MILITARY TECHNICAL COLLEGE

CAIRO - EGYPT

ANALYSIS OF FULLY DEVELOPED LAMINAR CHANNEL FLOW OF SUSPENSIONS WITH EXTERNAL ELECTRIC FIELD

S.M.ELDIGHIDY*, K.A.ALBIDWEIHY**

ABSTRACT

Deposition of suspensions of solid particles in channel and pipe flows causes serious technical problems due to blockage and contamination of flow passages. Such problems, for instance, occur in the fluidic devices (Laminar Proportional Amplifiers) used for flow control. In this work, the analysis of fully developed laminar channel flow of suspensions with external electric field is presented. The external electric field is obtained by a number of equally spaced parallel cylinders having the same potential and placed at the channel centre line.

A mathematical model for the flow of suspensions using the equations of diffusion, Poisson, external electric field and rate of deposition along with the proper boundary conditions is given. Numerical solution is obtained by using the finite difference technique.

The effects of the external electric field intensity and the number of charged cylinders on the rate of deposition of the solid particles are studied.

The analytical results show that an externally applied electric field obtained by an array of equally spaced parallel cylinders having the same voltage and placed at the channel centre can appreciably reduce the rate of particle deposition from laminar channel flow of suspensions if the potential of the charged cylinders has an opposite sign with respect to the polarity of the charge on the particles. In addition, increasing the number of cylinders for the same applied voltage results in more reduction in rate of deposition.

* Associate professor, Dept. of Mech. Eng., Faculty of Eng., Zagazig Univ., Zagazig, Egypt. ** Associate professor, Dept. of Elect. Eng., Cairo Univ., Giza, Egypt.

INTRODUCTION

Particle deposition from channel flow of suspensions is encountered in many practical applications such as fluidic devices and electrostatic precipitators. Deposition of particles in external flows of suspensions have been investigated by many authors [1-3]. In addition, a lot of research work has been done in deposition of particles in internal flows of suspensions [4-17]. The effects of diffusion, gravity and electric fields on rate of deposition in both laminar and turbulent flows for pipes and channels have been investigated [4-17].

The electrostatic precipitator is one of the conventional particulate control devices that effectively control the emissions of the particles [17]. The collection mechanism is dependent upon the electrical force that results from the action of an electric field on electrically charged particles. However, the electrostatic precipitator is concerned with collection of solid particles of small size from a low speed flow of suspensions. In an electrostatic precipitator it is required to collect the solid particles on the walls. On the other hand, particle deposition in fluidic devices causes serious technical problems due to blockage and contamination of flow passages. This particle deposition is primarily due to the electric field generated by the charges on the particles. It is worth noticing that the presence of electric charges on the particles[18] makes it possible to keep them far from the walls by means of an external electric field thus reducing the particle rate of deposition. This reduction is dependent on the external electric field intensity and configuration.

Theoritical reduction of rate of deposition of suspensions in turbulent flow over a tilted flat plate by applying an external electric field has been reported [3]. It has been found that the intensity of the external electric field, the number of charged cylinders and their height above the plate affect the rate of deposition of the solid particles.

In this paper, the analysis of fully developed laminar channel flow of suspensions with external electric field is given. Here, it is meant by channel the constant area one, i.e. the parallel plate channel. The effect of different external electric field configurations on the rate of deposition, particle concentration and electric field intensity is studied. The external electric field is obtained by a number of equally spaced parallel cylinders having the same voltage and placed at the channel centre line.

ANALYSIS

Consider a fully developped laminar flow of suspensions flowing into a parallel-plate channel as shown in Fig. 1. An external electric field is applied by means of an array of parallel cylinders whose centres lie on the centre line of the channel. The cylinders are equally spaced and symmetrically placed with respect to the channel plates, and are excited by the same voltage. The electric field geometry is shown in Fig. 2.

The following assumptions were considered to simplify and limit the analysis of the problem :

(1) Incompressible, steady flow.
(2) Two dimensional, fully developped laminar flow

(3) Dilute suspension.
(4) Particle-particle interaction is negligible.
(5) Fluid -particle interaction follows the Stoke's drag law.
(6) The thickness of the layer of deposit is much smaller than half the channel width a.
(7) The effects of gravity and surface adhesion are neglected.
(8) Negligible axial component of the induced electric field.
(9) The effect of the charged cylinders on the flow characteristics of the fluid is neglected since it is limited to small regions around the cylinders.
(10) The cylinders radii are much smaller than both the spacing between them and half the channel width.
(11) The effect of the particle charges in the calculation of the external electric field is neglected.

