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AVAILABLE ENERGY CONTOUR LINES ON T-S AND P-V PLANES

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ABSTRACT

The primary objectives of this paper are : 1-To present the available energy balance equation for a non-reacting closed system. 2- To develop and discuss a computational method for plotting the available energy contour-lines of that system on the temperature-entropy, and pressure-volume planes. 3 - To apply the method and present the contour-lines for three different cases for air : i - as a perfect gas (having perfect gas equation of state and const-ant specific heats), ii. as a semiperfect gas (having perfect gas equation of state and temperature-dependent specific heats), and iii- as a real gas (having Vander-Waals equation as an example for equation of state , and temperature-dependent specific heats).

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INTRODUCTION

Available energy has been a subject in classical thermodynamics that can be traced back over a period of more than 100 years. However, there have been significant developments within the last few years. In the course of this extensive treatment of the subject, some reproduction of well-known material is unavoidable if clarity and continuity of presentation are to be maintained. Parts of the present work are thought to be original, particularly in the concise nature of the general methodology developed for plotting the available energy contour-lines for real substances .

The method developed here can easily be extended to represent the available energy surface in three dimensions. The importance of this work lies in the fact that it enables appropriate evaluation and design criteria to be set-up for many work-producing and work - absorbing systems. Examples of using the contour - lines to analyze cycle performance of I.C. engines are given in $\lceil 1-2 \rceil$.

AVAILABLE ENERGY BALANCE EQUATION

The available energy a, for a non-reacting closed system communicating only with the surroundings is defined, (e.g.[4] and [8]), as the maximum work which could be obtained by bringing the system to a state of thermodynamic equilibrium with the surroundings - to its dead state at pressure P_0 and temperature T_0 . The value of a per unit mass of the system is given by :

$$a = u + KE + PE + P_0 v - T_0 s - (u_0 + P_0 v_0 - T_0 s_0) + a_0$$
(1)

where u, KE and PE are the values per unit mass of the internal, kinetic and potential energies, and v and s are specific volume and entropy of the system respectively. In the absence of KE and PE and with the available energy at dead state $a_0 = o$, Eq. (1) reduces to :

$$a = u - u_0 + P_0(v - v_0) - T_0(s - s_0)$$

The above equation is composed entirely of changes in properties and does not involve heat or work: It applies to all closed systems.

COMPUTATIONAL METHOD

The calculation of thermodynamic properties of any simple substance requires expressions of two types:

(1) an equation for constant volume specific heat at low density c_v^0 (T), and (2) a P-v-T equation of state, which can be put in the form:

$$P = \rho RT + F (\rho, T)$$

where R is the gas constant, and F is a function of ρ and T with $F=O(\rho^2)$ as $\rho \neq 0$, so that semiperfect behaviour is obtained at low densities(P = ρ RT). For the present work the above two expressions are assumed known. They will be used in the following manner.

The internal energy u is generally a function in T and v and its differential

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• p,T

integration is at zero density and the second is at constant temperature.

The entropy is determined from the Gibbs equation: $ds = (\frac{1}{T})du + (\frac{P}{T})dv$. Adding and subtracting $R(d_{P}/P)$, integrating over the path shown in Fig.1, and using Eq.(3), the last equality can be rearranged into: $s-s_0 = \int_{T}^{T} (\frac{c_V^0(T)}{T}) dT - R \ln(P) + \int_{0}^{P} \frac{1}{\rho^2} [(\frac{-\partial F(P,T)}{\partial T})] d\rho$ (7) Datums for u and s are chosen so that $u-u_0 = 0$ and $s-s_0 = 0$ for semiperfect gas at T_0 and P_0 . This is done by adding $[-\int_{0}^{P_0} (\frac{1}{\rho^2}) [F(P,T_0) - T_0 \frac{\partial F(P,T)}{\partial T}] dP] \& [Rn(P_0) - \int_{0}^{P_0} (\frac{1}{\rho^2}) [-\frac{(\partial F(P,T))}{\partial T}] dP]$ to the R.H.S. of Eqs (6) and (7) respectively. Also substituting $v-v_0 = \frac{1}{P} - \frac{1}{P_0}$ into Eq. (2)., it reduces to : $a = \int_{T}^{T} c_V^0(1 - \frac{T_0}{T}) dT + T_0R \ln(\frac{P}{P_0}) + \frac{P_0}{P_0}(\frac{P_0}{\rho} - 1) + \int_{0}^{P} (\frac{1}{\rho^2}) [F(P,T) + T_0(1 - \frac{T}{T_0}) - \frac{\partial F(P,T)}{\partial T}] dP - \int_{0}^{P^0} (\frac{1}{\rho^2}) [F(P,T)] dP$ (8 a)

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(8 b)

(9)

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(note that a=o at T_0 and P_0). Equation (8) can be written in the compact form:

$$z(a, \rho, T) = 0$$

Mathematically, the above equation is a level surface in the $z(a,\rho,T)$ -space, (since grad $z=(\Im z/\Im a, \Im z/\Im \rho, \Im z/\Im T) \neq (0,0,0)$ at every point in the $z(a,\rho,T)$ -space). The equation for the available energy contour-lines(level curves) is:

$$z(a^{,}, \rho, T) = 0, a^{,} = cons$$

Equation (9) is a nonlinear equation in ρ and T, and its degree of complexity depends on the expressions of c_V^0 (T) and P(ρ,T) used in it, and a stable nonlinear technique must be used to solve it .

