved simultaneously by successive iteration.

In the first iteration cycle, we assume a spanwise C_L - distribution very close to the (C_{Lmax}-M) separation boundary of the blade aerofoil. Assuming k=1.0, an approximate value for the optimum solidity can be estimated by equation (12). Equations (7) and (8) can then be solved simultaneously to get the induced velocity distribution and local pitch angle. Estimate k from equation (6) and s from equation (12). More precise values for v_i and θ distributions can then be obtained by solving again equations (7) and (8). Repeat the iteration cycle several times until the error is diminished.

In the second iteration cycle, we assume a practical value of linear twist, $\Delta\theta$, as linearly twisted blades are much used in practice. Use the proposed value of $\Delta\theta$ and the final value of the tip pitch angle obtained from the first iteration cycle to estimate a linear distribution of θ along the blade, i.e.

$$\theta = \theta_t + \Delta\theta\,(x-1) \quad \text{where} \quad \Delta\theta = \theta_t - \theta_r$$

As a first approximation use the final value of the optimum solidity obtained from the first iteration cycle to estimate the corresponding v_i - distribution using equation (7). Estimate k from equation (6) and the C_L - distribution from the relation, $C_L = a\alpha = a\,(\theta - v_i/V_t x)$. The corresponding optimum solidity can be estimated by eqn (12). Use the latter value of s to estimate more precise values for v_i, hence C_L and k. Repeat the estimation cycle of v_i several times to get its exact distribution along the blade that corresponds to the proposed linear twist. Be sure that the resulting C_L at tip region and most blade sections do not exceed C_L max. If blade stall appears on the outboard sections the assumed twist should be reduced. Estimate the corresponding optimum disc area from equation (5). Repeat the second iteration cycle for different values of linear twist, say $\Delta\theta = -4^\circ, -8^\circ, -12^\circ, -16^\circ, \ldots$, and estimate the corresponding $A_{opt.}$. The optimum linear twist that can be used is, therefore, which gives the minimum possible rotor diameter.

Tip Speed

This should be as high as possible since higher tip speed rotors are relatively small, light and less sensitive to gusts as smaller disc area can be used. In forward flight, high tip speed is also desirable as this reduces the reversed flow region and possibility of stall on the retreating blades, but its maximum possible value is limited by the critical Mach number in the tip region at maximum forward speed of the helicopter. The critical Mach number of the advancing blade aerofoil will depend upon its angle of attack. Using an appropriate blade twist, the tip Mach number can be chosen such that the product $M^2 C_{L\,max.}$ is a maximum for the aerofoil considered with no possibility of tip stall at high forward speeds.

On the other hand, the rotor aerodynamic noise [8] is strongly dependent upon tip speed. This may dictate a reduction in the chosen tip speed. Some degradation of the performance might be accepted to achieve a significant reduction in noise level. All sizes of helicopters in current use have tip speeds within a narrow range of 180-240 m/sec. Helicopters designed with noise reduction as a primary consideration have multi-bladed rotors with tip speeds in the order of 180 m/sec.

Rotational Speed

If the rotor diameter and tip speed have been decided, the rotor rotational speed is then obtained from the relation, $\Omega = V_t/R$. Since tip speeds are nearly the same for all sizes of helicopters (180-240 m/sec.), a small diameter rotor will have a higher rotational speed, thus permitting a lower torque-

drive system.
Degradation of the rotor thrust due to gyroscopic flapping in angular manoeuvres [2], say for example, pitching at hover, is given by

$$(\Delta C_L)_{precession} = 16aq/\gamma\Omega \qquad (9)$$

In this manoeuvre, the rotor angular-rate-damping or damping-in-pitch is therefore

$$M_q = 16hT/\gamma\Omega \qquad (10)$$

From eqns. (8) and (9) it follows that small diameter rotors, which operate at high rotational speeds, will tend to have low thrust degradation and rotor damping in angular manoeuvres. Low thrust degradation is desirable to increase the acceleration capability of the rotor. But on the contrary, low rotor damping may result in excessive control sensitivity and undesirable dynamic instability of the helicopter. However, the rotor damping and precessional loss of thrust can also be adjusted by the blade Lock's number as shown by equations (9) and (10).

