ASAT

COMPREHENSIVE THERMODYNAMIC INVESTIGATION OF ALL 3-PROCESS, WORK-PRODUCING CYCLES FOR NON-REACTING PERFECT GASES.

Y. M. ABDEL-RAHIM* , A. M. NASSIB*

ABSTRACT

This paper investigates the thermodynamic performance of all 3-process, work-producing cycles for non-reacting perfect gases. Out of the 18 possible combinations of cycles composed of 3 processes, there are only 8 cycles which can produce work. The analysis studies the effect of cycle parameters on cycle performance. The performance is reflected by the first and second law efficiencies η_I and η_{II} and dimensionless heat and available energy input $q_{in} (=Q_{in}/c_v T_{min})$ and $a_{in} (=A_{in}/c_v T_{min})$ respectively. The parameters in dimensionless form are: compression ratio r (=maximum to minimum volume ratio), specific heat ratio k, minimum to ambient temperature ratio T_{min}/T_0, maximum to minimum temperature and pressure ratios T_{max}/T_{min} and P_{max}/P_{min} respectively.

The study shows that in the suggested practical range of application (r $\leq$ 30, $T_{max}/T_{min} \leq$ 10, $P_{max}/P_{min} \leq$ 100 and T_{min}/T_0=1.0) η_I and η_{II} vary between 20 to 72% and 25 to 100% respectively.

The analysis methodology presented here could be a main tool in the performance investigation and design of more complicated cycles.

* Lecturer, Dept. of Mechanical Engineering, Assiut University, Assiut, Egypt.

INTRODUCTION

Most thermodynamic studies of the performance of theoretical work-producing and work-absorbing cycles (e.g.[3 - 5]) are applied to cycles composed of 4 or more processes (e.g. Otto cycle, Diesel cycle, Brayton cycle, etc...), and little attention has been given to 3-process cycles. In these studies the first law of thermodynamics (law of conservation of energy) has been the main tool in the analysis. Recently, the second law of thermodynamics (available energy analysis) was used as an another essential tool in cycle analysis (e.g.[1 - 2]).

The present work is a comprehensive study of the performance of work-producing, 3-process closed cycles for non-reacting perfect gases (having ideal gas equation of state and constant specific heats). There are 18 different 3-process cycles which can be formed from combinations of processes: constant volume (isochoric), constant pressure (isobaric), constant temperature (isothermal) and constant entropy (isentropic). Only 8 cycles (shown in Fig.1 on p-v and T-s planes) are work-producing cycles. The present work investigates the effect of cycle parameters on cycle performance. The analysis covers the range of parameters: $r=5$ to 30, $k=1.2$ to 1.4, $T_{min}/T_O=0.8$ to 1.2 and $T_{max}/T_{min} = 2$ to 10.

In the discussion of practical applicability of these cycles, limiting r-values (suggested not to exceed 30) which result in values of $T_{max}/T_{min} \geq 10$, and/or $p_{max}/p_{min} \geq 100$ are demonstrated.

It is believed that the present work could open new horizons for using 3-process cycles in real applications. Also the analysis methodology presented in this work could be a good tool in performance analysis and design of more complicated cycles.

ANALYSIS

The governing equations used in the analysis are:

Ideal gas equation of state $pv=RT$ (1)

Constant specific heats c_p, c_v = const.   (2)

The first law of thermodynamics $q= \Delta u+w$ (3)

The second law of thermodynamics $a_{in}= \int_a^b (1- \frac{T_o}{T})dq_{in}$ (4)

The relationship $T ds = du+p dv$ (5)

The above governing equations are used to determine n_I and n_{II} as follows:

n_I is defined as:

$$\eta_I = \frac{w_{net}}{q_{in}} = 1 - \frac{q_{out}}{q_{in}} \qquad (6)$$

Introducing the governing equations (1) through into Eq.(6) for each of the 8 cycles expresses η_I in terms of r and k as shown in Table 1.

