MILITARY TECHNICAL COLLEGE

CAIRO - EGYPT

1 # USING THE SEMIBENDING THEORIES IN SOLVING THIN
2 WALLED CYLINDRICAL SHELL CONSTRUCTIONS.

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4 ## ABSTRACT

The semibending theories of thin walled cylindrical shell construction neglect the increment in curvature of the shell in the axial direction. The basic differential equations of the shells using the semibending theories are derived for three different cases and are named as simplified semibending, semibending theory with incompressible middle surface and semibending theory with compressible middle surface. The differences between these theories depend on the values of the circumferential normal strain and the shear strain. The derived differential equations are solved and applied to the problems of influence lines of stresses and deformations for different loading conditions on a long shells.

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INTRODUCTION

One of the basic assumptions of the semibending theories of shells is the negligence of the increment in curvature of the shell in the axial direction. Thus, the longitudinal bending moments M_x, the transversal shearing forces Q_x and the torsional moments M_{xx} & M_{yy} are neglected. The stress resultants taken into consideration are, Fig. 1, N - longitudinal normal force, T - circumferential normal force, S - shearing force, $Q = Q_y$ - transversal shearing force and M_s - circumferential bending moment. Generally these theories can be applied to a shall of length $L > 2R$ where R is the shell radius. According to the values of the circumferential strain ϵ_s and shear strain γ the semibending theories are classified into : i- Simplified semibending theory (SS), considered that $\epsilon_s = 0$ and $\gamma = 0$ [3]. ii- Semibending theory with incompressible middle surface (SIM); considered that $\epsilon_s = 0$ and $\gamma \neq 0$, iii- Semibending theory with compressible middle surface (SCM); considered that $\epsilon_s \neq 0$ and $\gamma \neq 0$.

GOVERNING DIFFERENTIAL EQUATIONS

The principle of minimum potential energy is used for deriving the differential equations. The potential energy is given by

$$U = \int_{x_o}^{x_1} \Gamma(x, \xi, \xi', \xi'')dx \tag{1}$$

where Γ represents the potential energy per unit length of the shell along the axis x. It is necessary to find the function $\xi(x)$ such that U is minimum. We shall investigate parts of the shell where external loading does not act in order to avoid solution of non-homogeneous equations, its influence is taken in the bounding conditions. Correlate the shell to the orthogonal coordinate system x and S, Fig. 1, the equilibrium equations of the shell element are

$$\frac{\partial N}{\partial x} + \frac{\partial S}{\partial s} = 0, \quad \frac{\partial T}{\partial s} + \frac{\partial S}{\partial x} - \frac{Q}{R} = 0$$
$$\frac{\partial Q}{\partial s} + \frac{T}{R} = 0 \qquad Q - \frac{\partial M_s}{\partial s} = 0 \tag{2}$$

and using the strain displacement relations

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$$\epsilon_x = \frac{\partial u}{\partial x} \quad , \quad \epsilon_s = \frac{\partial v}{\partial s} - \frac{w}{R}$$

$$\gamma = \frac{\partial u}{\partial s} + \frac{\partial v}{\partial x} \quad , \quad \chi_s = \frac{\partial^2 w}{\partial s^2} + \frac{w}{R^2} \tag{3}$$

and the stress - strain relations

$$\epsilon_x = \frac{1}{Et}(N - \mu T) \qquad \epsilon_s = \frac{1}{Et}(T - \mu N)$$

$$\gamma = \frac{2(1+\mu)}{Et} S \qquad \chi_s = -\frac{12(1-\mu^2)}{Et^3} M_s \tag{4}$$

The solution will be much simpler if we solve the symmetric and the antisymmetric part separately, for symmetric load we can write

$$\begin{aligned}
N(x,\varphi) &= \sum N_n(x) \cos n\varphi, & S(x,\varphi) &= \sum S_n(x) \sin n\varphi \\
T(x,\varphi) &= \sum T_n(x) \cos n\varphi, & M_s(x,\varphi) &= \sum M_{sn}(x) \cos n\varphi \\
Q(x,\varphi) &= \sum Q(x) \sin n\varphi, & u(x,\varphi) &= \sum U_n \cos n\varphi \\
v(x,\varphi) &= \sum v_n(x) \sin n\varphi, & w(x,\varphi) &= \sum w_n(x) \cos n\varphi
\end{aligned} \tag{5}$$

Consider the longitudinal normal force $N_n(x)$ is the unknown function $f(x)$ and write the expression of the strain energy interms of $N_n(x)$ and its derivaties. The differential equations can be found using Euler's equation

$$\frac{\partial \Gamma}{\partial N_n} - \frac{d}{dx}\left(\frac{\partial \Gamma}{\partial N'_n}\right) + \frac{d^2}{dx^2}\left(\frac{\partial \Gamma}{\partial N''_n}\right) = 0 \tag{6}$$

Differential Equation of SCM

From equations (2) and (5) we get

$$M_{sn}(x) = -\frac{R^3}{n^2(n^2-1)} N''_n(x), \quad T_n(x) = -\frac{R^2}{n^2-1} N''_n(x)$$

$$Q_n(x) = \frac{R^2}{n(n^2-1)} N''_n(x), \qquad S_n(x) = -\frac{R}{n} N'_n(x) \tag{7}$$

