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MILITARY TECHNICAL COLLEGE

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A THERMODYNAMICAL ANALYSIS OF AN A/C PRESSURIZED CABIN

By

SALAMA MOHAMED*, REFAT EL-BANNA**

ABSTRACT

A thermodynamical analysis of the dynamic characteristics of an Air-Craft pressurized cabin, as a control volume, was carried out to establish an approach to the design of a microprocessor to control the atmospheric conditions inside it. The main controlled parameters are specified, the related differential equations are derived and possible solutions were shown. The obtained theoretical results were compared with the results of a model measurements.

I. INTRODUCTION

The comfort and normal functioning of an air-craft crew during altitude flight depends on the conditions inside the cabin. These conditions, sould satisfty certain physiological and medical requirements, and should be automatically maintained throughout the flight.

The main parameters which should be controlled are the cabin air pressure P_k, the cabin air temperature T_k, and the relative humidity of air inside the cabin. These parameters are effected by the atmospheric pressure and

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* Lecturer of Mechanical Engineering, MTC, Cairo, Egypt

**Lecturer of Electrical Engineering, MTC, Cairo, Egypt

temperature, and controlled by the inlet and outlet flow conditions of the air flowing through the cabin regulators.

In previous investigations [1], [2]; the analysis was concerning the change of one parameter, either P_k or T_k, while keeping other parameters unchanged. In this article, we perform analysis of the system characteristics for simultaneous changes in pressure and temperature of the cabin air.

2. Governing Equations :

A view of an A/C cabin, as a control volume, is represented in fig.1.

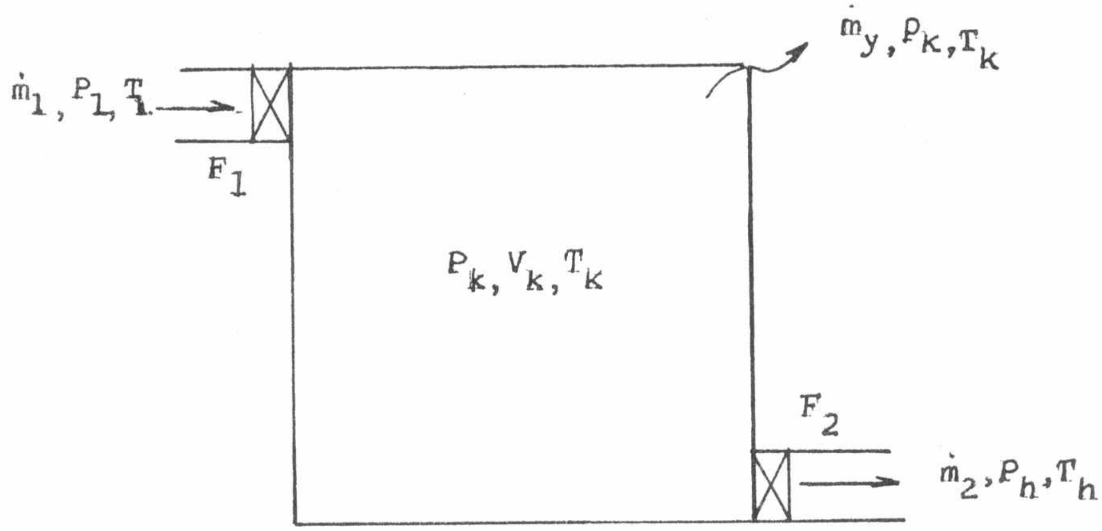

Fig (1)

Diagram of a pressurized cabin with controlled air parameters

The cabin has a total volume V_k and occupied by air of mass m_k at the pressure P_k and absolute temperature T_k. Air is allowed to flow to the cabin through a duct of an area F_1 ; from a pressurizing source, at a flow rate $\dot{m}_1$. Such pressurized air has controlled values of pressure and temperature denoted as P_1 and T_1 respectively. Air is flowing out from the cabin through an outlet valve of area F_2 at the flow rate $\dot{m}_2$, pressure P_h and temperature T_h. Air leakage from the cabin, due to incomplete sealing; has the flow rate $\dot{m}_y$, leakage pressure P_k and temperature T_k. In the presented analysis, we consider the air as an ideal gas and neglect the change in kinetic and potential energy of the flowing air.