Considering the aforementioned assumptions, the governing equations for the particulate phase will be [2, 3, 6] :

$$u\frac{\partial \rho_p}{\partial x} = D_p \frac{\partial^2 \rho_p}{\partial y^2} - \frac{1}{\bar{F}} \left(\frac{q}{m_p}\right) \frac{\partial\, \rho_p (E_i + E_e)}{\partial y} \tag{1}$$

$$\frac{\partial E_i}{\partial y} = \left(\frac{\rho_p}{\varepsilon_o}\right) \left(\frac{q}{m_p}\right) \tag{2}$$

$$E_e = \sum_{n=1}^{N} - K_n V\left\{ \sum_{m=o}^{\infty} (-)^m \varepsilon_m \left[\frac{2am-y}{r_{nm}^2} - \frac{2am+y}{r'^2_{nm}}\right] \right\} \tag{3}$$

$$\dot{m} = \sigma \left(\frac{\rho_{pw}}{\bar{F}}\right) \left(\frac{q}{m_p}\right) (E_{iw} + E_{ew}) \tag{4}$$

Where K_n, r_{nm} and r'_{nm} are given by

$$K_n = \frac{C_{ii}}{2\pi\varepsilon_o} = \frac{1}{\ln(4a/d)}$$

$$r_{nm}^2 = (X-X_n)^2 + (2ma - y)^2$$

$$r'^2_{nm} = (X-X_n)^2 + (2ma + y)^2$$

Equation (1) is the diffusion equation with an added term to account for the electric drift of the particles due to both the induced and the external electric field. Eq. (2) is the Poisson equation. The left hand side of Eq. (2) does not contain the external electric field since its divergence is zero except on the surfaces of the cylinders. Equations (3) and (4) represent the external electric field and rate of deposition of solid particles respectively. The fluid velocity for fully developed laminar channel flow is given by :

$$u = \frac{1}{2\mu} \frac{dP}{dx} (y^2 - a^2) \tag{5}$$

where dP/dx is the pressure gradient and μ is the fluid dynamic viscosity.

The boundary conditions are :

at X=0 (at intlet)

$$\left.\begin{aligned} \rho_p (0,y) &= \rho_{p_o} \\ E_i (0,y) &= 0 \end{aligned}\right\} \tag{6}$$

at y=0 (at channel center line)

$$\left.\begin{aligned} \partial\rho_p(X,0)/\partial y &= 0 \\ E_i (X,0) &= 0 \end{aligned}\right\} \tag{7}$$

at y=0 (at the wall)

$$D_p \; \partial\rho_p(x,a)/\partial y = (1 - \sigma)\left(\frac{\rho_{pw}}{\bar{F}}\right)\left(\frac{q}{m_p}\right)(E_{iw} + E_{ew}) \tag{8}$$

Using the normalized quantities as defined in the nomenclature, the above equations can be written as follows :

$$\beta K(1-Y^2) \frac{\partial\rho_p^*}{\partial x} = \frac{\partial^2\rho_p}{\partial Y^2} - \frac{\partial \rho_p^*(E_i^* + E_e^*)}{\partial Y} \tag{9}$$

$$\frac{\partial E_i^*}{\partial Y} = 4\ \alpha\ \rho_p^* \qquad (10)$$

$$E_e^* = \sum_{n=1}^{N} K_n\ V^* \left\{ \sum_{m=o}^{\infty} (-)^m\ \varepsilon_m \left[\frac{2m-Y}{R_{nm}^2} - \frac{2m+Y}{R'^2_{nm}} \right] \right\} \qquad (11)$$

$$\dot{m}^* = \sigma\ \rho_{pw}^*\ (E_{iw}^* + E_{ew}^*)/\beta \qquad (12)$$

with boundary conditions:

at X=0

$$\left.\begin{aligned} \rho_p^*\ (0,\ Y) &= 1 \\ E_i^*\ (0,\ Y) &= 0 \end{aligned}\right\} \qquad (13)$$

at Y = 0

$$\left.\begin{aligned} \partial \rho_p^*\ (X,\ 0)/\partial Y &= 0 \\ E_i^*\ (X,\ 0) &= 0 \end{aligned}\right\} \qquad (14)$$

at Y = 1

$$\partial \rho_p^*(X,\ 1)/\partial Y = (1-\sigma)\ \rho_{pw}^*\ (E_{iw}^* + E_{ew}^*) \qquad (15)$$

