1 # RESONANCE-FREE MECHANICAL MANIPULATORS

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4 ## ABSTRACT

MRAIC schemes have been proposed as an on-line vibration testing method for the identification and control of the dynamic characteristics of mechanical manipulators. The idea behind the method is to make use of input-output sampled time response data such as acceleration, velocity, or displacement at stations on the manipulator to determine the unknown natural frequencies and associated viscous damping ratios. The analysis procedure is based on the construction of a stochastic model for the manipulator from the experimental response data. A refrence model with an acceptable dynamic behaviour is then defined and the mechanical manipulator is forced to track this model using an MRAC algorithm. Also included in the analysis is a study of how to optimize the effects of different parameters introduced in the method of control, such as the starting value of the adaptation gain, the rate of decreasing this gain, and the ratio of the proportional to integral gain at each step.

The simulation study shows that the technique works, in principle, for manipulators when a finite number of modes are included in the response. The results obtained from the present investigation have indicated that the proposed method is adequate and not sensitive to measurement noise or round-off errors.

BACKGROUND AND OBJECTIVES

Robot dynamics and their control concepts have developed rapidly in the past few years. First, equations of motion for mechanical manipulators have been derived by various authors using Lagrangian approach [1-3]. In the face of the complexity of the closed form solution of Lagrangian formulation, which makes it too slow for real-time use, several alternative

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approaches have been proposed. Broadly these approaches fall into three categories: tabularization [4,5], simplification [6, 7], and recursive formulation [8]. In the same time, the Newton-Euler formulation has been developed in [9] and followed by its application to mechanical manipulators [10-12]. This approach yields a set of recursive equations which can be applied to the links sequentially. More recently a systematic recursive routine, which is also independent of the manipulator configuration and in which all input joint torques are referenced in their own local coordinates, is introduced [8].

On the other hand, many of controversies surrounding the relation between the dynamic characteristics of the manipulator and the other aspects of their control have been clarified due to recent progress. The independent joint control method [13], "Inverse Problem" technique [2], "Computed Torque" technique [6, 14], a form of quasi-linear multivariable control scheme called the resolved motion rate control method [15], a pseudo-linear open-loop feedback law which enjoys dynamic stability in the absence of measurement and parameters errors and termed resolved acceleration control technique [16], another pseudo-linear open-loop feedback law with nonlinear pre- and post processing of measurement and control signals [17], and a nonlinear control algorithm with full state feedback [18] are all introduced and analyzed in the last decade. Sliding mode theory has been applied to mechanical manipulators and a nonlinear switching control law with guaranteed tracking and stability properties is obtained [19]. Adaptive control law based on the model reference principle has been applied to manipulators [20,21].

The present investigation is aimed to developing an intelligent system which can assess the dynamic characteristics of mechanical manipulator, feels any excessive vibration, and control the dynamic behaviour of the manipulator to damp this vibration out in unknown environment. Besides that the analysis of this problem is terrifically complicated, the consideration of real-time requires that a simple method, which makes use of the rapid developments in micro-computer technology, be employed.

FUNDAMENTAL PRINCIPLES AND ANALYSIS

The first step in the development of the dynamic characteristics of a mechanical manipulator is the derivation of an analytical model of the spatial manipulator's elements keeping in mind that: 1- In order to maintain an adequate maneuverability of the manipulator usually a seven to nine or more degrees of freedom are required; 2- The inertia characteristics of the manipulator depends on the configuration and the payload which are often variable or unknown; 3- To simplify the analysis, the manipulator's elements are assumed to be rigid and effects such as connection clearances and motor or gear backlash are neglected. The payload is assumed to be grasped firmly by the manipulator end-effector so that the mass properties of the payload and wrist may be combined.

To obtain a mathematical model which describes the motion of a manipulator of N joints, an N-dimensional vector $\theta(t)$ representing the actual displacement of the N joints, generalized coordinates, is introduced. Multiple degree of freedom joints can be modelled as single degree of freedom joints with intermediate links of zero length and mass. Intutively, the inertia and gravitational forces are function of the configuration of the manipulator and hence depend on $\theta(t)$. The Coriolis force components ($i \neq j$) and centrifugal force components ($i=j$), however, depend on the joint velocities $\dot{\theta}_i \dot{\theta}_j$ for $i,j= 1,2,..,N$

In general, the equation describing the motion of a manipulator can be written as

$$J(\theta)\ \ddot{\theta}(t) + C\ \dot{\theta}(t) + D(\ \dot{\theta}_i,\dot{\theta}_j,\theta(t);\ i,j = 1,2,..,N) + G(\theta) = E\ Q(t) \qquad (1)$$

The inherent geometric nonlinearities of the set of coupled ordinary differential equations (1) make the system dynamics characterized by time varying unknown parameters which make it difficult to develop or solve such equations. It also shows that the applied torque at a joint depends on the state of movement at all the other joints.