By applying the first law of thermodynamics to the system, we may write :-

$$\dot{m}_1 \, i_1 = \dot{m}_2 \, i_2 + \dot{m}_y \, i_y + \dot{Q} + \dot{U} \qquad (1)$$

$\dot{Q}$: rate of heat losses from the cabin surface to the surrounding due to radiation and convection ;

$\dot{U}$: rate of change of stored energy in the cabin,

i_1: enthalpy of air flowing to the cabin$=C_p T_1$ J/kg

i_2: enthalpy of air flowing from the cabin$=C_p T_h$ J/kg

i_y: enthalpy of air leakage $= C_p T_k$ J/kg

The amount of heat loss is given by :

$$\dot{Q} = K_r . F_c \; . (T_k - T_h) \qquad W \qquad (2)$$

Where ;

F_c: total surface area of the cabin walls in m^2;

K_r: overall heat transfer coefficient determined from

$$K_r = \frac{1}{\frac{1}{h_c} + \frac{\delta}{k} + \frac{1}{h_o}} \qquad W/m^2.C \qquad (3)$$

Where

h_c: Coefficient of heat transfer, for heat exchange between the cabin air and the internal surface of the cabin in $W/m^2 °C$

δ : thickness of cabin shell in m;

k : Thermal conductivity of cabin walls in W/m°C

h_o: Coefficient of heat exchange between the external surface of the cabin walls and the ambient air $W/m^2 °C$

The coefficients h_c and h_o are calculated from :

$$h = \frac{k}{L} .C.(R_e)^a. (P_r)^b \qquad (4)$$

Where ;

Re,Pr : Reynolds and Prandtl numbers ;

L : reference dimensions of heat exchange by the cabin;

C,a,b : are constants found imperically [3];

The change of stored energy of the system per unit time $\dot{U}$ is given by

$$\dot{U} = \frac{d}{dt} (C_v. m_k. T_k) ; \qquad (5)$$

Where t represents the time.

Applying the equation of state of ideal gas on the air occupying the cabin volume, we get

$$m_k R T_k = P_k V_k \qquad (6)$$

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Where R is the gas constant for air (297 J/kg °K)

From the ideal gas relations ;

$$C_v = R/(\gamma-1) \qquad (7)$$

Where γ is the specific heats ratio.

Considering that the cabin volume V_k is constant and subtituting (6) and (7) in eqn. (5), we get :

$$U = \frac{V_k}{\gamma-1} \frac{d P_k}{dt} \qquad (8)$$

Denoting $\dot{m}_1 = G_1$, $\dot{m}_2 = G_2$, $\dot{m}_y = G_y$, and substituting eqns (2) and (8) into(1), we get

$$C_p G_1 T_1 = C_p (G_2+G_y).T_k + K_r F(T_k-T_h) + \frac{V_k}{\gamma-1} \frac{d P_k}{dt} \qquad (9)$$

Applying the principle of conservation of mass on the cabin, we get :

$$\frac{dm_k}{dt} = G_1 - (G_2+G_y) \qquad (10)$$

Substituting eqn (10) into equation (6) after differentiating its both sides, we get

$$T_k(G_1-G_2-G_y) + m_k \frac{d T_k}{dt} = \frac{V_k}{R} \frac{dP_k}{dt} \qquad (11)$$

Considering that the system is initially at steady state (no variation with time) , if the initial state parameters are denoted by the letter (0); so ; from eqn (10) we get

$$\frac{dm_k}{dt} = G_{10} - (G_{20} + G_{yo}) = 0$$

$$\text{i.e. } G_{10} = G_{20} + G_{yo} \qquad (12)$$

and from eqn (9), we get

$$T_{10} = T_{ko} + \frac{K_r F}{C_p G_{10}} (T_{ko} - T_{ho}) \qquad (13)$$

If the state of the system is disturbed from the initial state, so, each parameter gets a new value defined by a relation of the from :

$$y = y_o + \Delta y \qquad (14)$$

Where y stands for $P_1, P_h, P_k, T_1, T_h, T_k, G_1, G_2, G_y$, and m_k;y: represents any parameter value after its initial value y_o is disturbed by the difference Δy. By introducing such difinitions in equations (9) and (10), we get

$$G_{10}T_1 + T_{10}\Delta G_1 = G_{10}T_k + T_{ko}\Delta G_2 + \frac{K_r.F}{C_p}(T_k - T_h) + \frac{V_k}{C_p(\gamma - 1)}\frac{dP_k}{dt} \qquad (15)$$

$$\frac{V_k}{R}\frac{dP_k}{dt} = T_{ko}(G_1 - G_2) + m_{k_o}\frac{dT_k}{dt} \qquad (16)$$