NUMERICAL SOLUTION

Equations (9) to (12) with the boundary conditions given by Eqs. (13) to (15) can be solved for the unknowns ρ_p^*, E_i^* and $\dot{m}^*$ by applying the finite difference technique [19,20]. The finite difference equations take the form of the matrix equation AX=B which can be solved at any axial position using the standard Gauss elimination method. The quantity $\partial \rho_p^*\ E_i^*/\partial Y$ of the last term of Eq. (9) is nonlinear and it has been approximated by replacing the values of one of the variables ρ_p^*, E_i^* at a given position by its values at the preceding axial position.

RESULTS AND DISCUSSIONS

The results presented in this paper are based on typical values for the parameters (α, β and σ) obtained from previous researches [2, 6]. These values are $\alpha=1$, $\beta=1000$ and $\sigma=0.5$ and were considered constant throughout the computations since the main objective of this paper is to study the influence of the external electric field on the rate of deposition of the particles on the channel walls. The diameter of the charged cylinders was taken 1% of the channel width. The assigned range for the number of cylinders and voltages is N=1 to 4, V*= 0 to 300.

The effect of external electric field E_e^* on the rate of deposition $\dot{m}^*$

for the case of three cylinders with $V^*=25$ each (at $x=2.5$, 5, 7.5) is depicted in Fig. 3. It is obvious that the rate of deposition curve becomes wavy with regions of maxima and minma at the axial positions of the charged cylinders. Such regions are located around the negative peaks of the external electric field as can be seen from Fig. 4 which shows the distribution of the y-component of the induced and external electric fields along the channel wall. From Fig. 3 it is clear that the external electric field reduces significantly the deposition rate of the particles on the channel walls.

The effect of the number of charged cylinders N on the relative rate of deposition $\dot{m}_r^*$ for different values of voltage V^* ($V^*=25$, 50 and 100) is shown in Fig. 5. It can be noticed that as N increases, $\dot{m}_r^*$ decreases as expected.

Figure 6 shows the variation of the relative rate of deposition with voltage for different numbers of charged cylinders (N=1 to 4). It is clear that $\dot{m}^*$ decreases as V^* increases and that the decrease in $\dot{m}^*$ becomes more pronounced for larger N. Comparing the results of the flat plate [3] where N=3 and H=10 (H is the height of the cylinders above the plate normalized to the boundary layer thickness) with the results of the channel for n=3, one can see that $\dot{m}^*$decrease in the channel more than on the flat plate. This is attributed to the stronger electric field in the channel case because of the presence of the upper plate and the resulting multiple images.

Figure 7 indicates the distribution of the y-component of the total electric field ($E_i^*+E_e^*$) and the induced one (E_i^*) for N=3 and $V^*=25$. It is clear that the external elctric close to its negative peak at x=2 is strong enough to overcome the effect of the induced electric field resulting even in a negative total elctric field at the wall which keeps the particles away from the wall. Between the cylinders E_e^* decreases in magintude and in some locations as x=10 it is unable to overcome the effect of the induced electric field resulting in a smaller reduction in the rate of deposition. The distribution of the induced electric field is slightly different from its distribution without the external electric field.

CONCLUSIONS

The analytical results show that an externally applied electric field obtained by an array of equally spaced parallel cylinders having the same voltage and placed at the channel centre line can appreciably reduce the rate of particle deposition from laminar channel flow of suspensions if the potential on the charged cylinders has an opposite sign with respect to the polarity of the charge on the particles. In addition, increasing the number of cylinders for the same applied voltage results in more reduction in the rate of deposition. The particle deposition is almost negligible if the number of cylinders is such that the spacing between them is approximately equal to the channel width. The required voltage V is given by

$$V = -50 \left[\bar{F} D_p / (q/m_p) \right]$$

which corresponds to $V^*=50$.

6

The required voltage V depends on $\bar{F}$, D_p and (q/m_p), and is of the order of several kilovolts for a typical case.