Until recently there were two independent approaches towards deriving this set of dynamic equations of motion for mechanical manipulators and to find the explicit expression of J, C, D, G, and E. They are the Lagrangian formulation and the Newton-Euler formulation. Both formulations can be obtained in either a closed form, which unsuitable for real time use, or in a recursive form. One way to deduce the parameters in this set of equations is to follow the procedure proposed by Walters [8], and to express the linear and angular velocities $\dot{\theta}_i$ and accelerations $\ddot{\theta}_i$ of the links, i.e. solve for kinematics, in a recursive form, starting from the base, with the initial conditions θ_o, $\dot{\theta}_o$, and $\ddot{\theta}_o$ identically zero for a base at rest, and working towards the tip of the manipulator. Then the forces and torques are computed recursively from the tip to the base. Because the inertia tensor $J_i(\theta)$ and internal position vectors ℓ_i, s_i, and z_i (as shown in Fig. 1) are constant with respect to coordinate system attached to the i-th link, they do not have to be recomputed for each manipulator position. Therefore, the above procedure could be made efficient by referring the dynamics to local link coordinates rather to global coordinates [12]. Hence, by defining a local coordinate system at each joint according to Denavit and Hartenberg convention [22] there will be 3X3 transformation matrices H_i^{i-1} given by

$$H_i^{i-1} = \begin{bmatrix} \cos\phi_i & -\sin\phi_i\cos\psi_i & \sin\phi_i\sin\psi_i \\ \sin\phi_i & \cos\phi_i\sin\psi_i & -\cos\phi_i\sin\psi_i \\ 0 & \sin\psi_i & \cos\psi_i \end{bmatrix} \qquad (2)$$

which transform any vector with reference to joint i coordinates system to a new coordinate system whose coordintes are parallel to those at joint i-1 but its origin is at joint i, i.e. pure rotation. Therefore, the transformation matrix relative to the base coordinates (global coordinates) is given by:

$$H_i = H_i^o = H_1^o \, H_2^1 \, H_3^2 \dots H_i^{i-1} \tag{3}$$

Considering the kinetic and potential energies of the system, the recursive Lagrangian formulation can be casted in the following form

$$f_i = tr\left(\frac{\partial H_i}{\partial \theta_i} L_i\right) - G^T \frac{\partial H_i}{\partial \theta_i} b_i \tag{4}$$

where $L_i = H_{i+1}^i L_{i+1} + \ell_i e_{i+1} + a_i \ddot{d}_i^T + J_i \ddot{H}_i^T$, (5)

$$e_i = e_{i+1} + m_i \ddot{d}_i^T + a_i H_i^T , \tag{6}$$

$$a_i = s_i/m_i \text{ , and } b_i = m_i s_i + H_{i+1}^i b_{i+1} . \tag{7}$$

Another way to obtain inertia effects is to follow Newton-Euler approach in which each link is considered as a free body accelerating in space and obeying Newton's equation