The amount of air from the pressurizing source G_1 depends on the state of the inflowing air (P_1 and T_1) and on the pressure of the cabin air P_k. Hence, for variable flow area F_1 we may express G_1 as follows

$$G_1 = f_1(P_1, T_1, P_k, F_1) \qquad (17)$$

Similary the quantity allowed to flow out of the cabin depends on the state of the cabin air (P_k, T_k) and the atmospheric pressure P_h, so for variable flow area F_2, i.e.

$$G_2 = f_2(P_k, T_k, P_h, F_2) \qquad (18)$$

Expressing the finite differences $\Delta G_1, \Delta G_2$ in terms of the dependent variables of eqns (17) and (18) we get:

$$\Delta G_1 = \left(\frac{\partial G_1}{\partial P_1}\right)_o \Delta P_1 + \left(\frac{\partial G_1}{\partial P_k}\right)_o \Delta P_k + \left(\frac{\partial G_1}{\partial T_1}\right)_o \Delta T_1 + \left(\frac{\partial G_1}{\partial F_1}\right)\Delta F_1 \qquad (19)$$

$$\Delta G_2=\left(\frac{\partial G_2}{\partial P_k}\right)_o \Delta P_k+\left(\frac{\partial G_2}{\partial P_h}\right)_o \Delta P_h+\left(\frac{\partial G_2}{\partial T_k}\right)_o \Delta T_k+\left(\frac{\partial G_2}{\partial F_2}\right)\Delta F_2 \quad (20)$$

Substituting (19) and (20) into (15) and (16) and rearranging, we get :

$$\frac{d\phi_k}{dt} = M_1 \phi_k+M_2\theta_k +F(t) \quad (21)$$

and

$$\frac{d\theta_k}{dt} = M_3\theta_k+M_4\phi_k+R(t) \quad (22)$$

Where $\phi_k= \frac{\Delta P_k}{P_{k_o}}$; and $\theta_k=\frac{\Delta T_k}{T_{k_o}}$

and $F(t) =\sum_{i=1}^{6} A_i X_i$

$$R(t) = \sum_{i=1}^{6} B_i X_i \quad (23)$$

So ϕ_k represents the percentage deviation of the cabin air pressure from an initial value P_{k_o} and θ_k represents the percentage deviation in cabin air temperature from an initial temperature T_{k_o}. Sources of such deviations are represented by the vector parameters X_i for i=1,2,.. 6., where

$$X_1= \frac{\Delta P_1}{P_{1_o}}, \quad X_2=\frac{\Delta P_h}{P_{h_o}}, \quad X_3=\frac{\Delta T_1}{T_{1_o}}$$

$$X_4= \frac{\Delta T_H}{T_{H_o}}, \quad X_5=\frac{\Delta F_1}{F_{1_o}}, \quad X_6=\frac{\Delta F_2}{F_{2_o}}$$

The coefficients M_j (j=1,2,3,4) and A_i, B_i for i=1,2,..6., can be easily found from eqns (19) and (20).

3. General Solutions

The last three equations derived in the previous section, can be represented in the form [5]:

$$|\dot{\psi}| = |M| \cdot |\psi| + |L| \cdot |X_i| \qquad (24)$$

Where

$$|\psi| = \begin{vmatrix} \frac{d\emptyset_k}{dt} \\ \frac{d\theta_k}{dt} \end{vmatrix} ;$$

$$|M| = \begin{vmatrix} M_1 & M_2 \\ M_3 & M_4 \end{vmatrix}$$

$$|L| = \begin{vmatrix} A_1 & A_2 & A_3 & A_4 & A_5 & A_6 \\ B_1 & B_2 & B_3 & B_4 & B_5 & B_6 \end{vmatrix} \text{, and}$$

$$|X_i| = \begin{vmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \\ X_6 \end{vmatrix}$$

The general solution of equation (24) when the system is subjected to input disturbance $X_i(t)$, is given by [5] :

$$|\psi(t)| = |Y(t)| |\psi(o)| + \int_o^t |Y(t-\tau)| |L| |X_i(\tau)| \, d\tau \quad (25)$$

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Where $\left|\psi(o)\right| = \begin{vmatrix} \emptyset\ (o) \\ \theta\ (o) \end{vmatrix}$;

& $\left|Y(t)\right| = \begin{vmatrix} Y_{11}(t) & Y_{12}(t) \\ Y_{21}(t) & Y_{22}(t) \end{vmatrix}$; (26)