The general technique for the case of a perfect gas will be outlined.Equation(8), for a perfect gas (with $\rho = (\frac{1}{V})$, and a = a^{*} = const.), reduces to :

$$a^{*} = c_{V}^{T_{0}} \left[\left(\frac{T}{T_{0}} - 1 - \ln\left(\frac{T}{T_{0}} \right) \right] + RT_{0}^{T_{0}} \left[\left(\frac{V}{V_{0}} \right) - 1 - \ln\left(\frac{V}{V_{0}} \right) \right]$$
(10 a)

or

$$a^{*} = c_{V}T_{0}f\left(\frac{T}{T_{0}}\right) + R T_{0} f\left(\frac{V}{V_{0}}\right)$$
(10 b)
where $f(x) = \left[x-1-\ln(x)\right]$ is always a non-negative function for x>0. Since
 $\left(\frac{T}{T_{0}}\right) > 0$ and $\left(\frac{V}{V_{0}}\right) > 0$, then Eq.(10 b) gives bounds for $f\left(\frac{T}{T_{0}}\right)$ and $f\left(\frac{V}{V_{0}}\right)$ as :

$$o < f(\frac{T}{T_0}) < (\frac{a^*}{c_v T_0})$$
, and $o < f(\frac{v}{v_0}) < (\frac{a^*}{RT_0})$

This means that T and v can be defined only within these bounds. Also for any admissible value of v (or T), Eq.(10 b) gives two corresponding T(or v) values. The computational procedure , in a global sense , reduces to solving Eq. (10 b) by assuming admissible values for T (or v) and solving for the two roots of the other variable . The procedure is to be repeated till the admissible domain of T and v is covered .

CASE i. AIR AS A PERFECT GAS

Figures (2a) and (2b) show five constant available energy contour-lines on the p-v and T-s planes, along with lines of constant p,v,T and s. In the first figure, the line $T=T_0$ is the locus of points on the available energy contour-lines where $(dp/dv) = \infty$. Also in the second figure, lines of $P=p_0$ and $v=v_0$ are the loci of points on the contour-lines where $(dT/ds)=\infty$, and (dT/ds) = o respectively. In Fig. (2a), as T decreases (below T_0), the contour-lines approach each other very fast specially as p goes below p_0 and v goes above v_0 . Similar situation exists in Fig, (2b).

CASE ii. AIR AS A SEMIPERFECT GAS

Temperature- dependent specific heats, (shown in Fig. (3a) and (3b), and the perfect gas equation of state , are used to represent the air as a semiperfect gas . The temperature range covered by the specific heat equation is 273-3800 K, with maximum error 1.64 % . A comparison between the contour-lines



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of this case and that of case i shows that there is no difference on the p-v plane, while on the T-s plane, the difference, (case i is greater than case ii) increases as T increases. The maximum difference is a temperature shift of 12 K at T=680K for the contour-line a*=100.0 KJ/Kg.

CASE iii. AIR AS A REAL GAS

The Vander-Waals equation of state (given in Figs (4a) and (4b)) and the same temperature - dependent specific heat equations, used in case ii , are used to represent air as real gas . The contour - lines in this case are exactly the same as those of case ii .

CONCLUSIONS

The method presented in this paper for plotting the contour - lines is quite general and can be used for any non-reacting closed system . The importance of these contour-lines in second law (exergy) analysis of workproducing and work - absorbing processes and cycles is comparable to the use of steam-tables and other thermodynamic charts in first law(energy) analysis . Finally, it is worth to mention that the graphes preseated here are computer plotted , and the computer program can be easily used to plot the contours for any non-reacting material , provided that c_v^0 (T) and p- p - T relation are known .

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NOMENCLATURE

sp. available energy а cv^{,c}p const. vol. and const. press. sp. heats const. vol. sp. heat at very low density сV F,f, z functions KE, PE sp. kinetic and potential energies Ρ pressure gas constant and molecular wt. for air R.M universal gas const. Ru sp. entropy S absolute temperature Τ sp. internal energy u sp. volume ٧ density ρ Subscript:

o condition at reference (dead) state.

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Fig. 2b. Available energy contour-lines on the T-s plane for air as a perfect gas, $v_{\rm O}$ = 0.844 ${\rm m}^3/{\rm kg}$

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