η_{II} is defined as:

$$\eta_{II} = \frac{w_{net}}{a_{in}} \qquad (7a)$$

Introducing Eqns.(4) and (6) into the last equality, it reduces to:

$$\eta_{II} = \frac{\eta_I}{\left[\int_a^b (1 - \frac{T_o}{T}) \, dq_{in}\right] / q_{in}} \qquad (7b)$$

where a-b is defined as the heat input process of the cycle. The above equation is reduced using the governing equations to the simple form $\eta_I / \eta_{II} = 1 - T_o/T_h$, where:

$$T_h = T_a = T_b \quad \text{if} \quad \text{a-b is isothermal, and}$$

$$T_h = \frac{T_b - T_a}{\ln(T_b/T_a)} \quad \text{if a-b is either isobaric or isochoric}$$

i.e. T_h=LMT, the logarithmic mean temperature of process a-b. using the last conclusion, Eq.(7b) reduces to:

$$\eta_{II} = \eta_I / \eta_c , \quad \eta_c = 1 - \frac{T_o}{T_h} \qquad (8)$$

where η_c is the efficiency of an equivalent Carnot cycle - working between temperature limits T_o and T_h. Expressions of η_{II} in terms of r,k and T_{min}/T_o for the 8 cycles are shown in Table 1. Table 2 shows η_I and η_{II} as functions of k, T_{min}/T_o and $x (=T_{max}/T_{min})$.

        DISCUSSION

The preceding analysis shows that cycle parameters r,k, T_{min}/T_o and x greatly influence η_I , η_{II} , q_{in} and a_{in}. The analysis studies the effect over ranges of cycle parameters: r=5 to 30, k=1.2 to 1.4, T_{min}/T_o=0.8 to 1.2 and x=2 to 10. Sample curves are displayed in Figs.2 through 5. Limiting values of r (given in Table 3) which result in $x \geq 10$, and $P_{max}/P_{min} \geq 100$ are shown in these figures by symbols O and ● respectively.

The above figures show that cycles 1 and 8 (beyond r=5.18) and cycles 2 and 6 (beyond r=10) are not practical for real appli-cations (T_{max} is in the order of 3000K=10 x T_{min}). The largest values of η_I and η_{II} in Figs.(2) and (3b) are those of cycles 5 and 3, and the lowest are those of cycles 8 and 6 respec-tively. Figure 3 shows that decreasing T_{min}/T_o below 1.0

results in an overall decrease in η_{II} of all cycles except
cycles 4 and 7. Cycle 4 has an overall increase in η_{II}.
Cycle 7 has values of η_{II} exceeding 100% (not shown in the
figure) which means it is impossible to operate it with
$T_{min}/T_O < 1.0$ (it violates the second law of thermodynamics).
Similar discussion holds for the effect of r on q_{in} and a_{in}
shown in Fig.4.

The effect of x (shown in Fig.5) shows that cycles 5 and 7 have
the largest η_I and η_{II} (larger by about 15% and 35% than
those of cycles 3 and 4 at x=10 respectively). Table 3 summar-
izes the discussion of the 8 cycles at k=1.4 and $T_{min}/T_O=1.0$.

CONCLUSIONS

From the preceding sections, the following conclusions are
drawn:

1-The performance of the 8 cycles is greatly affected by r,k,
T_{min}/T_O and T_{max}/T_{min}.

2-Increasing the value of k (from 1.3 to 1.4) and/or T_{min}/T_O
(above 1.0) results in an asymptotic increase in η_I and η_{II}
of the 8 cycles.

3-It is impossible to operate cycle 7 with T_{min}/T_O less than
1.0, since η_{II} is larger than 100% which violates the second
law of thermodynamics.

4-Cycles efficiencies in the suggested practical range of
application (r $\leq$ 30, $T_{max}/T_{min} \leq$ 10, $p_{max}/p_{min} \leq$ 100 and
$T_{min}/T_O \simeq$ 1.0) vary between 20 to 72% for η_I and 25 to 100%
for η_{II}. The highest values are those of cycle 5 and the
lowest are those of cycle 8.