The strain energy has the form (1) where the strain energy per unit length is

$$\Gamma = \int_{R-t/2}^{R+t/2} \oint \Gamma_{ko} \; R \, d\varphi \, dR \tag{8}$$

R,t are shell radius and thickness respectively while
Γ_{ko} is strain energy per unit volume.

$$\Gamma_{ko} = \frac{1}{2E}\left(\sigma_x^2 + \sigma_s^2\right) - \frac{\mu}{E}\sigma_x \sigma_s + \frac{1}{2G}\left(1,2\,\tau_{Rs}^2 + \tau_{sx}^2\right) \tag{9}$$

Determining Γ from (8) and applied to Euler's equation (6): we get

$$N_n^{lv}(x) - 2\, a_n N_n''(x) + b_n^2 N_n(x) = C_n \tag{10}$$

Primes indicate differentiation with respect to x and

$$a_n = \frac{t^2}{R^4} \frac{n^2\left[1+\mu - n^2(2+\mu)+n^4\right]}{12+\delta^2 n^2\left[n^2+12/5\cdot(1+\mu)\right]}$$

$$b_n = \frac{t}{R^3} \frac{n^2(n^2-1)}{\sqrt{12+\delta^2 n^2\left[n^2+12/5\cdot(1+\mu)\right]}} \tag{11}$$

$$c_n = \frac{\mu t^2}{\pi R^6} \frac{n^4(n^2-1)^2}{12+\delta^2 n^2\left[n^2+12/5\cdot(1+\mu)\right]} \, \phi^N(x)$$

$$\delta = t/R$$

where $\phi^N(x) = \oint T_s \cos n\varphi \, d\varphi$ (12)

C_n corresponding to the case of n=1, which can be solved individually considering the shell as a beam with rigid cross-section. The differential equation for $n \geqslant 2$ is

$$N_n^{lv}(x) - 2\, a_n N_n''(x) + b_n^2 N_n(x) = 0 \tag{13}$$

These differential equation are also applicable for cases of antisymmetrical loading.

Differential Equation of SIM

The same steps as for the case of SCM and using :
$\epsilon_s = 0$ we obtain the same form of the differneital equation (10) or (13)

with different coefficients where

$$a_n = \frac{5\ \delta^2 (1+\mu)\ n^2 (n^2-1)^2}{12\left[5\ R^2 + (1+\mu)t^2\ n^2\right]}$$

$$b_n = \frac{\delta\ n^2 (n^2-1)\sqrt{5(1-\mu^2)}}{R^2\sqrt{12\left[5 + (1+\mu)\delta^2\ n^2\right]}} \tag{14}$$

SOLUTION FOR INFINITELY LONG CIRCULAR CYLINDRICAL SHELL

Consider a circular cylindrical shell of radius R, thickness t and sufficiently great length, subjected to the following loads of a small load angle ϑ .

Radial Line Load

The load can be expressed by a Fourier series as , Fig. 2.

$$P(\varphi) = \frac{P_o}{2} + \sum_{n=1}^{\infty} P_n \cos n\varphi \tag{15}$$

where $P_o = \frac{Q'}{\pi R}$, $P_n = \frac{Q'}{\pi R}\ \frac{\sin n\vartheta}{n \sin\vartheta}$

Solving the differential equation (13) and applying the boundary conditions we get

for $b_n^2 > a_n^2$

$$N_n(x) = \frac{n^2\ P_n}{4R}\ \frac{e^{-\omega x}}{(r^2+\omega^2)}\left[\frac{r^2-3\,\omega^2}{\omega} \cos r\,x + \frac{3r^2-\omega^2}{r} \sin r\,x\right] \tag{16}$$

$$\text{where} \quad r = \sqrt{(b_n - a_n)/2}\ , \qquad \omega = \sqrt{(b_n + a_n)/2} \tag{17}$$

for $b_n^2 < a_n^2$

$$N_n(x) = \frac{n^2\ P_n}{2R(m_1^2-m_2^2)}\left[\frac{m_2^2}{m_1}\ e^{-m_1 x} - \frac{m_1^2}{m_2}\ e^{-m_2 x}\right] \tag{18}$$

$$\text{where} \quad m_1 = \sqrt{a_n + \sqrt{a_n^2 - b_n^2}}\ , \quad m_2 = \sqrt{a_n - \sqrt{a_n^2 - b_n^2}} \tag{19}$$

Radial Surface Load

The shell Fig. 3, is subjected to a uniform radial surface load. $N_n(x)$, determined for a line load, are integrated over the range of the load and gives for x = 0 :

for $b_n^2 > a_n^2$

$$N_n(0) = \frac{n^2 P_n}{2R\,(r^2+\omega^2)^2}\left[\frac{r^4-6r^2\omega^2+\omega^4}{r\omega}\, e^{-\omega b} \sin rb - 4(r^2-\omega^2)e^{-\omega b} \cos rb + 4(r^2-\omega^2)\right] \tag{20}$$

and for $b_n^2 < a_n^2$

$$N_n(0) = \frac{n^2\ P_n}{R(m_1^2-m_2^2)}\left[\frac{m_2^2}{m_1^2}(1-e^{-m_1 b}) - \frac{m_1^2}{m_2^2}(1-e^{-m_2 b})\right] \tag{21}$$

