MILITARY TECHNICAL COLLEGE

CAIRO - EGYPT

1 APPLICATION OF VARIATIONAL TECHNIQUE FOR SOLUTION OF

2 THIN-WALLED MULTICELL STRUCTURE

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4 ABSTRACT

The variational technique is applied to calculate the stresses for a thin-walled structure composed of a closed cell connected to a four-flanges open cell. The structure has one built-in end, the other end is free, loaded by torque moment. a rectangular model is constructed for application of the obtained general results found by variational technique. Experimental measurements are carried out on it using semi-conductor strain gauges.

The theoretical and experimental results of normal stress distribution in flanges are in good agreement.

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INTRODUCTION

Thin-walled structures are widely used nowadays in machinery as well as in aviation due to their relatively light weight. A continuous effort is paid to decrease their weight. This led to development of methods achieving more accurate results in stress and deformation distributions, carrying capacity and stability. The torsion rigidity of thin-walled structure is always the critical one from point of view of transfering loads, specially when it affects the performance, like in wings of aircraft. Mostly wing structure is composed of several cells connected to four flanges thin-walled structure. In many cases, there is a necessity to interrupt the structure by a cut-out for assembly or function reasons. Then from the view point of stress analysis the structure can be divided into two types of multicell, connected to an open or closed fourflanges thin-walled cell. If such structure was under torsion and restrained to warp, a system of normal & shear stress were produced due to the so called secondary bending. The solution of such type of structure using restrained warping theory (which assume that normal stresses are carried only by flanges and shear stresses by thin-walls) was given for idealized shapes [3] . Recently the variational technique was used for formulation of the governing differential equation [2]

VARIATIONAL TECHNIQUE

The total strain energy in structure under torsion can be written as

$$U = \int_{o}^{L} F(z,B,B',M_{t1},...)dz \tag{1}$$

Since the system must by in equilibrium, it must satisfy Euler's equations of the variational calculus to obtain minimum strain energy. The basis of this method is the determination of the functional F which is the strain energy per unit length of the structure. For a structure composed of multicell connected to four flanges thin-walled open cell Fig. 1, the functional F is in the form:

$$F = F(z,B,B',M_{t1},...M_{tn}, M_{t\bar{1}},...,M_{t\bar{n}}) \tag{2}$$

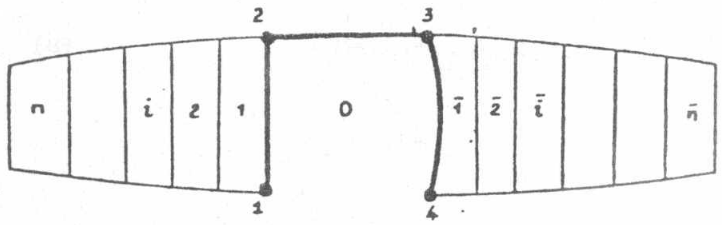

Fig.1, Multicell wing structure

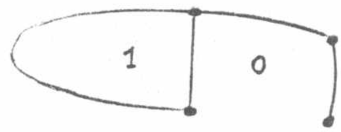

Fig. 2.

Therefore Euler's equations are

$$\frac{\partial F}{\partial B} - \frac{d}{dz}\left(\frac{\partial F}{\partial B'}\right) = 0, \quad \frac{\partial F}{\partial M_{ti}} = 0, \quad \frac{\partial F}{\partial M_{t\bar{i}}} = 0 \tag{3}$$

The strain energy per unit length of the structure is formed by the strain energy of its parts.

$$\text{i.e. } F = \sum_{i=1}^{n} F_{is} + \sum_{\bar{i}=1}^{\bar{n}} F_{\bar{i}s} + F_{so} + F_{fo} \tag{4}$$

Where : F_s is strain energy in sheets

F_f is strain energy in flanges

in general

$$F_{is} = \left(q_t^2 \frac{s}{2Gt}\right)_i , \quad q_t = \frac{M_t}{2U} \tag{5}$$

$$F_f = \frac{1}{2}\frac{B^2}{(EI)_f} = \frac{1}{2}\frac{B^2}{4U}\left[\frac{\alpha_1^2}{EA_1} + \frac{(\alpha_1-\alpha_2)^2}{EA_2} + \frac{(1-\alpha_2)^2}{EA_3} + \frac{1}{EA_4}\right] \tag{6}$$

Where: α_1, α_2 are constants depend on cross-section configuration.