REFERENCES

1.Abdel-Rahim, Y.M. and Clark, J.S., "Comprehensive Analysis of the
Air Standard Otto Cycle Using the Second Law of Thermodynamics",
International Meeting for Energy Sources Technology, New Orleans,
Louisiana, U.S.A.(1984).

2.Gaggioli, R.A., "The Concept of Available Energy", Chem. Engg.
Science, Vol.16, (1961).

3.Keenan, J.H., "Thermodynamics", John Wiley and Sons, (1941).

4.Obert, E.F., "Concepts of Thermodynamics", McGraw-Hill,(1960).

5.Sonntag, R.E. and Van Wylen, G. "Introduction to Thermodynamics",
John Wiley & Sons, (1982).

6.William, V.G."Thermodynamic Aspects of Energy Conservation",Inter.
Journal of Energy, Vol.5 pp 783-792, Pergamon Press (1980).

7.Wilson, S.S. and Radwan, M.S."Appropriate Thermodynamics for Heat
Engine Analysis and Design", IMech E Vol.15 No.1, (1977).

Table 1. Expressions of η_I and η_{II} in terms of r,k and T_{min}/T_o shown in Figs.2-5.

cycle	η_I	η_I/η_{II}
1	$1 - \dfrac{k(r-1)}{(r^k-1)}$	$1 - \dfrac{k \ln(r)}{(T_{min}/T_o)(r^k-1)}$
2	$1 - \dfrac{k(r-1)}{r-1 + r(k-1)\ln(r)}$	$1 - \dfrac{k \ln(r)}{(T_{min}/T_o)\left[r-1 + (k-1)r\ln(r)\right]}$
3	$1 - \dfrac{1 - r^{1-k}}{(k-1)\ln(r)}$	$1 - \dfrac{1}{(T_{min}/T_o)r^{k-1}}$
4	$1 - \dfrac{k\left[1-(r)^{1/k-1}\right]}{(k-1)\ln(r)}$	$1 - \dfrac{1}{(T_{min}/T_o)\, r^{(k-1)/k}}$
5	$1 - \dfrac{(k-1)\ln(r)}{r^{k-1} - 1}$	$1 - \dfrac{(k-1)\ln(r)}{(T_{min}/T_o)(r^{k-1} - 1)}$
6	$1 - \dfrac{r-1 + (k-1)\ln(r)}{k(r-1)}$	$1 - \dfrac{\ln(r)}{(T_{min}/T_o)(r-1)}$
7	$1 - \dfrac{(k-1)\ln(r)}{k(r^{(k-1)/k} - 1)}$	$1 - \dfrac{(k-1)\ln(r)}{k(T_{min}/T_o)(r^{(k-1)/k} - 1)}$
8	$1 - \dfrac{(r^k - 1)}{k(r^k - r^{k-1})}$	$1 - \dfrac{\ln(r)}{(T_{min}/T_o)(r^k - r^{k-1})}$

Table 2. Expressions of η_I and η_{II} in terms of x, k and T_{min}/T_o shown in Figs. 2-5.

cycle	$x = T_{max}/T_{min}$	η_I	η_I/η_{II}
1	r^k	$1- \dfrac{k\,(x^{1/k} - 1)}{x - 1}$	$1- \dfrac{\ln(x)}{(T_{min}/T_o)\,(x - 1)}$
2	r	$1- \dfrac{k(x - 1)}{x-1 + x(k-1)\,\ln(x)}$	$1- \dfrac{k\,\ln(x)}{(T_{min}/T_o)\left[x-1+x(k-1)\ln(x)\right]}$
3	r^{k-1}	$1- \dfrac{x - 1}{x\,\ln(x)}$	$1- \dfrac{1}{(T_{min}/T_o)\,x}$
4	$r^{(k-1)/k}$	$1- \dfrac{x - 1}{x\,\ln(x)}$	$1- \dfrac{1}{(T_{min}/T_o)\,x}$
5	r^{k-1}	$1 - \dfrac{\ln(x)}{x-1}$	$1- \dfrac{\ln(x)}{(T_{min}/T_o)\,(k - 1)}$
6	r	$1- \dfrac{x-1 + (k-1)\,\ln(x)}{k\,(x - 1)}$	$1- \dfrac{\ln(x)}{(T_{min}/T_o)\,(x - 1)}$
7	$r^{(k-1)/k}$	$1- \dfrac{\ln x}{(x - 1)}$	$1- \dfrac{\ln(x)}{(T_{min}/T_o)\,(x - 1)}$
8	r^k	$1- \dfrac{(x - 1)}{k(x - x^{(k-1)/k})}$	$1- \dfrac{\ln(x)}{(T_{min}/T_o)\,k(x-x^{(k-1)/k})}$