REFERENCES

1. Forstrom, R.J., Voss, G.O. and Blackshear, P.L., "Fluid Dynamics of Particles (platelet) Deposition for Filtering Walls; Relationship to Atherosclerosis," J. Fluids Engr., ASME Trans.(1973).
2. Eldighidy, S.M. and Fatahalah, K.A., "Deposition of Suspensions in Turbulent Flow over a Tilted Flat Plate," presented in the ASME Meeting of the Fluids Engineering in Boulder, Colorado, U.S.A. (1981).
3. Eldighidy, S.M. and Albidweihy, K.A., "Theroretical Reduction of Rate of Deposition of Suspensions in Turbulent Flow over a Tilted Flat Plate by Applying an External Electric Field," Journal of Heat and Fluid Flow, England (under publication)
4. Chua, J.H. and Wang, C.S., "Deposition of Sub-micron Particles from Steady Flows in a Branched Tube," J. Res. Assoc. Powder Tech., Vol.9 (1972).
5. Davies, C.N., "Diffusion and Sedimentation of Aerosol Particles from Poiseuille Flow in Pipes, " J. Aerosol Sci., Vol. 4 (1973)
6. Eldighidy, S.M., "Deposition of Suspensions in Laminar Flow in the Enterance Region of a Channel and in a Diffuser," Ph.D. thesis, New Jersey Inst. of Tech., U.S.A. (1975).
7. Eldighidy,S.M., Chen, R.Y. and Comparin, R.A., "Deposition of Suspensions in the Enterance of a Channel," J. Fluids Eng., ASME Trans., Vol. 99, Series 1, No. 2 (1977).
8. Eldighidy,S.M., Chen, R.Y. and Comparin, R.A., "Laminar Flow of Suspensions in the Enterance Region of a Diffuser," J. Fluids Eng., ASME Trans., Vol. 101, No. 3 (1979).
9. Eldighidy, S.M., "Channel Flow of Suspensions in Tubulent Fluid Diffusion and Electrostatic Effects," (HMT) Heat and Fluid Flow in Power System Components, Pergamon Press Ltd., Vol. 3 (1979).
10. Ingham, D.B., "Diffusion and Sedimentation of Aerosol Particles from Poiseuille Flow in Rectangular Tubes," J. Aerosol Science, Vol. 7 (1976).
11. Williams, J.C. and Jackson, R., "The Motion of Solid Particles in an Electrostatic Precipitator," Proceedings of the Symposium on the Interaction between Fluids and Particles, London (1962).
12. Stock, D.E. and Eshbach, E.J., "Predicting Particle Motion and Efficiency of Electrostatic Precipitators," Presented at the Symposium on Polyphase Flow and Transport Technology, San Francisco, California (1980).
13. Soo, S.L. and Tung, S.K., "Pipe Flow of Suspensions in Turbulent Fluid, Electrostatic and Gravity Effects," Appl. Sci. Res., Vol. 24 (1971).
14. Soo, S.L. and Tung, S.K., "Deposition and Entrainment in Pipe Flow of a Suspension," Powder Technology, Vol. 6 (1972).
15. Stukel, J.J. and Soo, S.L., "Turbulent Flow of a Suspension into a Channel," Powder Technology, Vol. 2 (1968/1969).
16. Wang, C.S., Altshuler, B. and Palmes, E.D., "The Distribution and Deposition of Particles Suspended Between Parallel Plate Surfaces," J. Colloid and Interface Science, Vol. 26 (1968).
17. Nichols, G.B., "Particulate Emission Control from Pulp Mill Recovery Boilers with Electrostatic Precipitators," IEEE Trans. on Industry Appl., Vol. IA-13, No. 1 (1977).

18. Kunkel, W.B., "The Static Electrification of Dust Particles on Dispersion into a Cloud," J. Appl. Phys., Vol. 21 (1950)
19. Cohen, A.M., "Numerical Analysis," Halsted Press (1972)
20. Carnahan, B., Luther, H.A., Wilkes, J.O., "Applied Numerical Methods," John Wiley & Sons Inc., Ch. 7 (1969)
21. Smythe, W.R., "Static and Dynamic Electricity," McGraw Hill Book Co. (1968)