Tangential Circumferential Line Load

The load, Fig. 4, can be expressed as

$$F(\varphi) = \frac{F_o}{2} + \sum_{n=1}^{\infty} F_n \cos n\varphi$$

$$F_o = \frac{F'}{\pi R}\,, \quad F_n = \frac{F'}{\pi R}\ \frac{\sin n\vartheta}{n \sin\vartheta} \tag{22}$$

Applying the boundary conditions to the solution of the differential equation (13) we get

For $b_n^2 > a_n^2$

$$N_n(x) = -\frac{n\ F_n}{4R(r^2+\omega^2)}\left(\frac{r^2-3\omega^2}{\omega}\, e^{-\omega x} \cos r x + \frac{3r^2-\omega^2}{r}\, e^{-\omega x} \sin rx\right) \tag{23}$$

and for $b_n^2 < a_n^2$

$$N_n(x) = \frac{n\ F_n}{2R(m_2^2-m_1^2)}\left(\frac{m_2^2}{m_1}\, e^{-m_1 x} - \frac{m_1^2}{m_2}\, e^{-m_2 x}\right) \tag{24}$$

Tangential Circumferential Surface Load

The load is shown in Fig. 5, integrating $N_n(x)$, eqns.(23) & (24), over the range of the load gives for $b_n^2 > a_n^2$

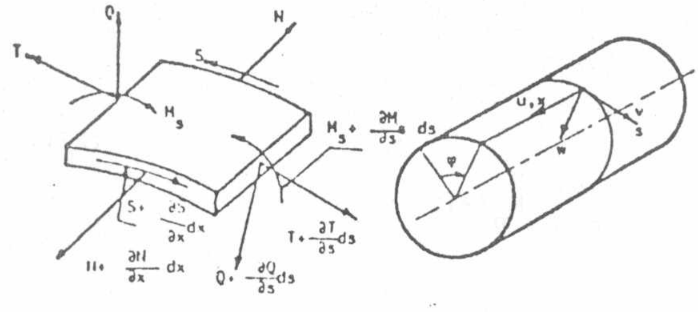

Fig. 1.

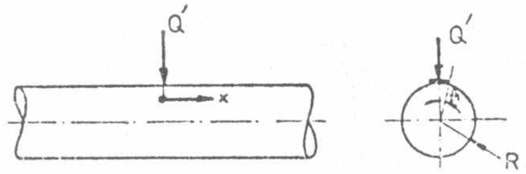

Fig. 2.

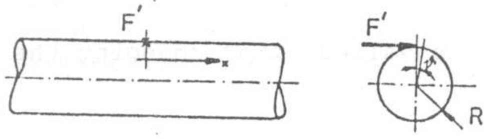

Fig. 4.

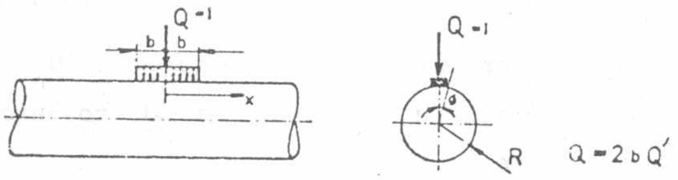

Fig. 3.

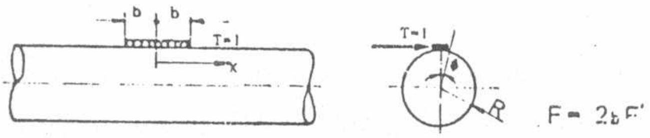

Fig. 5.

$$N_n(0) = \frac{n F_n}{R(r^2+\omega^2)^2}\left[\frac{r^4 - 6r^2\omega^2 + \omega^4}{r\omega} e^{-\omega b} \sin rb + 4(r^2-\omega^2)(1-e^{-\omega b}\cos rb)\right] \quad (25)$$

and for $b_n^2 < a_n^2$

$$N_n(0) = \frac{n F_n}{R(m_1^2-m_2^2)}\left[\frac{m_2^2}{m_1^2}(1-e^{-m_1 b}) - \frac{m_1^2}{m_2^2}(1-e^{-m_2 b})\right] \quad (26)$$

INFLUENCE LINES

The stresses and deformations of infinitely long circular cylindrical shell subjected to surface load of width 2b with small load angle ϑ are found by substituting the corresponding expression of $N_n(x)$ and its derivatives into the corresponding expressions of stresses and deformations. For the stresses, $\sigma_x(x) = N(x)/t$ and $\sigma_s(x) = T(x)/t + 12M_s(x)\, y/t^3$ where N(x), T(x) and $M_s(x)$ are given by equations (5) and (7). For the deformations; for SCM, using equations (3), (4) & (5) we get for symmetric loading

$$W_n(x) = -\frac{12(1-\mu^2)}{E\,t^3}\;\frac{R^5}{n^2(n^2-1)^2}\;N''_n(x)$$

$$V_n(x) = -\frac{R^3}{Et}\;\frac{1+A\,n^2(n^2-1)}{A\,n^3(n^2-1)^2}\;N''_n(x) - \frac{\mu R}{Etn}\;N_n(x) \tag{27}$$

where $A = \dfrac{\delta^2}{12(1-\mu^2)}$

and for antisymmetric loading equations (27) are valied after changing the sine of $V_n(x)$.