Table 3. Summary of cycle performance at k=1.4 and $T_{min}/T_o=1.0$.

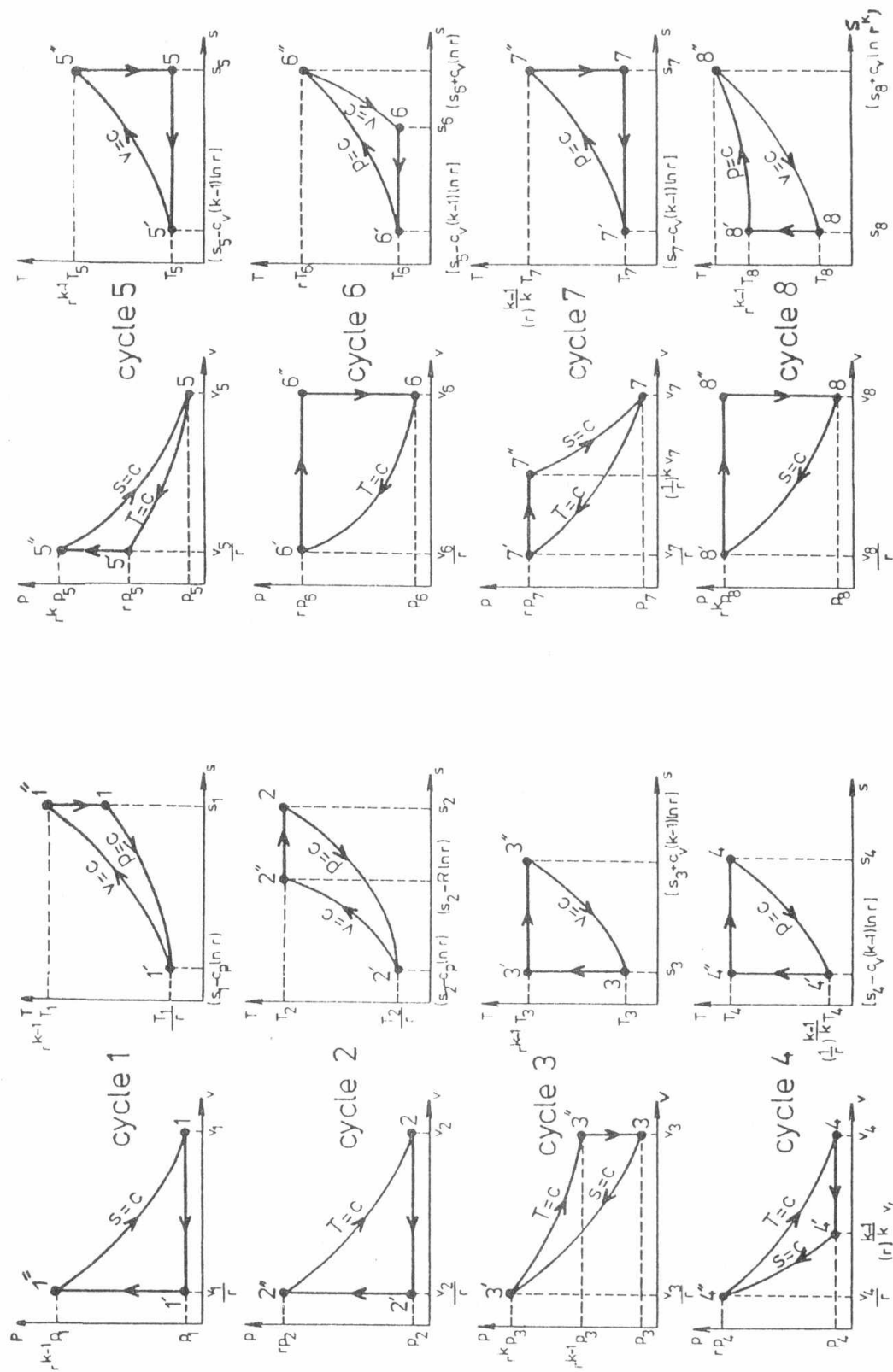| cycle	critical values of r		limiting r-value*	values at limiting r			
	$\dfrac{P_{max}}{P_{min}}=100$	$\dfrac{T_{max}}{T_{min}}=10$		$\dfrac{P_{max}}{P_{min}}$	$\dfrac{T_{max}}{T_{min}}$	$\eta_I\%$	$\eta_{II}\%$
1	26.80	5.18	5.18	10	10	37	47
2	100	10	10	10	10	30	38
3	26.80	316.20	26.80	100	3.73	51	85
4	100	3162.30	30*	30	2.64	36	58
5	26.80	316.20	26.80	100	3.73	72	100
6	100	10	10	10	10	21	27
7	100	3162.30	30*	30	2.64	40	100
8	26.80	5.18	5.18	10	10	20	25