NOMENCLATURE

a	half width of channel
d	diameter of the charged cylinder
D	dimensionless diameter of the charged cylinder, $D = d/a$
D_p	particle diffusivity
E_i	induced electric field intensity, $E_i = E_{iy}$
E_i^*	dimensionless induced electric field intensity, $E_i^* = (\frac{q}{m_p})\ (a/\bar{F}\ D_p)\ E_i$
E_e	the y-component of the external electric field intensity Eq. (3)
E_e^*	dimensionless y-component of the external electric field intensity given by Eq. (11)
$\bar{F}$	inverse of relaxation time for momentum transfer [6]
K	constant of fluid phase velocity [$U=K(1-Y^2)$], $K = -(a^2/2u_{av})\ dp/dx$
m	order of image of charged cylinder
m_p	mass of particle
$\dot{m}$	mass flow rate of the particles deposited on the plate, given by Eq. (4)
$\dot{m}^*$	dimensionless rate of deposition of particles given by Eq. (12)
$\dot{m}_r^*$	relative rate of deposition of particles, defined as the ratio of $\int_0^L \dot{m}^*\, dx$ with external electric field to its value without E_e , where L is the normalized channel length in the flow direction
N	number of equally charged cylinders.
q	electric charge per particle
r,r'	distance from a point (x,y) to a charged cylinder and its image respectively as shown in Fig. 2
R,R'	dimensionless distances, $R=r/a$, $R'= r'/a$
u,v	axial and vertical components of fluid velocity
u_p,v_p	axial and vertical components of particle velocity
u_{av}	average velocity
U	dimensionless velocity of fluid, $U=u/u_{av} =K(1-Y^2)$
V	applied voltage on each cylinder
V^*	normalized voltage, $V^* = -(q/m_p)\ (V/\bar{F}\ D_p)$
x,y	axial and vertical coordinates respectively
X,Y	dimensionless axial and vertical coordinates, $X=x/a$. $Y=y/a$

Greek Letters

α	electrostatic charge parameter $\quad \alpha = (\frac{\rho_{po}}{4\ \varepsilon_o})\ (\frac{q}{m_p})^2\ (\frac{a^2}{\bar{F}\ D_p})$

6

β diffusive peclet number, $\beta = u_{av} a / D_p$

ε_o permittivity of free space, $= 8.85434 \times 10^{-12}$ Coul²/N.m²

ρ_p density of particle cloud (concentration)

ρ_{po} inlet particle concentration (uniform)

ρ_{p*} dimensionless particle concentration,

σ sticking probability accounts for electr-viscous and gravity forces

Superscript

* dimensionless quantities as defined

Subscripts

c for centre line condition
e for external electric field
i for induced electric field
o initial condition
p for particle phase
w for wall condition

