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MAXIMUM FINAL ENERGY CHANDELLE FOR SUPERSONIC AIRCRAFT

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ABSTRACT

The chandelle is a very important three dimensional maneuver used during aircombat. The final total energy per unit weight of the airplane is an important criterion required to be maximum at the end of the maneuver. A simple criterion function leads to the increase of the time of performance of the chandelle. This paper introduces a compound criterion function that includes the time of performance of the chadelle. This approach enables to solve the maximum final energy chadelle with a limitation upon the time of performance of the maneuver.
The gradient method is used to solve this nonlinear trajectory optimzation problem. The obtained optimal control laws have different strategies for the supersonic aircraft and for the jet trainer. The gain of energy for supersonic fighter is very high compared to the nominal solution. This paper shows that classical solution of the chadelle is not suitable in this case.

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INTRODUCTION

The chandelle was solved in Ref. [1] as a three dimensional moneuver assuming an exchange of kinetic and potential energies of the aircraft. A modified solution was given in Ref [2] which took into consideration the existance of longitudinal component of the load factor. In Ref [2] the optimum chandelle was solved. In Ref [3] maximum final energy chandelle was solved for a jet trainer. When applying the method on a supersonic fighter the time of per formance of the chandelle is increased. This fact suggests the modified work of this paper. The objective here is to find that control law leading to the maximum final energy and simultaneously keeping the time of performance of the chandelle at a required low values.

MATHEMATICAL MODEL

The mathematical model expressing this maneuver is a system of nonlinear differential equations of the form

$$\dot{x} = f(x, u) \qquad , \quad u \in U \qquad (1)$$

where x is a four dimensional state vector with the components (h, v, γ, ψ) and u is a two dimensional control vector with the components (n, Ø). A complete analysis of this model is given in Ref [4] . The above mathematical model assumes that the aircraft is represented as a mass point, the forces acting upon an airplane performing three dimensional maneuver is shown on fig. (1)

PROBLEM FORMULATION

The objective of this paper is to find a control law n (t) and Ø (t) such that the energy per unit weight of the airplane at the end of the maneuver E_f is maximum and the time of performance is kept at required low values. So a new criterion function is defined as

$$P_1 = - B\, E_f \,.C + (1 - C).\, t_f \qquad (2)$$

Where B is a number used to give both terms of the above equation values of the same order of magnitude.
This modefied criterion function represents the extension of the work of Ref [3]. If C = 1. the problem is to maximize the final energy without any restriction on the time of performance of the chandelle t_f . If C = 0 , then the problem is transfered to minimum time chandelle.
Since the gradient method will be used to solve this problem it is better to use the penalty function approach to change the fixed end point problem to a free end point one. Finally the problem can be formulated as follows:

6 Find n(t) and Ø (t) which minimize

$$P = - B\, E_f . C + (1 - C)\, t_f + k\, (\gamma(t_f) - \gamma_f)^2 \qquad (3)$$

subject to

$$\dot{h} = V \sin \gamma \qquad (4a)$$

$$\dot{V} = (T - D - W \sin \gamma)/m \qquad (4b)$$

$$\dot{\gamma} = \frac{g}{V} (n \cos \text{Ø} - \cos \gamma) \qquad (4c)$$

$$\dot{\psi} = \frac{g}{V} \frac{n \sin \text{Ø}}{\cos \gamma} \qquad (4d)$$

with the following boundary conditions

$h(0) = h_0$	$h(t_f) = \text{free}$
$V(0) = V_0$	$V(t_f) = \text{free}$
$\gamma(0) = 0$	$\gamma(t_f) = \text{free}$
$\psi(0) = 0$	$\psi(t_f) = 180^{\circ}$

Equations (4a) through (4d) represent the mathematical model of a three dimensional maneuver.