* Limiting value of r not to exceed 30.

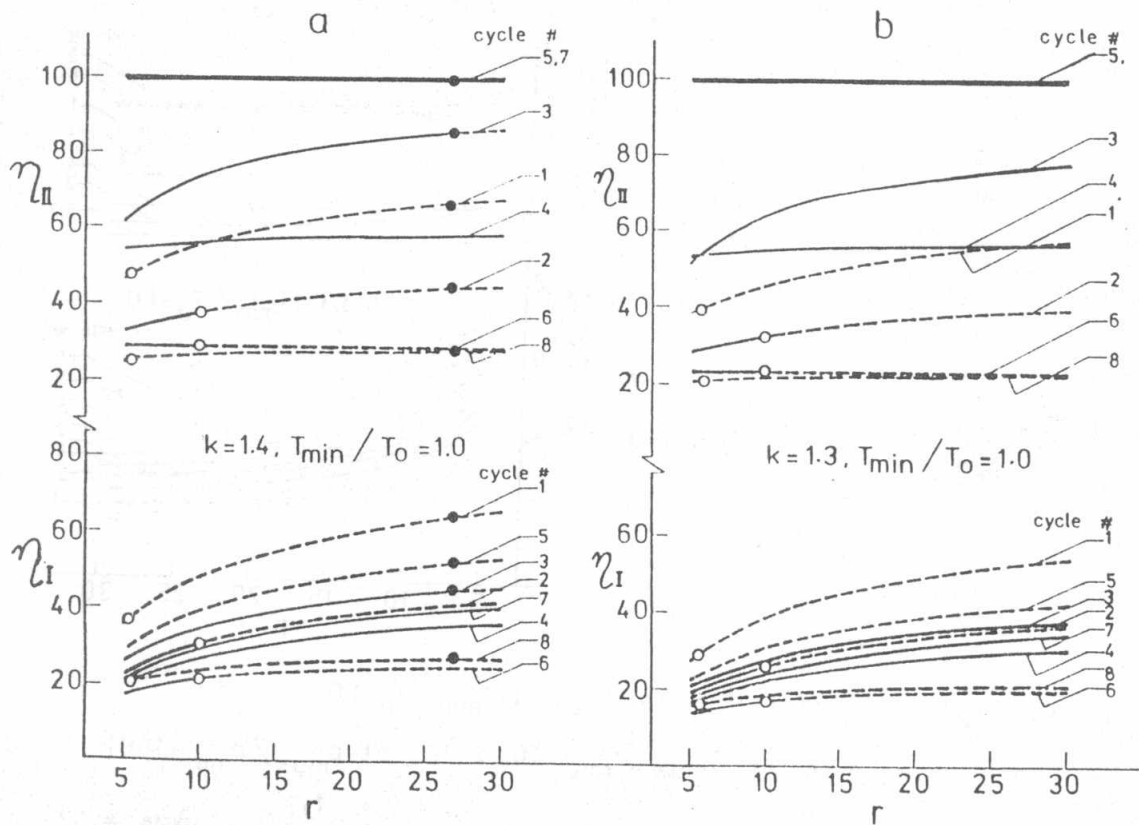Fig. 1. Representation of the eight 3_process cycles on the p_v and T_s planes.