$$f_i = m_i \ddot{r}_i \tag{8}$$

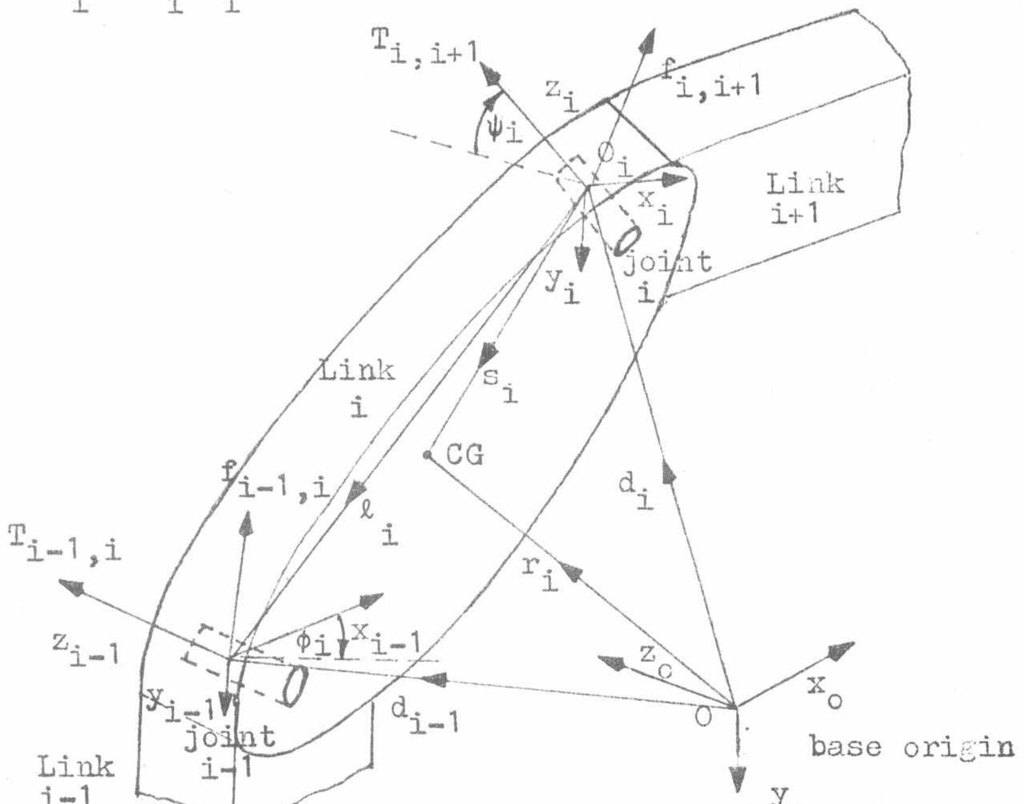

Fig.1. Free-body diagram of the i-th link showing local and global coord., position vectors, forces, & torques.

for linear movement, and Euler's equation

$$T_i = J_i \dot{\omega}_i + \omega_i \times J_i \omega_i \quad (9)$$

for angular rotations. To solve for the inertia force f_i and inertia torque T_i, the angular velocity, angular acceleration of each link and the acceleration of the center of the mass $\ddot{r}_i$ were found from kinematics as before, then the joint torques required for the movement were found from the static equations

$$f_i = f_{i-1,i} - f_{i,i+1} + m_i g \quad (10)$$

$$T_i = T_{i-1,i} - T_{i,i+1} - (\ell_i + s_i) \times f_{i-1,i} + s_i f_{i,i+1} \quad (11)$$

Assuming that some how the current values of the system parameters J, C, D, G, and E are found, however, in the proposed technique they need not to be known explicitly. And if the rate of change of these parameters was found relatively slow, then the manipulator equation of small oscillations about the nominal position can be obtained by the following linearized perturbation equations of motion:

$$\bar{J}\, \delta\ddot{\theta} + \bar{C}\, \delta\dot{\theta} + \bar{K}\, \delta\theta = \bar{E}\, \delta Q(t) \quad (12)$$

where $\bar{J} = \partial/\partial\theta\, [J(\theta)\, \ddot{\theta}]_{\bar{\theta}}$; $\bar{C} = \partial/\partial\theta\, [C + D]_{\bar{\theta}}$;

and $\bar{K} = \partial/\partial\theta\, [G + D - EQ]_{\bar{\theta}}$ (13)

are constant matrices derived from the derivative of the force terms with respect to θ and evaluated at nominal conditions. It may be useful to note that the stiffness matrix $\bar{K}$ does depend on $G(\bar{\theta})$, as with a pendulum.

Identification

The idea behind the present technique is to implement an adaptation identification algorithms which may use present and past measurement input-output data in order to update the estimated manipulator's model as it moves. As a large number of digital-computer controlled manipulators have been used in many industrial areas in recent years, and this number is expected to increase in the near future, the present investigation is motivated to consider the implementation of discrete-time adaptive scheme. The required algorithms for such implementation can be developed based on the discrete time MRAS theory, To apply the discrete time MRAS theory, the following discrete-time model for the manipulator is obtained by considering a quasi-linear model which assumes that the changes of the system parameters $\bar{J}$, $\bar{C}$, $\bar{K}$, and $\bar{E}$ are slow relative to the speed of adaptation and treats these parameters as time invariant matrices. Therefore, introducing state space representation, the equations of motion (12) could be written in the following form by replacing $\delta\theta$ by x