The initial values $\emptyset_k(o)$ and $\theta_k(o)$ are zeros at time t=o, since at this time there is no input disturbance, in other words, considering that $X_i(o) = o$, we reduce the general solution in equation (25) to the form

$$\psi(t) = \int_o^t \left| Y\ (t-\tau) \right| \left| L \right| \left| X_i\ (\tau) \right| d\tau \qquad (27)$$

The solution of the matrix function Y(t) is found to depend on the characteristic roots of an auxiliary equation of the form[5]:

$$\begin{vmatrix} \lambda - M_1 & M_2 \\ M_3 & \lambda - M_4 \end{vmatrix} = o$$

or the form :

$$(\lambda - M_1)(\lambda - M_4) - M_2\ M_3 = o$$

finally :

$$\lambda^2 - (M_1 + M_4) + (M_1 M_4 - M_2 M_3) = o \qquad (28)$$

The two roots of equation (28) depends on the value of the coefficients M_1, M_2, M_3, and M_4. So, these two roots λ_1 and λ_2 of this equation can be real ; imaginary, or equal. The case to be considered for our analysis will assume real and different roots of equation (28); according to the practical values of coefficients M_1, M_2, M_3, and M_4 in most of the studied cabins. Laplace transformation was used for this case to obtain the system solution in the form of a matrix function Y(t) , which is found as :

$$\left|Y(t)\right| = \begin{vmatrix} C_{11}\, e^{\lambda_1 t} - b_{11}\, e^{\lambda_2 t} & C_{12}(e^{\lambda_1 t} - e^{\lambda_2 t}) \\ C_{21}(e^{\lambda_1 t} - e^{\lambda_2 t}) & C_{22} e^{\lambda_1 t} \; b_{22} e^{\lambda_2 t} \end{vmatrix} \quad (29)$$

Where :

$$C_{11} = \frac{\lambda_1 - M_4}{\lambda_1 - \lambda_2}, \quad C_{12} = \frac{M_2}{\lambda_2 - \lambda_1}$$

$$C_{21} = \frac{M_3}{\lambda_2 - \lambda_1}; \quad C_{22} = \frac{\lambda_1 - M_1}{\lambda_1 - \lambda_2} \quad (30)$$

$$b_{11} = \frac{\lambda_2 - M_4}{\lambda_1 - \lambda_2}; \quad b_{22} = \frac{\lambda_2 - M_1}{\lambda_1 - \lambda_2}$$

Substituting equation (29) into equation (28); we get the final solution by numerical integration.

So finally the deviation of the cabin air pressure $\emptyset_k(t)$ and temperature $\Theta_k(t)$ can be found as function of the input disturbances $X_i(t)$ or as function of time. This integration is done by using the numerical integration-computational techniques and introducting an IBM, PC computer.

4. A Case Study

As explained in section 3, the deviation in air pressure $P_k(t)$ and temperature $T_k(t)$ of an air-craft cabin are function of the input disturbances. A cabin model had been fabricated and the associated test rig had been elaborated where the dynamic charachteristics of the cabin model in presence to three input disturbances were measured. These disturbances are namely