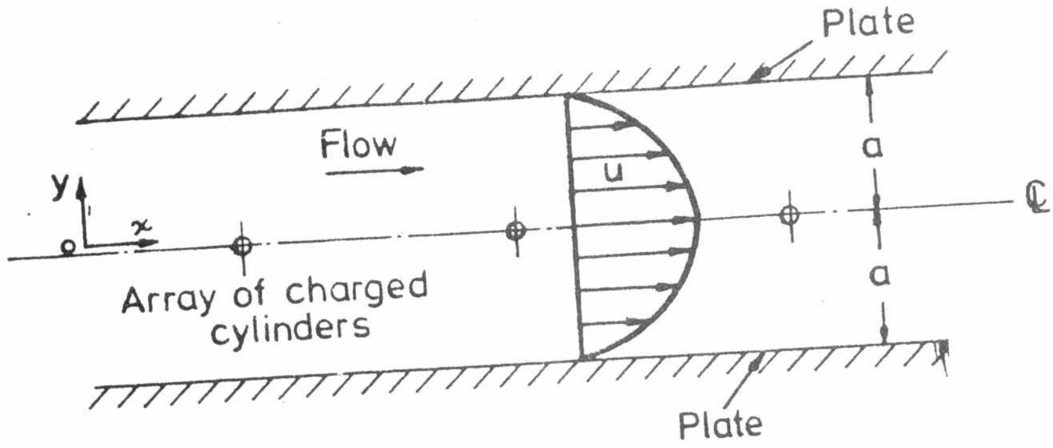

Fig. 1 Flow Configuration

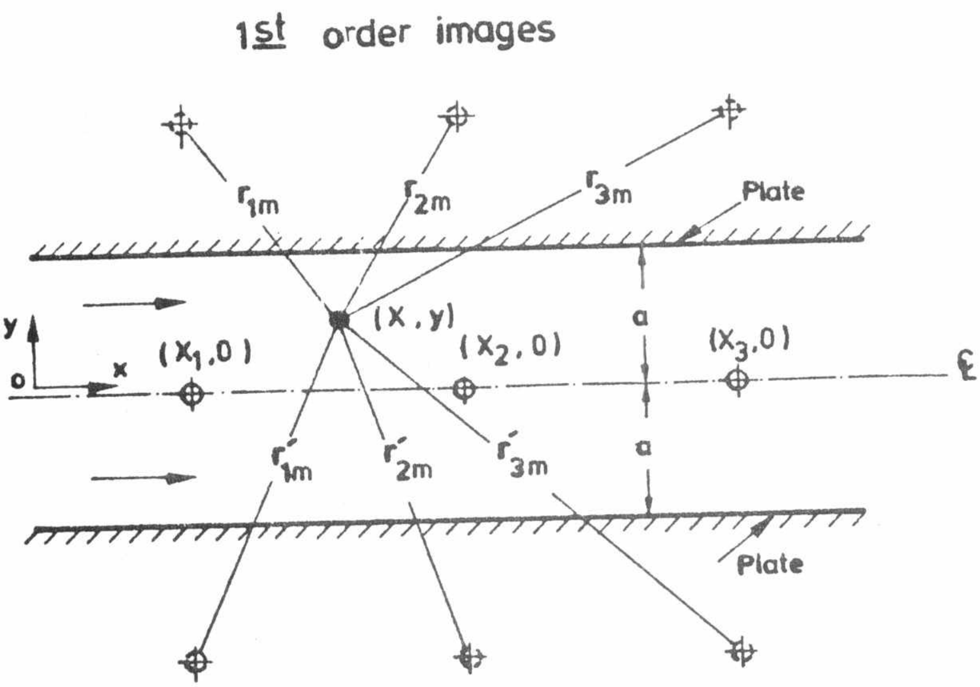

Fig. 2 External electric field geometry .

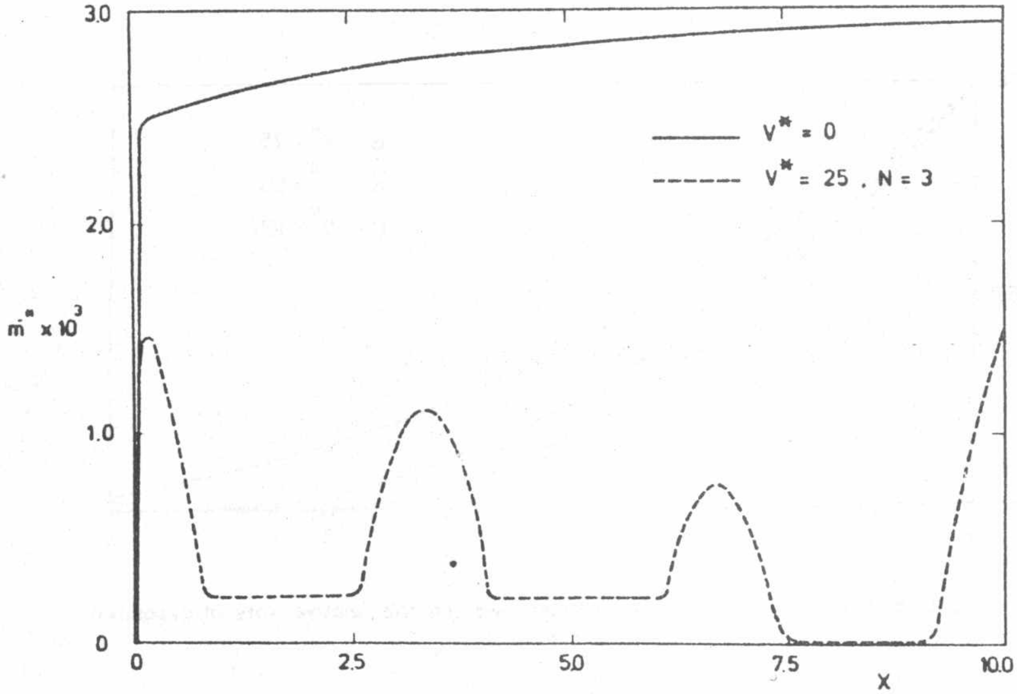

Fig. 3 Effect of external electric field on rate of deposition .