For SIM $W_n(x)$ has the same form as above while for symmetric loading $V_n(x) = \dfrac{W_n(x)}{n}$ and for antisymmetric loading $V_n(x) = -\dfrac{W_n(x)}{n}$, Substituting the corresponding expressions of $N_n(x)$ and $N''_n(x)$ into the above equations and considering that x tends to zero, after rearrangement we get for unit radial surface load

$$W_R(0,\varphi) = \frac{1}{E'R}\left(\frac{R}{E}\right)^{5/2} f_{w_R}$$

$$V_R(0,\varphi) = \frac{1}{E'R}\left(\frac{R}{t}\right)^{5/2} f_{v_R} \tag{28}$$

$$\sigma_{s_R}(0,\varphi) = \frac{1}{t^2}\left(\frac{t}{R}\right)^{1/2} f_{s_R}$$

and for unit tangential circumferential surface load

$$W_T(0,\varphi) = \frac{1}{E'R}\left(\frac{R}{t}\right)^{5/2} f_{w_T}$$

$$V_T(0,\varphi) = \frac{1}{E'R}\left(\frac{R}{t}\right)^{5/2} f_{v_T} \tag{29}$$

$$\sigma_{s_T}(0,\varphi) = \frac{1}{t^2}\left(\frac{t}{R}\right)^{1/2} f_{s_T}$$

where $E' = \dfrac{E}{1-\mu^2}$. , For SCM and $b_n^2 > a_n^2$

$$f_{w_R} = \frac{6}{\pi k}\left\{\sum_{n=2}^{\infty}\frac{\cos n\varphi}{(n^2-1)^2}\lambda_{1n} + \frac{1}{4}\left[\left(\frac{\varphi^2}{2} - \pi\varphi + \frac{\pi^2}{3} - \frac{3}{4}\right)\cos\varphi + (\pi-\varphi)\sin\varphi - 2\right]\right\} \tag{30}$$

$$f_{VR} = \frac{6}{\pi k}\left\{\sum_{n=2}^{\infty} \sin n\varphi \left[\frac{1+A\, n^2(n^2-1)}{n(n^2-1)^2}\right]\lambda_{1n} + \frac{\mu\,\delta}{12(1-\mu^2)}\sum_{n=2}^{\infty} n\lambda_{2n}\sin n\varphi\right.$$

$$+\frac{1}{4}\left[1+2\mu A(1+\mu)\right]\left[\left(\frac{\varphi^2}{2}-\pi\varphi+\frac{\pi^2}{3}-\frac{11}{4}\right)\sin\varphi - 2(\pi-\varphi)\cos\varphi\right. \qquad (31)$$

$$\left.+2(\pi-\varphi)\right] - \frac{A}{4}\left[1+2\mu(2+\mu)\right]\left[\left(\frac{\varphi^2}{2}-\pi\varphi+\frac{\pi^2}{3}+\frac{1}{4}\right)\sin\varphi\right]+$$

$$\left.+\frac{A}{4}(1+2\mu)\left[\left(\frac{\varphi^2}{2}-\pi\varphi+\frac{\pi^2}{3}-\frac{3}{4}\right)\sin\varphi - 2(\varphi-\pi)\cos\varphi\right]\right\}$$

$$f_{SR} = \frac{3}{\pi k}\left\{\sum_{n=2}^{\infty}\frac{\cos n\varphi}{(n^2-1)}\lambda_{1n} + \frac{1}{2}\left[1+\frac{1}{2}\cos\varphi - (\pi-\varphi)\sin\varphi\right]\right\} \qquad (32)$$

$$f_{WT} = -\frac{6}{\pi k}\left\{\sum_{n=2}^{\infty}\frac{\sin n\varphi}{n(n^2-1)^2}\lambda_{1n} + \frac{1}{4}\left[\left(\frac{\varphi^2}{2}-\pi\varphi+\frac{\pi^2}{3}-\frac{11}{4}\right)\sin\varphi -\right.\right.$$

$$\left.\left.-2(\pi-\varphi)\cos\varphi + 2(\pi-\varphi)\right]\right\} \qquad (33)$$

$$f_{VT} = \frac{6}{\pi k}\left\{\sum_{n=2}^{\infty}\cos n\varphi\left[\frac{1+A\,n^2(n^2-1)}{n^2(n^2-1)^2}\right]\lambda_{1n} + \frac{\mu\,\delta}{12(1-\mu^2)}\sum_{n=2}^{\infty}\lambda_{2n}\cos n\varphi +\right.$$

$$+\frac{1}{4}\left[1+2\mu A(1+\mu)\right]\left[\left(\frac{\varphi^2}{2}-\pi\varphi+\frac{\pi^2}{3}-\frac{23}{4}\right)\cos\varphi + 3(\pi-\varphi)\sin\varphi -\right.$$

$$\left.-2\pi\varphi+\varphi^2+\frac{2\pi^2}{3}-4\right] - \frac{A}{4}\left[1+2\mu(2+\mu)\right]\left[\left(\frac{\varphi^2}{2}-\pi\varphi+\frac{\pi^2}{3}-\frac{3}{4}\right)\cos\varphi\right.$$