Higher rotor speed is also advantageous for autorotative performance. With high rotor speed the pilot possesses greater time margin to lower his collective pitch to start a steady autorotative flight from lower heights above terrain. The higher the rotor speed the greater the height from which the helicopter can descend and land safely using the main-rotor kinetic energy when suffering a total power failure near to the ground. This may place a lower limit on the main-rotor speed and polar moment of inertia.

It is also found that the rotor vibrating forces and moments transmitted to the hub in forward flight and angular manoeuvres are only harmonics having frequencies which are integral multiples of the rotor speed and number of blades [1-2]. In order to reduce the amplitude of helicopter vibration, the exciting frequencies can be increased by increasing the rotor speed and number of blades. The bases on which the number of blades can be chosen are given later on.

Rotor Solidity

Using the blade element theory [1-2], the thrust generated by a hovering rotor is given by

$$T = \rho A V_t^2 . s\bar{C}_L/6 \qquad (11)$$

If the rotor diameter have been decided by size requirements the required solidity can be estimated by eqn. (11). In case of no size constraints, an optimum value for the rotor solidity can be obtained by solving equations (5) and (11) simultaneously for s, thus giving

$$s_{opt.} = 27(\bar{C}_D)^2/\ \bar{C}_L^3\ k^2 \qquad (12)$$

where $$\bar{C}_L = 3 \int_0^1 C_L x^2 \, dx \tag{13}$$

C_L is the maximum useable local lift coefficient as it varies along the blade. The optimum distribution of the section lift coefficient that gives the maximum thrust is the closest to the ($C_{L\ max.}$-M) separation boundary of the blade aerofoil. This may be achieved by the appropriate blade twist.
However the rotor solidity is related to the number of blades and their aspect ratio, since by definition

$$s = bc/\pi R = b/\pi AR \tag{14}$$

Number of Blades

Factors governing the choice of the number of blades of a helicopter rotor include vibration characteristics, aerodynamic noise, tip losses, mechanical complexity and rotor weight. Increasing the number of blades, the amplitude of the helicopter vibration decreases since the magnitudes of the rotor input forces and moments decrease and the input frequencies increase as illustrated before. Lower thrust per blade also reduces the rotor noise and this may be the primary aerodynamic consideration in selecting the number of blades. The rotor tip losses also decreases if the number of blades is increased since the tip-loss factor [1] is given by

$$B = 1 - \sqrt{2C_T}/b \tag{15}$$

where BR is the effective rotor radius and $C_T = T/\rho A V_t^2$ is the rotor thrust coefficient. Highly increased number of blades introduces again vibration problems due to the large number of trailing vortices and their closer proximity to each other. This also increases complexity of the hub and pitch controls, and may increase the rotor weight. Fixing the rotor solidity at its optimum value given by eqn. (12), the number of blades and aspect ratio should be varied in proportion.
Typical number of blades of helicopter rotors in current use ranges from 2 to 8.

Blade Aspect Ratio

Choice of blade aspect ratio is a trade-off between aerodynamic and structural requirements. As $C_T = s\bar{C}_L/6$, the tip loss factor given by eqn (15) can be put in the form

$$B = 1 - \sqrt{\bar{C}_L/3\pi AR\, b} \tag{16}$$

This means that the higher the aspect ratio of the blades, the smaller the rotor tip thrust loss.

From the structural point of view, the blade bending and torsional stiffness is proportional to the aspect ratio as the thickness /chord ratio is nearly the same for blade aerofoils in current use. This can put an upper limit on the blade aspect ratio. Typical aspect ratios are in the range of 15-20 for main-rotors and 5-8 for tail-rotor blades.

Lock's Number

The parameter which governs the amount of flapping in any flight mode is the blade mass factor or Lock's number as it relates the aerodynamic forces to the inertia forces acting on the blade. Lock's number is defined as

$$\gamma = \rho a c R^4 / I_\beta \qquad (17)$$

In forward flight the flapping coefficients [2] are related to Lock's number by the relations

$$a_o = (1/2)\gamma \left[\frac{\theta_o}{4} (1+\mu^2) + \frac{\lambda}{3} \right] \qquad (18)$$

$$a_1 = \frac{\mu(8\theta_o/3 + 2\lambda)}{1-\mu^2/2} \qquad (19)$$

$$b_1 = \frac{4\mu a_o/3}{1+\mu^2/2} \qquad (20)$$

In angular manoeuvres, say for example pitching from hover, the precessional flapping coefficients are