Fig. 2. η_I and η_{II} vs. r at k =1.4 ,1.3 and $T_{min}/T_o = 1.0$
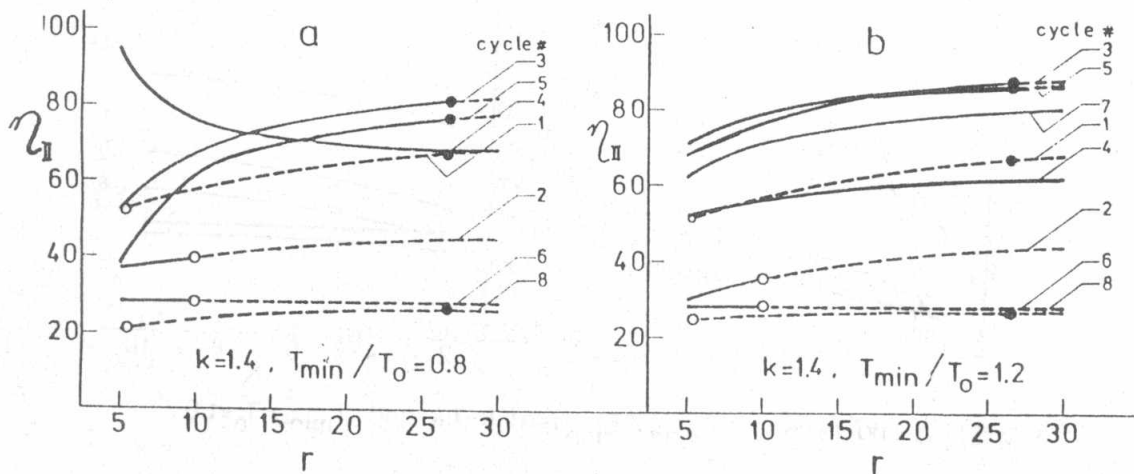(o r_{cr} at $T_{max}/T_{min}=10$, • r_{cr} at $p_{max}/p_{min}=100$)