$$\left.+(\pi-\varphi)\sin\varphi - 2\right] + \frac{A}{4}(1+2\mu)\left[\left(\frac{\varphi^2}{2}-\pi\varphi+\frac{\pi^2}{3}+\frac{1}{4}\right)\cos\varphi +\right.$$

$$\left.\left.+(\varphi-\pi)\sin\varphi\right]\right\} \qquad (34)$$

$$f_{ST} = -\frac{3}{\pi k}\left\{\sum_{n=2}^{\infty}\frac{\sin n\varphi}{n(n^2-1)}\lambda_{1n} + \frac{1}{2}\left[\frac{3}{2}\sin\varphi - (\pi-\varphi)(1-\cos\varphi)\right]\right\} \qquad (35)$$

where :

$$\lambda_{1n} = \frac{r_1^2-\omega_1^2}{2\,r_1\omega_1}\, e^{-k\omega_1}\sin k\,r_1 - e^{-k\omega_1}\cos k\,r_1 \qquad (36)$$

$$\lambda_{2n} = \frac{2(r_1^2-\omega_1^2)}{(r_1^2+\omega_1^2)^2}\, e^{-k\omega_1}\cos k\,r_1 - \frac{r_1^4-6r_1^2\omega_1^2+\omega_1^4}{2\,r_1\omega_1(r_1^2+\omega_1^2)^2}\, e^{-k\omega_1}\sin k\,r_1 \qquad (37)$$

$$\omega_1 = \frac{R}{\sqrt{\delta}}\,\omega\,, \qquad r_1 = \frac{R}{\sqrt{\delta}}\,r\,, \qquad k = \frac{b}{R}\sqrt{\delta} \qquad (38)$$

r and ω given by eqn. 17 , A given by 27. For SCM always $b_n^2 > a_n^2$ for all values of δ and n. For SIM, $b_n^2 > a_n^2$ for smaller n, say up to $n=\bar{n}$, while for $n > \bar{n}$ we have $b_n^2 < a_n^2$, then the influence line coefficients take the following forms.

$$f_{WR} = \frac{6}{\pi k}\left\{\sum_{n=2}^{\bar{n}} \frac{\cos n\varphi}{(n^2-1)^2}\lambda_{1n} + \sum_{n=\bar{n}+1}^{\infty} \frac{\cos n\varphi}{(n^2-1)^2}\lambda_{3n} + \frac{1}{4}\left[\left(\frac{\varphi^2}{2} - \pi\varphi + \frac{\pi^2}{3} - \frac{3}{4}\right)\cos\varphi + \right.\right.$$
$$\left.\left. + (\pi - \varphi)\sin\varphi - 2\right]\right\} \tag{39}$$

$$f_{VR} = \frac{6}{\pi k}\left\{\sum_{n=2}^{\bar{n}} \frac{\sin n\varphi}{n(n^2-1)^2}\lambda_{1n} + \sum_{n=\bar{n}+1}^{\infty} \frac{\sin n\varphi}{n(n^2-1)^2}\lambda_{3n} + \frac{1}{4}\left[\left(\frac{\varphi^2}{2} - \pi\varphi + \frac{\pi^2}{3} - \frac{11}{4}\right)\sin\varphi\right.\right.$$
$$\left.\left. -2(\pi - \varphi)\cos\varphi + 2(\pi - \varphi)\right]\right\} \tag{40}$$

$$f_{SR} = \frac{3}{\pi k}\left\{\sum_{n=2}^{\bar{n}} \frac{\cos n\varphi}{n^2-1}\lambda_{1n} + \sum_{n=\bar{n}+1}^{\infty} \frac{\cos n\varphi}{n^2-1}\lambda_{3n} + \frac{1}{2}\left[1 + \frac{1}{2}\cos\varphi - \right.\right.$$
$$\left.\left. - (\pi - \varphi)\sin\varphi\right]\right\} \tag{41}$$

$$f_{WT} = -\frac{6}{\pi k}\left\{\sum_{n=2}^{\bar{n}} \frac{\sin n\varphi}{n(n^2-1)^2}\lambda_{1n} + \sum_{n=\bar{n}+1}^{\infty} \frac{\sin n\varphi}{n(n^2-1)^2}\lambda_{3n} + \frac{1}{4}\left[\left(\frac{\varphi^2}{2} - \pi\varphi + \frac{\pi^2}{3} - \frac{11}{4}\right)\sin\varphi\right.\right.$$
$$\left.\left. -2(\pi - \varphi)\cos\varphi + 2(\pi - \varphi)\right]\right\} \tag{42}$$

$$f_{VT} = \frac{6}{\pi k}\left\{\sum_{n=2}^{\bar{n}} \frac{\cos n\varphi}{n^2(n^2-1)^2}\lambda_{1n} + \sum_{n=\bar{n}+1}^{\infty} \frac{\cos n\varphi}{n^2(n^2-1)^2}\lambda_{3n} + \right.$$
$$\left. + \frac{1}{4}\left[\left(\frac{\varphi^2}{2} - \pi\varphi + \frac{\pi^2}{3} - \frac{23}{4}\right)\cos\varphi + 3(\pi - \varphi)\sin\varphi - 2\pi\varphi + \varphi^2 + \frac{2\pi^2}{3} - 4\right]\right\} \tag{43}$$