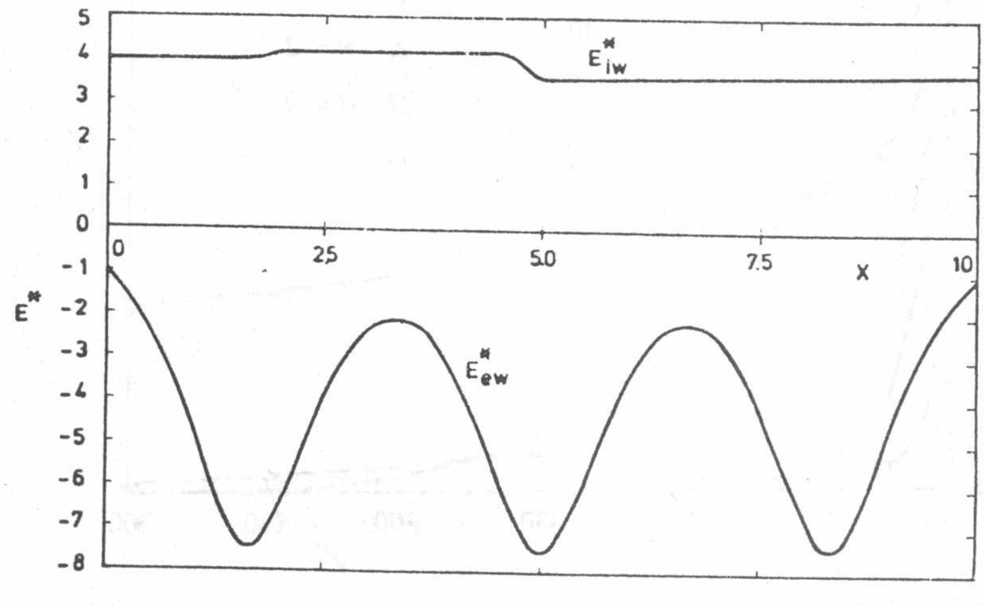

Fig. 4 Induced and external electric fields along the channel wall (V^*= 25 , N= 3) .

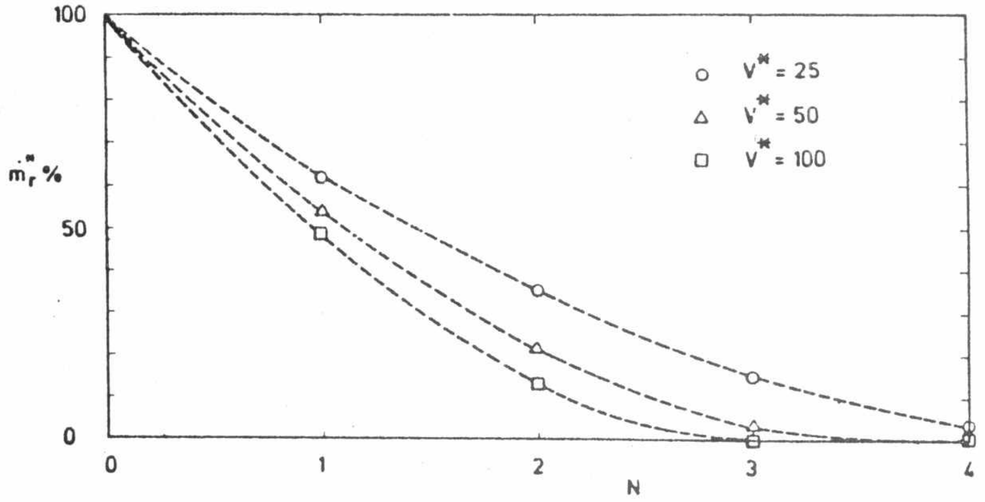

Fig. 5 Effect of number of charged cylinders on the relative rate of deposition .