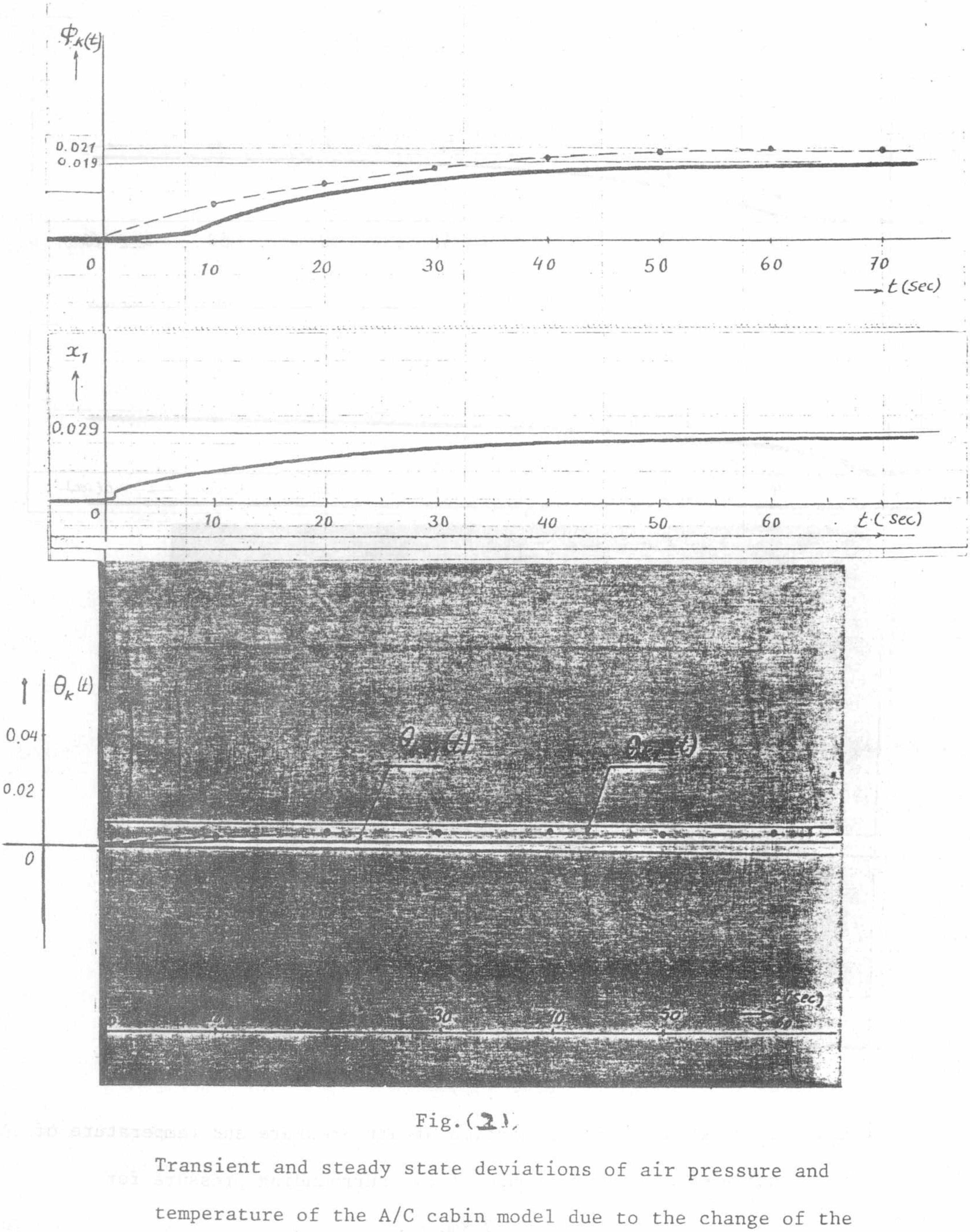

Fig.(2).

Transient and steady state deviations of air pressure and temperature of the A/C cabin model due to the change of the inlet pressure for H=5Km.