NECESSARY CONDITIONS

Applying the maximum principle and introducing the system Hamiltonian

$$H = \sum_{i=1}^{4} \lambda_i f_i \qquad (5)$$

then the necessary conditions are the Euler-Lagrange equations given by

$$\dot{\lambda}_h = -\frac{\partial H}{\partial h} \qquad (6a)$$

$$\dot{\lambda}_V = -\frac{\partial H}{\partial V} \qquad (6b)$$

$$\dot{\lambda}_\gamma = -\frac{\partial H}{\partial \gamma} \qquad (6c)$$

$$\dot{\lambda}_\psi = -\frac{\partial H}{\partial \psi} \qquad (6d)$$

The boundary conditions for the above equation are according to Ref [4]:

$$\lambda_h(t_f) = - B\, C \qquad (7a)$$

$$\lambda_V(t_f) = - B\, C\, V(t_f)/g \qquad (7b)$$

$$\lambda_\gamma(t_f) = 2k\, (\gamma(t_f) - \gamma_f) \qquad (7c)$$

$$\lambda_\psi(t_f) = -\frac{\dot{P}}{\dot{\psi}}\bigg|_{t=t_f} \qquad (7d)$$

The second part of the necessary conditions is

$$\frac{\partial H}{\partial n} = 0 \quad , \text{ and } \quad \frac{\partial H}{\partial \phi} = 0$$

RESULTS AND DISCUSSIONS

The method is applied on a supersonic fighter, its data is given in Ref [6]. The least square method is used to find analytical expression for the given graphical data. The engine thrust T and the drag coefficient C_D are obtained as follows

$$T = f(h, M) \quad , \text{ and}$$

$$C_D = C_D(M) + K(M)\, C_L^2(\alpha)$$

The gradient method Ref [5] is used to solve this nonlinear trajectory optimization problem. A nominal control n(t) and Ø (t) is chosen. Then the state equations (4a) through (4d) are integrated forward till ψ = 180 . Then the boundary values of λ's are found according to equations (7a) through (7d), hence the costate equations (6a) through (6d) are integrated backwords till t =0. Then the control vector is modified according to

$$u_{i+1} = u_i + w\, H_{u\, i}$$

where i is the number of iterations, and
w is a negative definite diagonal matrix of the gradient step sizes.
The procedure is repeated up to the required dagree of accuracy.

The Parameter C:

This parameter determines the type of the problem to be solved. If C = 1, then it is a pure maximum final energy maneuver. The time of performance of the chandelle is increased. This parameter represents a comporomize between the final time and the final energy. A dependence between this parameter and the final time of both supersonic fighter and jet trainer is given on figure (2). This parameter is closely connected to the penalty function constant k, which is very important for the convergence of the solution. If C =0 then the problem is transformed to a minimum time one. The final energy as function of the number of iterations is shown on fig.(3). It is seen that the final energy increases very rapidly at the beginning and the convergence is better for smaller values of the parameter C.

Comparison Between The Optimal Control Laws:

As seen on fig.(4) and fig.(5) the obtained optimal control laws are basically different for the two used aircrafts. The bank angle control law for supersonic fighter is changed slightly during

6 the maneuver, but for the jet trainer it starts with low values of bank angle and increuses twards the end of the chandelle. The obtained optimal load factor control for the supersonic fighter is increased in the middle of the maneuver and decreased again at the end, but for the jet trainer it starts at certain value and decreases continuously till the end of the maneuver. Both control laws could be easily performed by pilots.

Comparison Between Optimal Trajectories:

As shown on figure (6), the optimal trajectory of the supersonic fighter is shifted to a region of high energy altitude. The gain of total energy for supersonic airplane is very high compared to the nominal solution. It could be easily seen that classical solution of the chandelle is very far from reality. The airplane gains above 18 km of energy altitude during the performance of the chandelle, but classical solution assumes exchange of kinetic and potential energies. Concerning the jet trainer the optimal trajectory is shifted to higher energy altitude too, but the shift is very small compared to the supersonic fighter. The optimum flight path angle for both airplanes is shown on figure (7). The flight path angle for supersonic fighter is more shallow than that of the jet trainer.

The convergence of the method is good. This is shown on figure (8), the two defined parameters q_n and q_ϕ approach very small values after 10 iterations. This means that $\frac{\partial H}{\partial n}$ and $\frac{\partial H}{\partial \phi}$ are approaching zero.