$$\delta_1 = -16\, q/\gamma\Omega \qquad (21)$$

$$\delta_2 = -q/\Omega \qquad (22)$$

It can be seen that the amount of periodic flapping and the associated periodic increment in the blade angle of attack increase with forward speed and angular velocity of the helicopter. The maximum limit of forward speed and manoeuvrability is reached when the blade approaches stall, which in turn depends upon the amount of flapping generated. In order to reduce flapping in forward flight a low Lock's number is called for. Unfortunately, this is opposite to the requirement for angular manoeuvrability since gyroscopic flapping increases on lowering the Lock's number. Rotors with low Lock's number blades will have a higher angular-rate-damping and reduced sensitivity (eqn. 10). Necessary compromises should then be made at the choice of γ , in order to cope with different requirements concerning forward flight, manoeuvrability and angular-rate damping .

For a given type of blade construction it can be seen from eqn (17) that Lock's number is directly proportional to the blade aspect and lift curve of the blade aerofoil. It is then expected that Lock's number of the main rotor blades is much higher than that of the tail rotor. However, one could easily alter Lock's number by changing the blade weight and mass distribution if the blade aerofoil and aspect ratio have been decided. Existing helicopters have values of γ ranging from 8 to 15 for main-rotor blades and from 2 to 5 for tail rotors.

Blade Aerofoil

The principal feature desired of a rotor blade aerofoil section

is a high maximum lift coefficient and stalling angle of attack at the operating Mach and Reynolds numbers.
Low profile drag coefficient throughout the range of lift coefficients and zero or nearly zero pitching moments are also desired but are secondary in importance to the stalling characteristics. High pitching moment coefficients lead to undesirable periodic stick forces, to vibrations, and to undesirable control-position gradients. Aft camber may succeed in giving an overall improvement in the ($C_{L\ max}$ -M) separation boundary of the aerofoil but at the expense of rapidly growing pitching moments. The permissible level for control loads defines the maximum useful camber that may be used.

Conventional aerofoils used in most rotor blades are the NACA low-drag symmetrical aerofoils with thickness ratio ranging from 9 to 20%. A reduction in thickness/chord ratio would be effective in reducing the blade profile drag and the rotor noise but it should be optimized for the best $C_{L\ max.}$ capability. The most commonly used aerofoils in rotor blades are NACA 0012 and NACA 23012 having thickness ratio of 12%. Structural limitations may be put on the minimum thickness/chord ratio. Recently, the NPL and RAE in Great Britain have developed the "96" series of aerofoil sections, and Boeing-Vertol in the United States have developed the "VR" series. In both cases the idea has been to modify the nose portion of the NACA 0012 aerofoil in order to produce a peaky pressure distribution near the leading edge. Such pressure distributions have the effect of either eliminating the shock wave in the rear of the supersonic region or weakening it sufficiently for shock-induced separation not to occur.

Twist and Taper

The use of negative twist (washout) increases the inboard blade loading and prevents tip stall, thus offering a potential improvement in thrust. That part of the induced power loss arising from nonuniformity of the induced velocity distribution can also be reduced by negative twist. Ideally twisted blades for which, $\theta = \theta_t/x$, have the minimum induced power loss (k=1.0) as they give uniform downwash. The application of increasing amounts of linear twist approaches the benefits obtained by ideal twist. Highly twisted blades may produce negative thrust over the outer part of the blades at low thrust coefficients, or may cause stall at the inboard sections at high thrust coefficients. On the other hand, small amount of linear twist may not be sufficinet to prevent stall in the tip region at high thrust coefficients and Mach numbers. The optimum amount of linear twist which gives the maximum possible thrust, is that offers the closest local lift coefficients to the ($C_{L\ max}$-M) separation boundary of the blade aerofoil at the maximum collective pitch. The linear twist normally used on main rotors is $-8°$. As the tail rotor diameter is oftenly limited by the overall size of the helicopter and minimum ground clearance the induced velocity of a tail rotor is expected to be higher than that of the main rotor and hence the twist required will be greater than that used on main rotors, typically $-16°$.

Negative twist also reduces the rotor profile power. The rotor thrust efficiency is effectively increased.

Beneficial effects of taper on the induced and profile power losses are similar to those of twist. As negative twist is more profitable and might not entail additional production cost, it is preferred to use twist rather than taper. For structural reasons, the tapering of large diameter blades may be desirable.