$$f_{ST} = -\frac{3}{\pi k}\left\{\sum_{n=2}^{\bar{n}} \frac{\sin n\varphi}{n(n^2-1)}\lambda_{1n} + \sum_{n=\bar{n}+1}^{\infty} \frac{\sin n\varphi}{n(n^2-1)}\lambda_{3n} + \frac{1}{2}\left[\frac{3}{2}\sin\varphi - \right.\right.$$
$$\left.\left. - (\pi - \varphi)(1 - \cos\varphi)\right]\right\} \tag{44}$$

where
$$\lambda_{3n} = \frac{m^2}{1-m^2}\left(e^{-bm_2} - 1\right) - \frac{1}{1-m^2}\left(e^{-bm_1} - 1\right) - 1 \tag{45}$$

and $m = \frac{m_1}{m_2}$, m_1 and m_2 given by eqn. 19.

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RESULTS AND DISCUSSION

The expressions of the influence lines derived on the assumption that the load angle ϑ is small have relatively slow convergence n=1000 up to 2500. However considering, theoretically, that ϑ tends to zero, their convergence is more rapid, n = 175 upto 300, and they have a maximum numerical difference of 1 % more than the accurate influence lines derived on the basis that the load angle ϑ is very small.

The influence lines of the SCM and SIM are calculated numerically for k= 0,005 and δ = 0,01 and compared with those calculated according to SS for k= 0,005 Fig. 6-7 and 8. The differences between SCM and SIM are very small. The influence lines determined according to the SS depend on one shell parameter k, while those determined according to the SCM and SIM depend on two shell parameters k and δ . The values of the influence line coefficients calculated according to the SCM for k= 0.005 with different values of δ are given in Figs. 9-10 and 11 together with those calculated according to the SS for k = 0.005. f_{SR}, Fig. 9, calculated according to the SCM for δ = 0,0001 are coincident with those calculated according to the SS but the differences between them increases with increasing δ . At $\varphi = 0$ and δ = 0.05 the difference is 43,5 %. An increment in δ leads to increasing the differences and increasing φ decreasing the differences.

Similar statments arise for f_{WR} Fig. 10. The differences for f_{VR} are relatively small for all values of δ and φ Fig. 11. Generally the influence lines calculated according to SS are comparable with those calculated according to the SCM for $\delta < 0,02$. The dependences of the circumferential stress coefficient f_{SR} calculated according to SCM at $\varphi = 0^{o}$ on the shell parameter δ for different values of k is given in Fig. 12.

The semibending theories of shells are the suitable theories for studying many of the thin walled cylindrical shell problems: These theories have simple equations, fourth-order differential equations. They use single and quick convergence series and gives a sufficients accurate results. For very thin shells ($\delta \leq 0,001$) the simplified semibending theory can be used, while for $\delta > 0,001$ it is suitable to use the semibending theory with compressible middle surface.

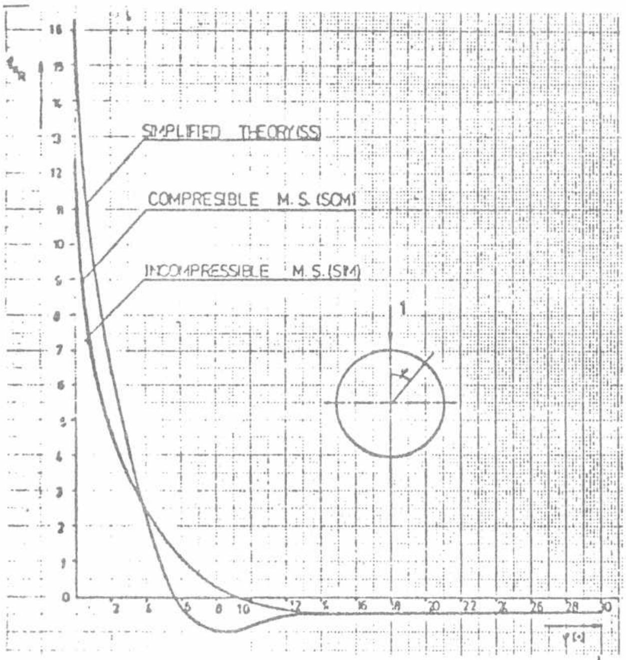

Fig.6. f_{SR} for SCM, SIM with $k=0,005$ and $\delta = 0,01$ and for SS with $k = 0,005$

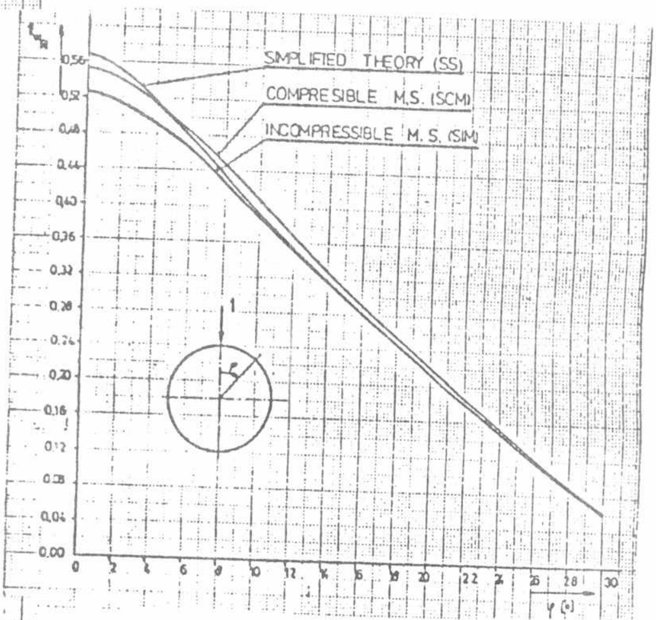

Fig.7. f_{WR} for SCM, SIM with $k=0,005$ and $\delta=0,01$ and for SS with $k=0,005$.