CONCLUSION

An optimal chandelle for supersonic fighter is obtained. The obtained optimal control law is simple enough to be performed by pilots. The classical solution of the chandelle is very far from reality it is seen that the gain of total energy for supersonic fighter is very high compared with the nominal solution. The convergence of the method is good and the used gradient method is efficient is this case.

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NOMENCLATURE

B	numerical coefficient
C	numerical parameter
C_D	drag coefficient
C_L	lift coefficient
D	drag force
E	energy altitude
g	acceleration of gravity
H	variational Hamiltonian
h	flight altitude
L	lift force
M	Mach number
m	aircraft mass
n	normal load factor,a control input
P	criterion function
T	thrust force
t	time
u	control variable
V	flight speed
W	aircraft weight
w	gradient step size
α	angle of attack
γ	flight path angle
ϕ	bank angle,a control input
λ	adjiont variable
ψ	heading angle

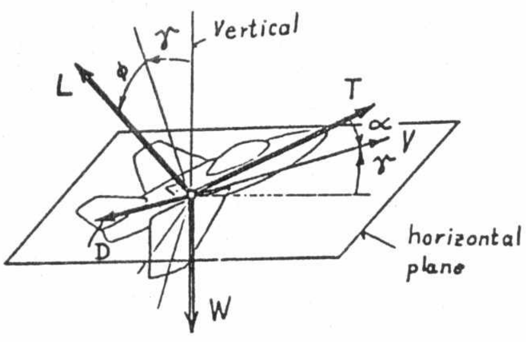

Fig.(1),Forces Acting upon an Airplane during three dimensional maneuver

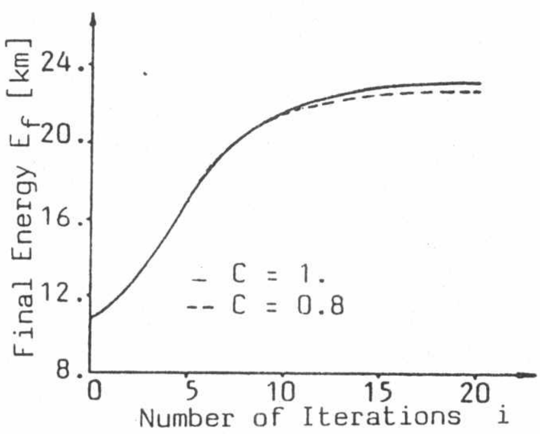

Fig.(2), Final Energy for Supersonic Airplane

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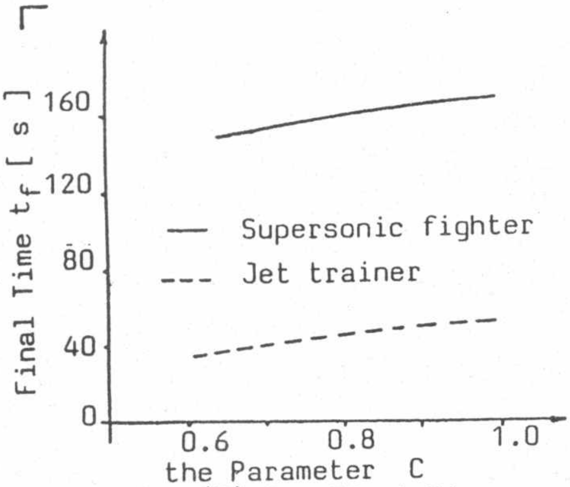