Also the following condition of equilibrium must be fulfilled.

$$M_t = B' + \sum_{i=1}^{n} M_{ui} + \sum_{\bar{i}=\bar{1}}^{\bar{n}} M_{u\bar{i}} \tag{7}$$

Note: St. Venant shear stresses are neglected (it is less than 2%). Under these basis the governing differential equations of bimoment B and moment distribution M_{u1}, M_{u2},..., are derived. They have the general form as follows.

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$$B''a_{11} - Ba_{12} + M'_{t2}\, a_{13} + M'_{t3} a_{14} \ldots + M'_{t\bar{1}}\, a_{1\bar{1}} + M'_{t\bar{2}} a_{1\bar{2}} + \ldots = m_t n_1$$

$$B'a_{21} + M_{t2} a_{22} + M_{t3} a_{23} + \ldots\ldots + M_{t\bar{1}} a_{2\bar{1}} + M_{t\bar{2}} a_{2\bar{2}} + \ldots = M_t n_2 \qquad (8)$$

$$B'a_{31} + \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad + \ldots = M_t n_3$$

$$\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot$$

Where all the coefficients $a_{11}, a_{12}, \ldots a_{ij} \ldots n_i$ can be determined from the structure cross-section characteristics.

APPLICATION OF THE GENERAL FORMULAE

The system of differential equations (8) are applied to a single cell connected with four flanges open cell structure Fig.2, by putting:

$$M'_{ti} = 0 \quad \text{for } i=2..n \;, \quad M'_{t\bar{i}} = 0 \quad \text{for } \bar{i}=\bar{1}..\bar{n} \qquad (9)$$

Thus the first differential equation only will be needed and it will be of the form:

$$B'' - \frac{1}{a^2} B = n m_t \qquad (10)$$

where:

$$\frac{1}{a^2} = \frac{1}{(EI)_f} \; \frac{1}{\frac{1}{(GI_t)_1} + \frac{2}{(GI_t)_{10}} + \frac{1}{(GI_t)_{\bar{u}10}} + \frac{1}{(GI_t)_{\bar{u}}} + \frac{1}{(GI_t)_{\bar{u}\bar{1}0}}} \qquad (10.a)$$

$$n = \frac{\frac{1}{(GI_t)_1} + \frac{1}{(GI_t)_{10}}}{\frac{1}{(GI_t)_1} + \frac{2}{(GI_t)_{10}} + \frac{1}{(GI_t)_{\bar{u}10}} + \frac{1}{(GI_t)_{\bar{u}}} + \frac{1}{(GI_t)_{\bar{u}\bar{1}0}}} \qquad (10.b)$$

Where: $(EI)_f$: torsional rigidity of flanges see equation (6)

$(GI_t)_i$: torsional rigidity of the cell i for its moment M_{ti}

$(GI_t)_{i,j}$: torsional rigidity of common walls between cells i,j

$(GI_t)_{\bar{u}}$: torsional rigidity of cell (0) for the moment B'

$(GI_t)_{\bar{u}i,j}$: torsional rigidity of common walls for the moment B'

The solution of this equation will be of the form

$$B = C_1 \sinh Z/a + C_2 \cosh Z/a + B_p \qquad (11)$$

The problem is solved for a beam having one built-in end and the other end is free loaded by a constant torque moment M_t. Since $m_t = dM_t/dz$ then m_t will be zero and consequently the particular integral $B_p=0$. The boundary condition can be expressed as :