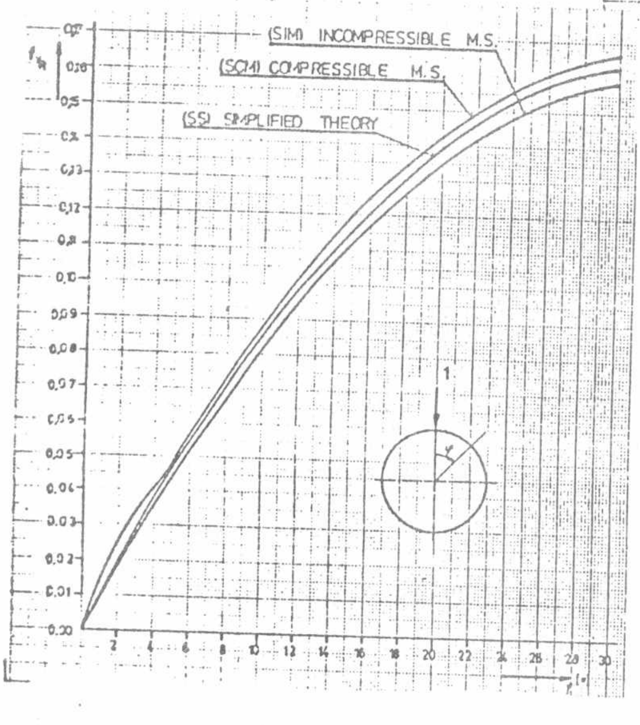

Fig.8. f_{VR} for SCM , SIM with $k=0,005$ and $\delta=0,01$ and for SS with $k=0,005$.

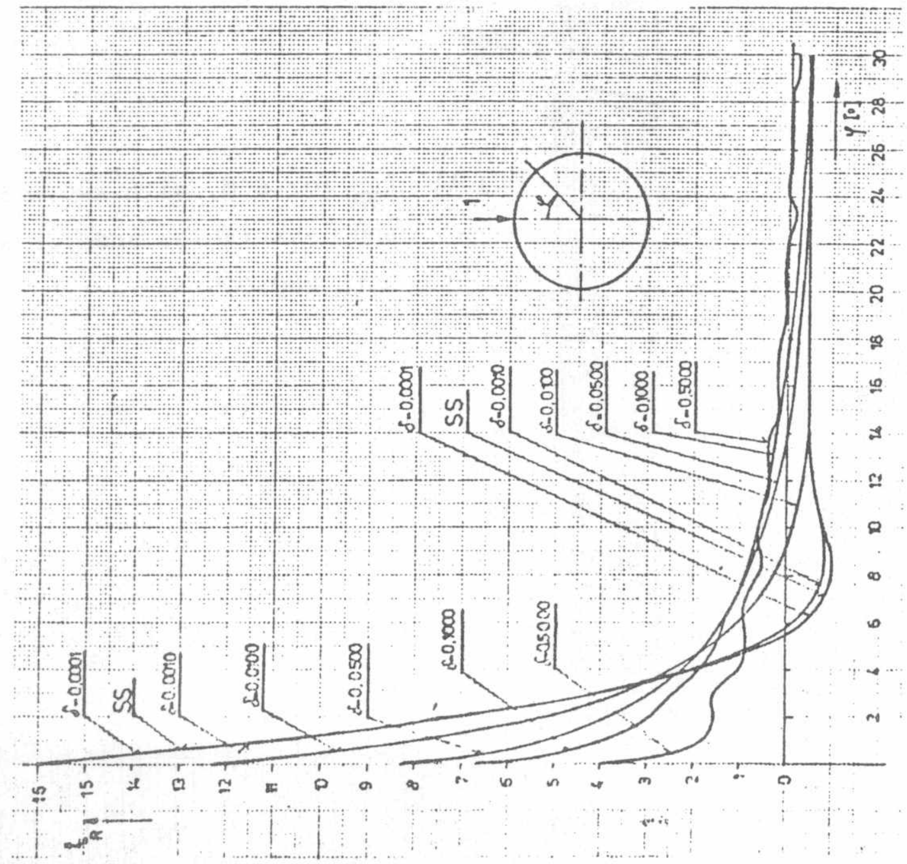

Fig.9. f_{SR} for SCM, with k=0,005 and δ -variable, and for SS with k=0,005.