Fig.(3) The Final Time

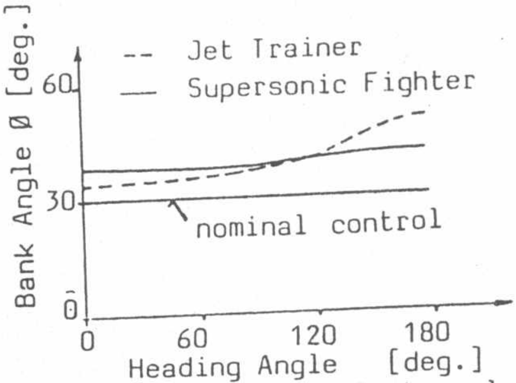

Fig.(4),Optimum Bank angle

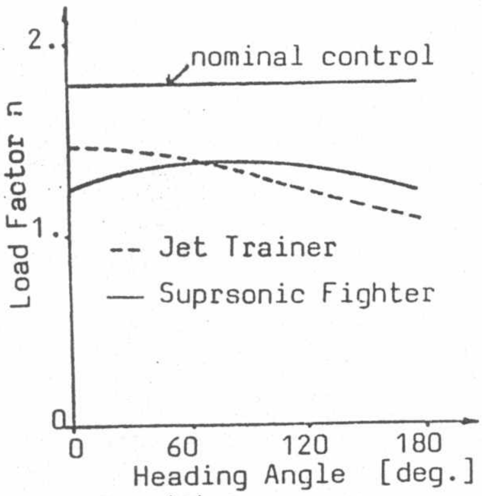

Fig.(5),Optimum Load Factor

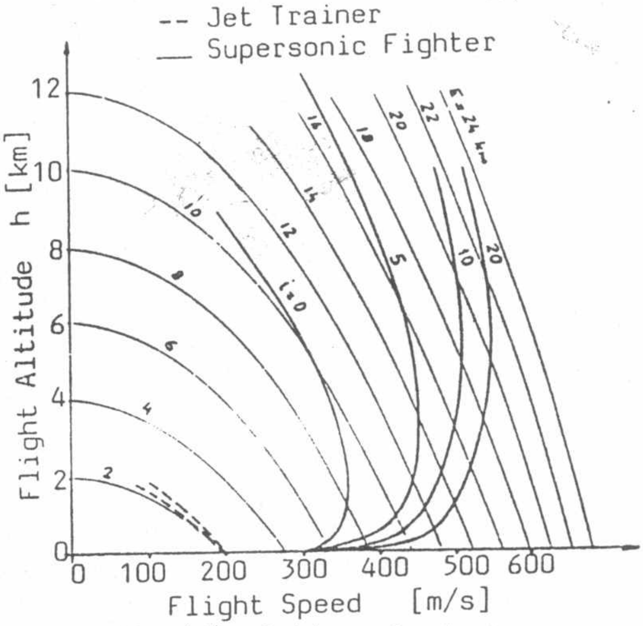

Fig.(6), Optimum Trajectory

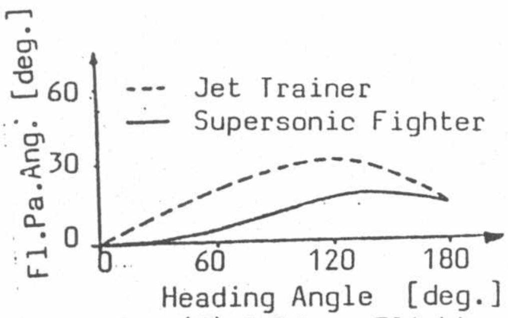

Fig.(7),Optimum Flight Path Angle

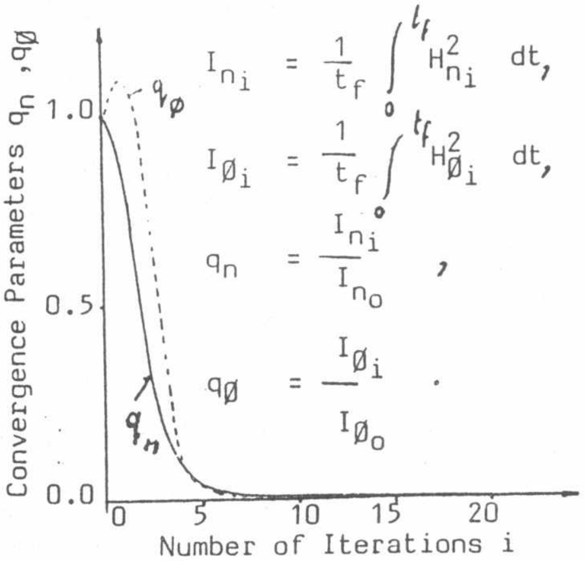

Fig.(8) Convergence of the Solution