Hub Design

In fully-articulated rotors, the dynamic response of the blades as well as the rotor forces and moments acting on the hub can effectively be changed by the amount of flap-hinge and drag-hinge offsets. The principal effect of flap-hinge offset is to increase the rotor control power and angular-rate damping in proportion. This effect increases the angular acceleration capability of the rotor with no change in the control sensitivity. For this reason offset flapping hinges permit an increase in the allowable C.G. range of single-rotor helicopters. Height of the hub above the helicopter C.G. have similar effects as can be shown from eqn(10).

The dynamic and aerodynamic behaviour of the blades can also be modified by the δ_3-flapping hinge, giving a pitch-flap coupling factor, $d\theta/d\beta=-\tan\delta_3$. Conventional pitch-flap coupling reduces the maximum values of flapping and angles of attack which occur around the azimuth in forward flight and angular manoeuvres, thus decreasing degradation of the rotor thrust margin. In this case the rotor damping is reduced to

$$M_q = 16hT\cos\delta_3/\gamma\Omega \qquad (23)$$

Pitch-flap coupling is used in many tail rotors to reduce the tilt of the rotor plane relative to the control plane in forward flight and angular manoeuvres. The majority of tail rotors use a δ_3 of 45° and $d\theta/d\beta=-1$. In some large helicopters, the main rotor has also δ_3 hinge, giving a pitch-flap coupling $d\theta/d\beta=-0.5$.

Great simplification in the hub design has been achieved with hingeless rotors. The controlled-stiffness elements used instead of the flapping and lagging hinges could achieve much more improvement in the control power, angular-rate-damping and C.G. range.

Shaft Tilt

A small forward tilt of the rotor shaft may be used to eliminate blade/vortex interference stall in forward flight. By this inclination the angle at which the wake departs from the rotor disc is increased such that the trailing vortices will be displaced away from the rotor plane. The large increase in the rotor noise resulting from blade/vortex interference in forward flight can also be alleviated by the forward tilt of the shaft. The rotor angle-of-attack instability can also be reduced by tilting the shaft. This also improves the helicopter pitch attitude in forward flight. Typical shaft inclination used in main and tail rotors is 5° .

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NOMENCLATURE

A	rotor disc area $=\pi R^2$
$A\!\!R$	blade aspect ratio (R/c)
a	section lift-curve slope
a_o	rotor coning angle
a_1	longitudinal flapping
B	tip loss factor
b	number of blades
b_1	lateral flapping
C_D	section profile-drag coefficient
$\bar{C}_D$	mean profile-drag coefficient
C_L	section lift coefficient
$\bar{C}_L$	mean lift coefficient
$C_{L\,max}$	max. section lift coefficient
c	blade chord
h	height of hub above C.G.
I_{zz}	helicopter yaw moment of inertia
I_β	blade flapping moment of inertia
k	induced power factor
M	free-stream Mach number
M_q	rotor damping-in-pitch
P	power required by a rotor
P_i	induced power
P_{io}	ideal induced power
P_p	profile power
P_{MR}	main-rotor power
R	rotor radius
r	local radial distance
s	rotor solidity $(bc/\pi R)$
T	rotor thrust
V	helicopter forward speed
V_t	tip speed $=\Omega R$
v_i	local induced velocity
$\bar{v}_i$	mean induced velocity
W	helicopter gross weight
X	distance between main-rotor axis and tail-rotor
x	non-dimensional radial distance (r/R)
α	section angle of attack
α_D	rotor angle of attack
γ	blade Lock's number
δ_1	longitudinal flapping due to pitching velocity of helicopter
δ_2	lateral flapping due to pitching velocity of helicopter
δ_3	pitch-flap coupling angle
θ	blade section pitch-angle
θ_o	collective pitch angle
θ_r	root section pitch angle
θ_t	tip section pitch angle
λ	non-dimensional inflow parameter $(V \sin\alpha_D - \bar{v}_i)/V_t$
μ	tip-speed ratio $V\cos\alpha_D/V_t$
ρ	local air density
ϕ	downwash angle $v_i/V_t\, x$
$\ddot{\psi}$	helicopter yaw acceleration
Ω	rotor rotational speed
Ω_{MR}	Main-rotor speed