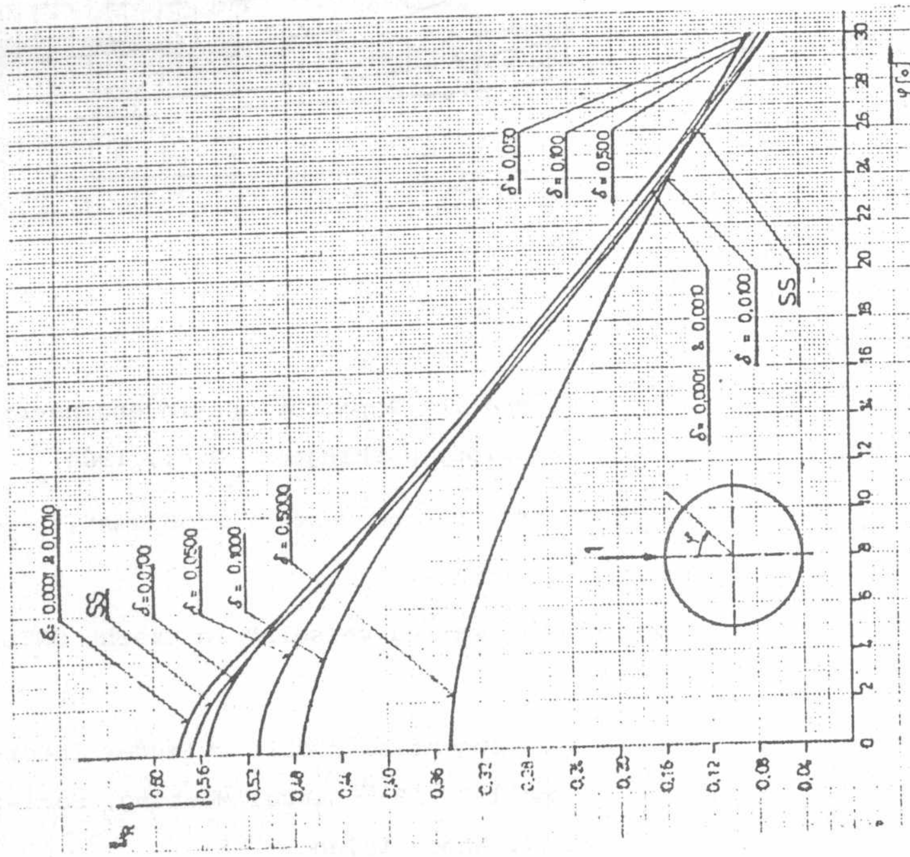

Fig.10. f_{WR} for SCM, with k=0,005 and δ -variable, and for SS with k=0,005.

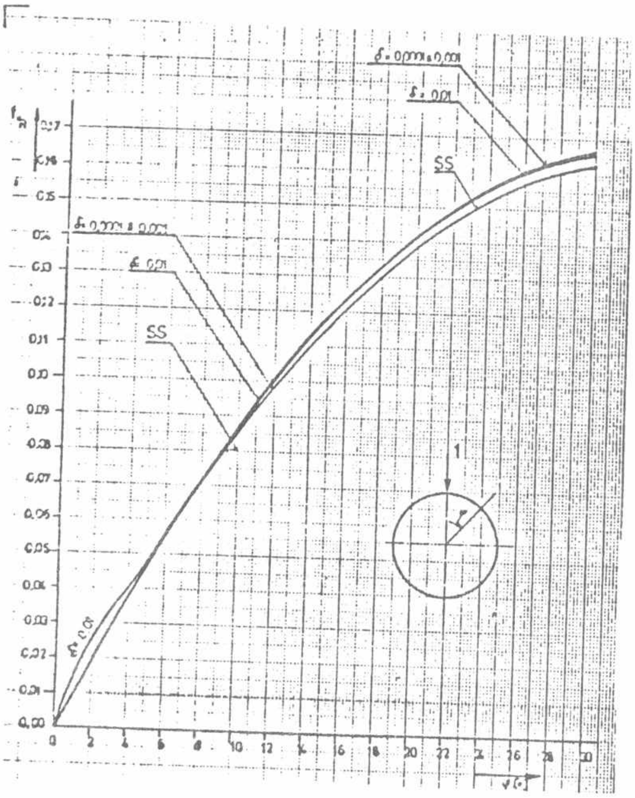

Fig.11. f_{VR} for SCM, with k=0,005 and δ-variable, and for SS with k=0,005

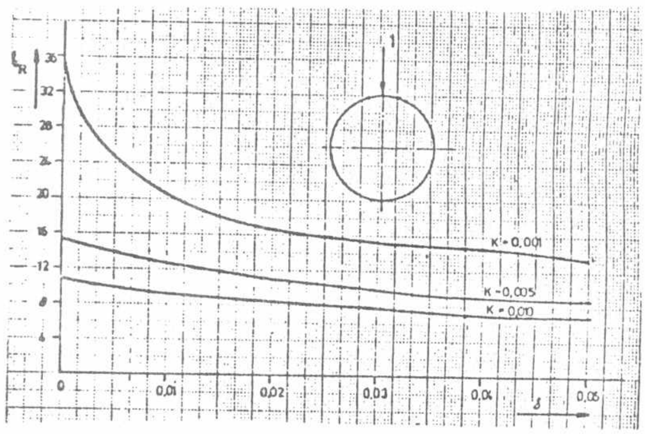

Fig.12. Dependence of f_{SR} on δ and k.

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