At free end(for $z=l$) , zero normal stress

$$B(l)=0 \quad , \quad \text{then} \quad C_2=C_1 \tanh l/a \tag{12.a}$$

At built-in end (for z=0) , zero warping

$$\frac{\partial U}{\partial B} = 0 \ , \quad \text{or} \quad \frac{\partial U}{\partial B}\left(\frac{\partial U}{\partial z} dz\right) = 0$$

$$\text{i.e.} \quad \left.\frac{\partial F}{\partial B'}\right|_{z=0} = 0 \ , \quad \text{then} \quad C_1 = M_t \, na \tag{12.b}$$

Hence

$$B = M_t \, n \, a \left[\sinh z/a - \tanh z/a \ \cosh z/a\right] \tag{13}$$

APPLICATION TO RECTANGULAR MODEL

To verify this method a rectangular cross-section model Fig.3, is chosen to compare theoretical calculations with the experimental results. For the given configuration we have :

$$\alpha_1 = 1 \ , \quad \alpha_2 = 0 \ , \quad \bar{U} = hb/2 \tag{14.a}$$

$$\frac{1}{(EI)_f} = \frac{4}{h^2 b^2 EA} \tag{14.b}$$

$$\frac{1}{(GI_t)_1} = \frac{\sum_{i=1}^{4} \frac{s_i}{G_i t_i}}{2U^2} = \frac{h+b}{4h^2 \, b^2 \, Gt} \tag{14.c}$$

$$\frac{1}{(GI_t)_{10}} = \frac{1}{4U \, \bar{U}} \ \frac{s_{10}}{G_{10} \ t_{10}} \alpha_1 = \frac{h}{2h^2 \ b^2 \ Gt} \tag{14.d}$$

$$\frac{1}{(GI_t)_{\bar{u}10}} = \frac{1}{4\bar{U}^2} \ \frac{s_{1n}}{G_{10} \ t_{10}} \alpha_1^2 = \frac{h}{h^2 \ b^2 Gt} \tag{14.e}$$

$$\frac{1}{(GI_t)_{\bar{u}}} = \frac{1}{4\bar{U}^2} \frac{s_{Ou}}{G_{Ou}\ t_{Ou}} \propto_2 = 0 \qquad (14.f)$$

$$\frac{1}{(GI_t)_{\bar{u}0\bar{1}}} = \frac{1}{4\bar{U}^2} \frac{s_{O1}}{G_{O1}\ t_{O1}} = \frac{h}{h^2\ b^2\ Gt} \qquad (14.g)$$

Then substituting in equation (10-a, 10-b) we obtain :

$$B'' - \frac{8Gt}{E\ A}\ \frac{1}{b+7h}\ B = \frac{b+2h}{b+7h}\ m_t \qquad (15)$$

whose solution, in the form of force dimensionless parameter of normal stress distribution in flanges σ/M_t is given by :

$$\frac{\sigma}{M_t} = \frac{1}{bhA}\ .n.a.\ \left[\sinh z/a - \tanh 1/a\ \ \cosh z/a\right] \qquad (16)$$

where: $n = \frac{b+2h}{b+7h}$, $a = \frac{EA(b+7h)}{8Gt}$

applying equation (16) on the carried out model Fig.4, in which we have b = 400 mm, h= 200 mm, t= 0.5 mm, A= 2x145 = 290 mm^2 where E,G of the used material were tested before use. They were $E=2.05x10^5 N/mm^2$ for flange material (low steel), and $G=2.315x10^4\ N/mm^2$ for sheet material (duralumine), the model span 1=900 mm. Thus $a= 1.074996x10^{-5}$ mm, n= 0.44444 and the relative length 1/a= 0.8372. Then substituting in (16) we obtain (17) which was calculated for different values of z and plotted directly on diagram Fig. 7.

$$\frac{\sigma}{M_t} = 2.0594\ x10^{-5}\ \left[\sinh z/1075 - \tanh 0.84\ \cosh z/1075\right] \qquad (17)$$

MODEL TESTING

A model composed of duralumine sheets and constructional steel angles was carried out in laboratory. Before using these materials the homogenety, isotropy of sheets and mechanical properties E,G of them were tested, using an accurate micrometer and wire strain gauge, on several specimen and then considering the mean values for each property.