Fig. 3. η_{II} vs. r at $T_{min}/T_o = 0.8$, 1.2 and k = 1.4 , 1.3
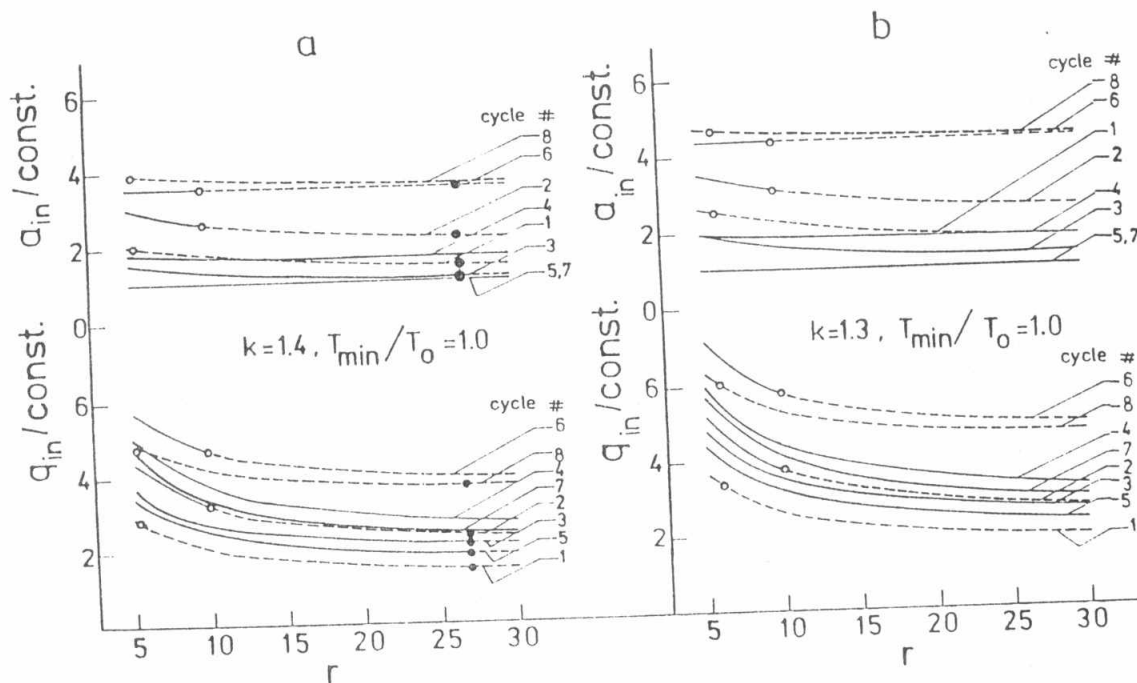(o r_{cr} at $T_{max}/T_{min}=10$, • r_{cr} at $p_{max}/p_{min}=100$)

Fig. 4. q_{in} and a_{in} vs. r at $k = 1.4$, 1.3 and $T_{min}/T_0 = 1.0$

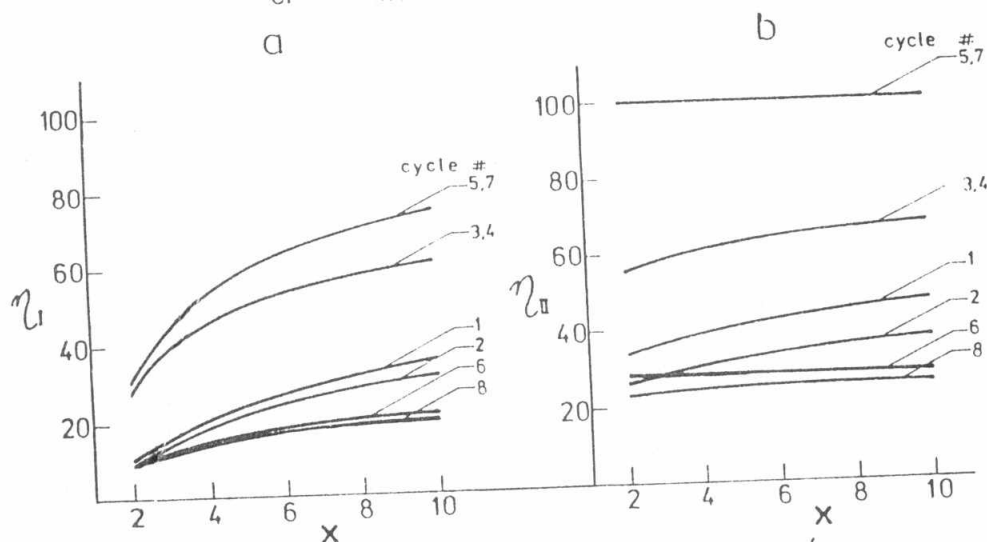(o r_{cr} at $T_{max}/T_{min} = 10$, • r_{cr} at $p_{max}/p_{min} = 100$)

Fig. 5. η_I and η_{II} vs. x ($= T_{max}/T_{min}$) at $k = 1.4$ and $T_{min}/T_0 = 1.0$.