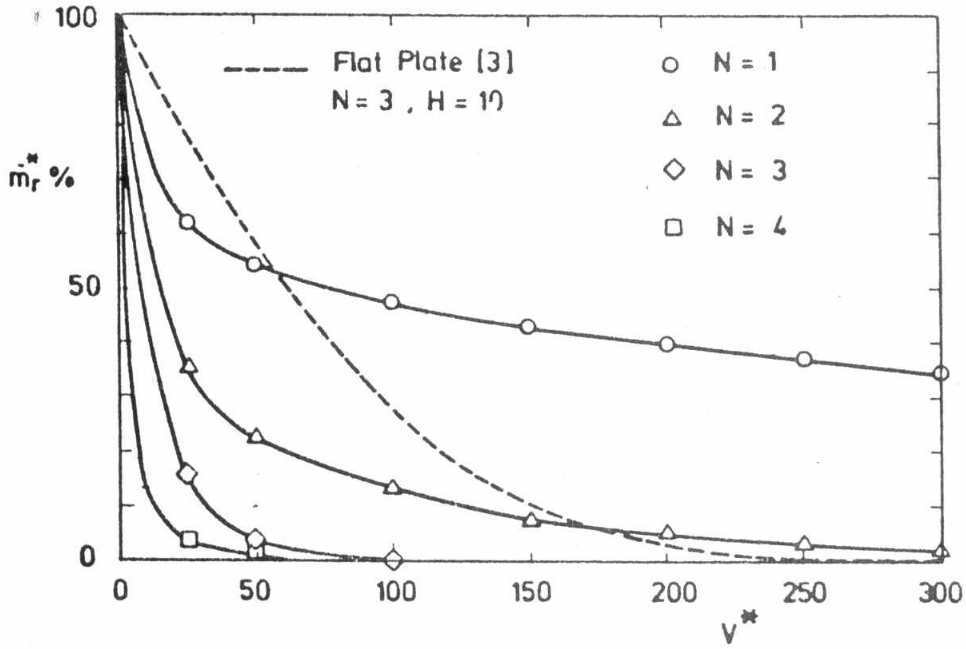

Fig. 6 Variation of the relative rate of deposition with voltage .

FIRST A.S.A.T. CONFERENCE

14-16 May 1985 , CAIRO

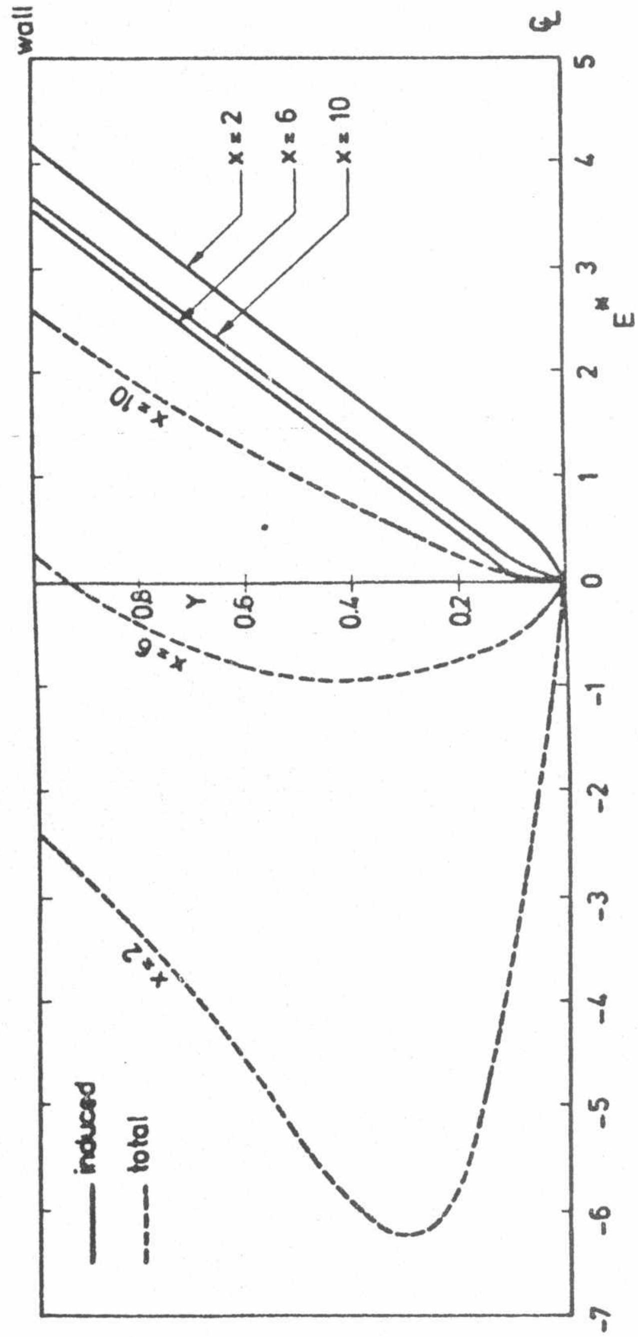

Fig. 7 Induced and total electric field distributions (V^* = 25 , N = 3).