The model has cross-section configuration as shown in Fig. 4. Verification of built-in end was done by doubling the model symmetrically, Fig.5. which gives equal and opposite warping on each side (cancelling each other) at plane of symmetry.

To have free warping at free end the model ends were supported by sliding supports. All the relative rotation between the plane of symmetry and the ends will be considered at the free end.

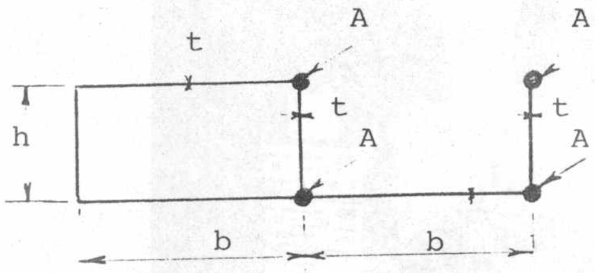

Fig. 3.

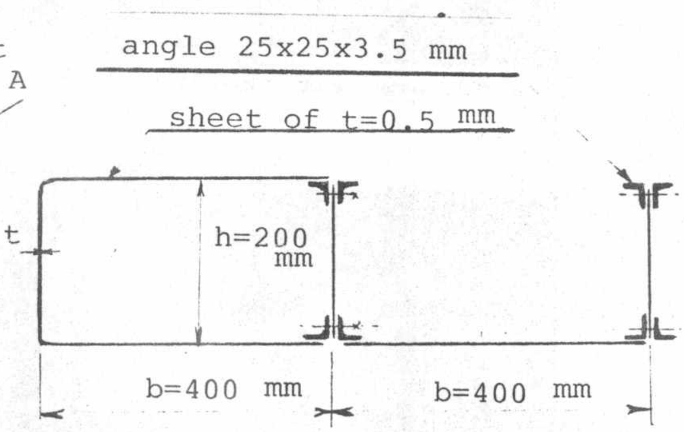

Fig. 4.

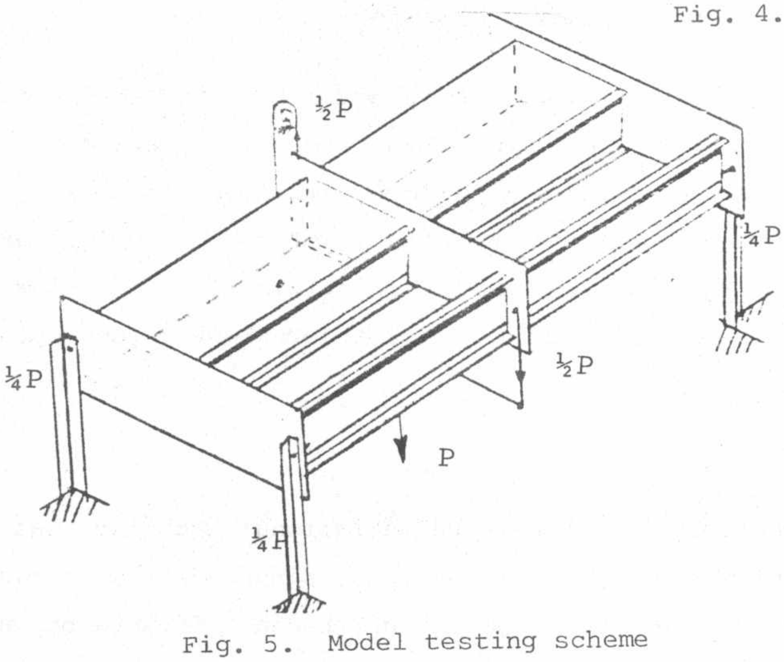

Fig. 5. Model testing scheme

The reaction at the sliding supports (at the free end) will form the loading torque moment. This model was loaded by different values of torque (P) Fig.5, which are measured by a calibrated dynamometer for several cycles. The loading cycle is formed by loading steps starting from zero load up to its maximum value, then unloading till the zero value once more. Normal stress distribution in flanges (σ) is measured by semiconductor strain gauges distributed on one flange of the model Fig.6.