——— measured

– – – – computed

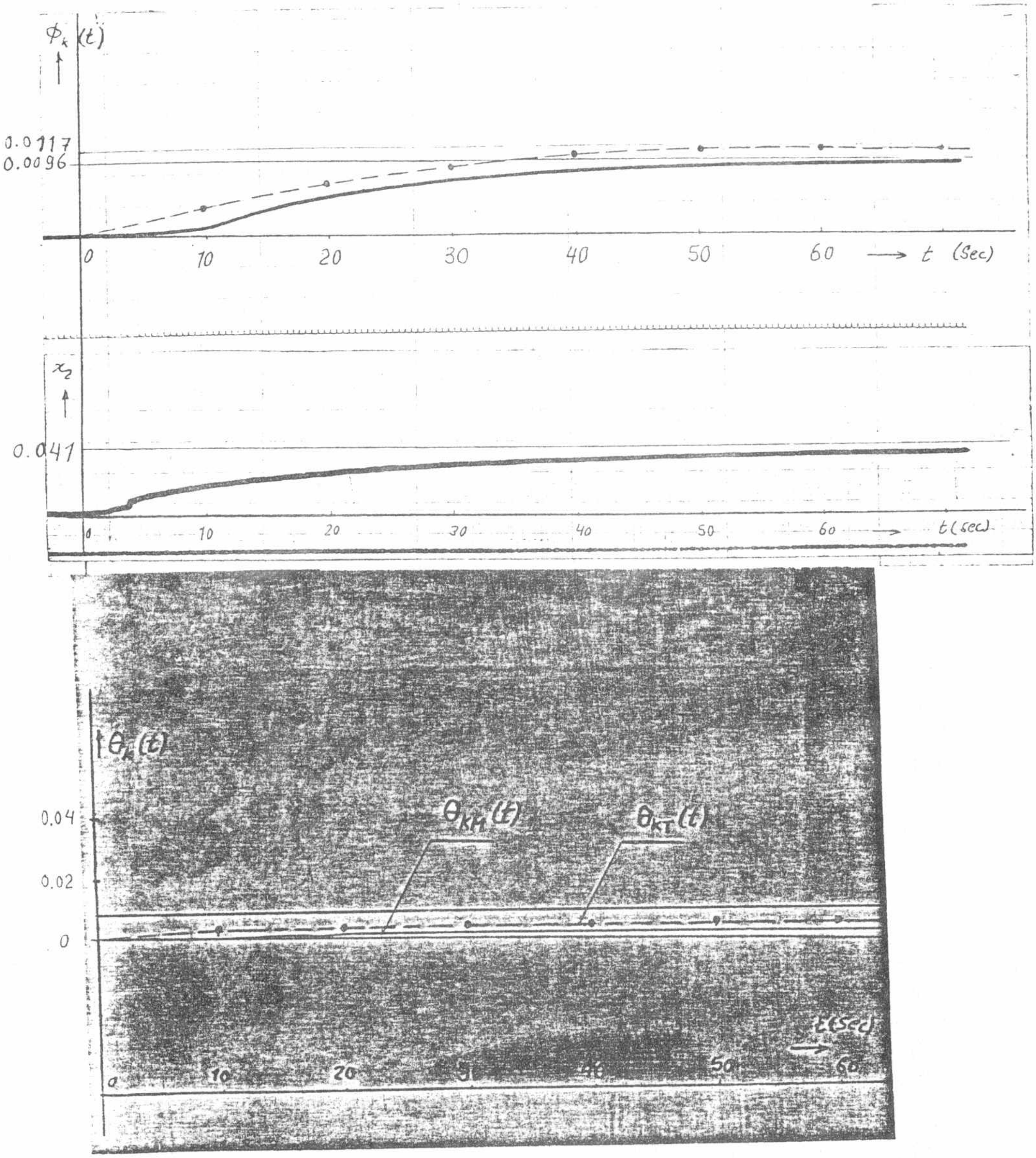

Fig. (3)

Transient and steady state deviation of air pressure and temperature of the A/C cabin model due to change of the surrounding pressure for

H = 7 Km.

———— measured

– – – – – computed

$P_l(t)$, $P_h(t)$, and $T_l(t)$ with values corresponding to the changes in the associated input conditions at flying altitudes of 3,4,5,9,14 km. These input disturbances were introduced in the differential equation and the deviation in cabin conditions were found by using of simple computational techniques. The computational and measurment results for the transient and steady state deviation in the cabin air pressure and temperature were compared as seen in figs 2, and 3 for selected cases.

These represented results are obtained for simulated altitudes of 5 and 7 km and with the following initial parameters :

H (km)	P_{10} (bar)	P_{ko} (bar)	P_{ho} (bar)	G_{10} (l/min)	G_{20} (l/min)	t_{10} (°C)	t_{ko} (°C)	t_{ho} (°C)
5	1.02	0.83	0.63	23.0	18.0	36	27	26
7	0.91	0.61	0.31	33.0	29.0	39	26	25

In figs (2) and (3) are also represented the transient and steady deviations of the cabin air pressure and temperature due to changes in the inlet pressure. The difference between measurement and computional results as seen, is in order of 3 - 5%, which have no vital influence on the cabin enviromental conditions and which is acceptable in such field of application.

Conclusion

The agreement found between the measurement and computational results leads to the conclusion that equs 22-26 are accepted as a base for design of a microprocessor. Such microprocessor will be capable of controlling the enviromental conditions inside the pressurized cabins through the change of another six input parameters.

Nomenclature

A,B,C : different

C_p : Specific heat at constant pressure

C_r : Specific heat at constant volume

F : Cross-Sectional area

G : mass flow rate

h : heat transfer convection coefficient

i : enthalpy

K : thermal conductivity

K_r : Overall heat transfer coefficient

m : mass flow rate

M : I matrix coefficients

P : Pressure

P : Prandtl Number

Q : heat flow rate

R : Gas constant

Re: Reynold's Number

T : temperature

t : time

U : internal energy

X_i : input disturbance

Θ : Percentage deviation in cabin pressure

Ø : Percentage deviation in cabin temperature

: root of the system-characteristic equation

: time

: thickness of the cabin walls

Suffixes :

1 : inlet cabin conditions

2 : outlet cabin conditions

h : outside-atmospheric conditions

i : for vector parameters (1-6)

K : inside cabin conditions

O : initial conditions

y : air leakage conditions.

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