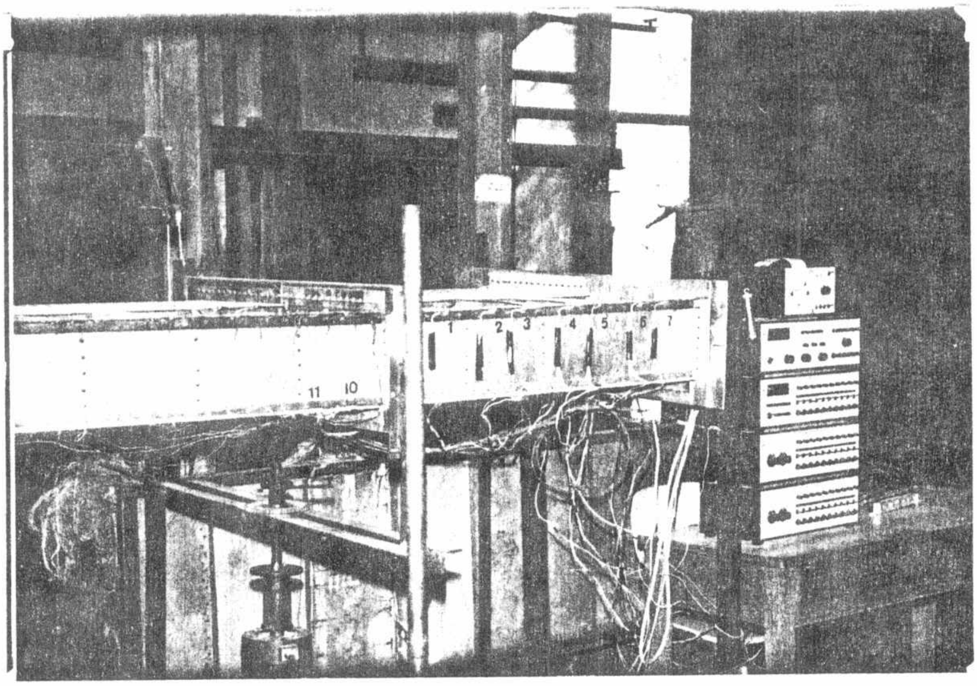

Fig.6. A picture of model testing

At every point, the obtained value of σ is divided by the corresponding M_t (for each reading, i.e. for each loading step). Repeating the loading cycle we obtain a lot of values, of force dimensionless parameter σ/M_t for every measuring point along the model span. The calculated mean values of σ/M_t (for every measuring point) is plotted against the model longitudinal axis z together with the theoretical values obtained from equation (17) on Fig.7.

ANALYSIS AND DISCUSSION

For checking of theoretical results, the variational technique was applied for a very simple case (that of box beam, single cell with four flanges which was discussed before [3] . The obtained formula by variational technique is (19) and the before was (18)

$$\frac{\sigma}{M_t} = \frac{\lambda}{8hbGt}(h-b)(\tanh \lambda l.\ \cosh \lambda z - \sinh \lambda z) \qquad (18)$$

where : $\lambda^2 = \frac{8Gt}{AE(h+b)}$

$$\frac{\sigma}{M_t} = na\ \frac{1}{hbA}\left[\sinh\ z/a - \tanh\ l/a.\ \cosh\ z/a\right] \qquad (19)$$

where : $a^2 = \frac{EA(h+b)}{8Gt}$, $n = \frac{b-h}{b+h}$

It can be proved by rearrangement that equations (18) and (19) are the same. Thus application of the variational technique, with taking into account the effect of restrained warping, (which can be applied for general cross-section shapes) gives good theoretical results.

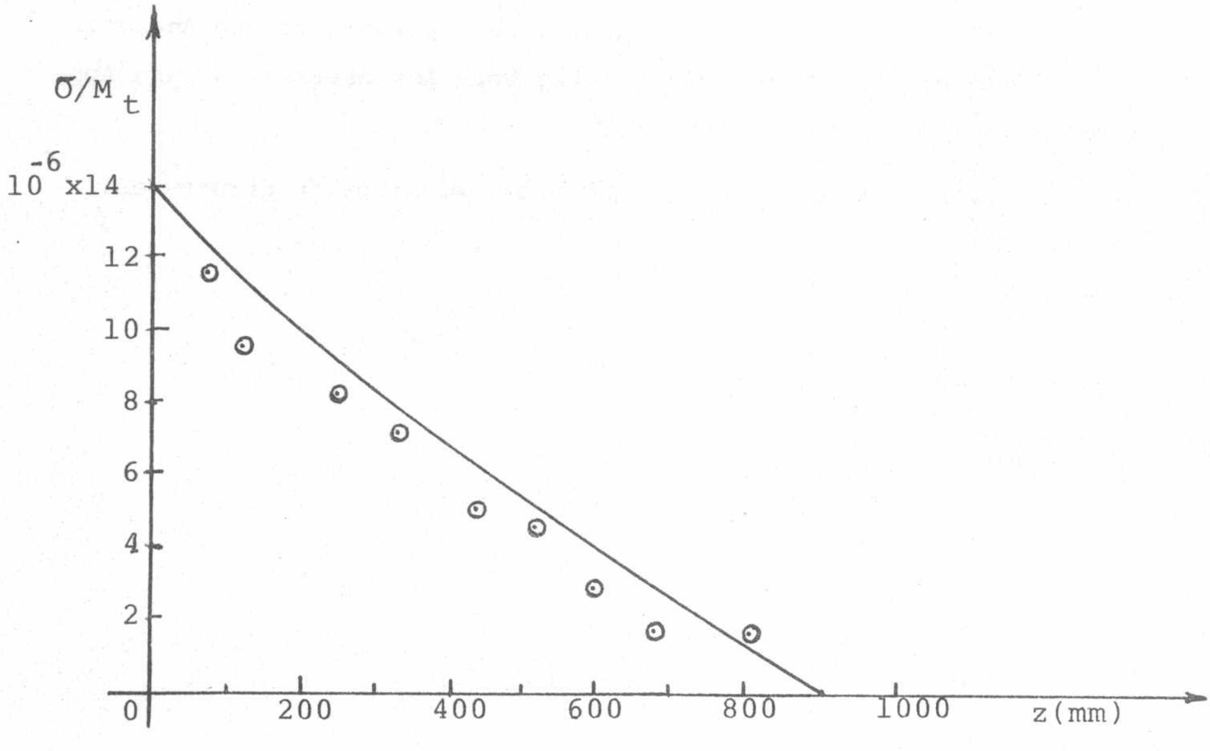

Fig.7. Theoretical and experimental results.

Fig.7. shows the obtained theoretical results from applying the variational technique on the used model, given by equation (17) together with the obtained measurement results. It is clear that the calculated values are near to the measured ones and little higher. Thus it is possible to

say that the variational method is safe from point of veiw of maximum stresses in flanges and consequently in thin sheets (normal stresses in flanges are produced by shear flow in adjacent sheets).

Finally the variational technique has the possibility of expressing general shape configurations without any need for idealizing the structure or simplifying it. This method needs only to divide the structure carefully into parts, which will certainly give better theoretical results than idealizing the structure. The experimental results proved that the obtained theoretical results are of good agreement.

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