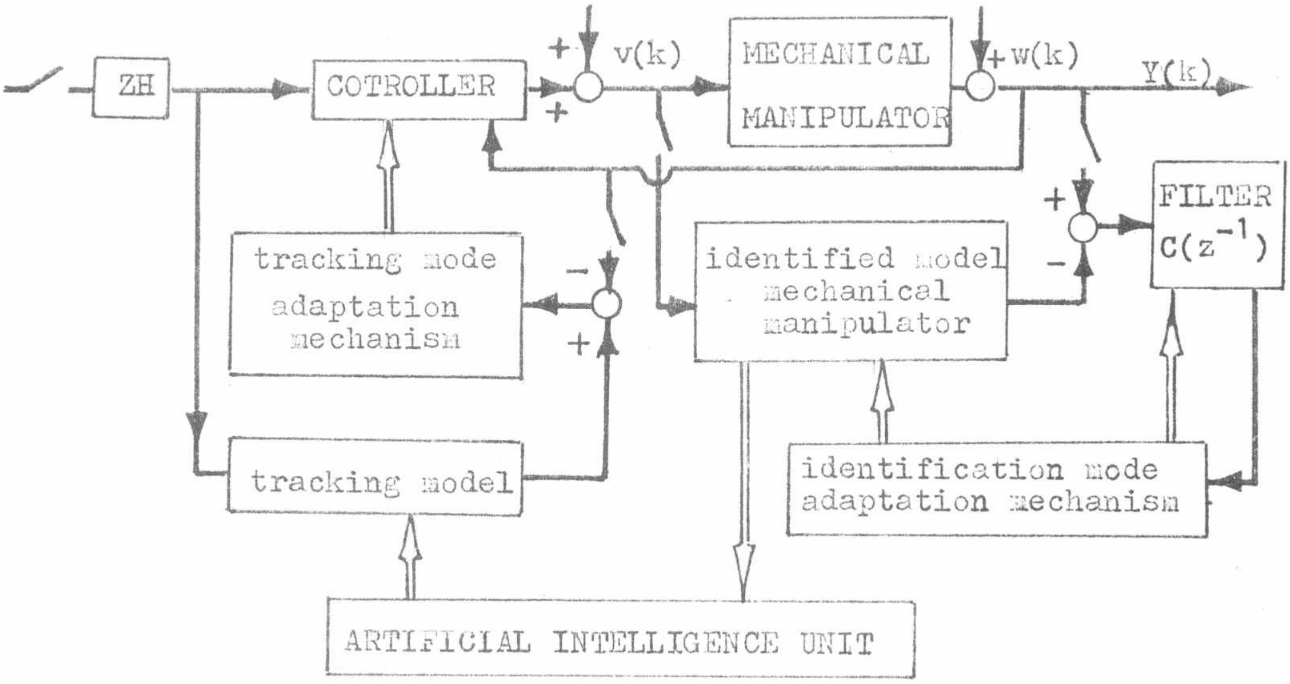

Fig.2. MRAIC block diagram for identification and control of mechanical manipulator.

$$\begin{bmatrix} \dot{x} \\ \ddot{x} \end{bmatrix} = \begin{bmatrix} & I & & 0 \\ -J(\bar{\theta})^{-1} & \bar{K} & -J(\bar{\theta})^{-1} & \bar{C} \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \end{bmatrix} + \begin{bmatrix} 0 \\ J(\bar{\theta})^{-1} \ \bar{E} \end{bmatrix} \delta Q(t) \qquad (14)$$

or $\quad d[X]/dt = [A][X] + [B][U(t)] \qquad (15)$

Taking samples of the signals at discrete intervals of time with a sampling rate satisfies Nyquist sampling theorem, eq. 15 can be written in difference form as:

$$[X(k+1)] = [P][X(k)] + [Q]\ [U(k)] \qquad (16)$$

where $\quad [P] = e^{[A]T}$

and if U(t) is sufficiently smooth such that the staircase approximation does not cause any serious error, then

$$[Q] = [P - I]\ [A]^{-1}\ [B]$$

Using similarity transformation, eq. 16 can always be put into either controllable or observable canonical form which yields an efficient algorithm due to the reduction of the number of the unknowns which need to be identified to the number of unknowns in the transfer function and to those required for unique solution. The observable canonical form for the system is given below, howevere, it should be noted that all of the above transformations are conceptual and do not require physical implementation since the response-excitation relationship is the same for a physical manipulator in specific configuration regardless of the choice of a set of coordinates for mathematical manipulation. Therefore, the response-excitation relationship is given by [23]

$$L(z^{-1})\ Y(k) = M(z^{-1})\ U(k) + \varepsilon(k) + h \tag{17}$$

where $\varepsilon(k)$ are sequence of independent and identically distributed, iid, random variables with zero mean value represent some unmeasurable noise and the parameter h is added to represent the DC component when it exists.

Therefore, our identification strategy is to fit each pair of the observed data into a parametric model(n,m) given by the characteristic polynomials L and M of the order n and m respectively where n and m are often a priori known. This model uses orthogonal decomposition to express the dynamic response Y(k) in terms of three parts. The first part is a dynamic memory capable to express the dependence of the response Y(k) on its n proceeding values Y(k-1), ..,Y(k-n); i.e. autoregressive dependence of order n [AR(n)] given by the polynomial L. The second part includes the dependence on the present and m preceeding excitations U(k),..,U(k-m); i.e. moving average model of order m [MA(m)] given by the polynomial M. The third part or the residuals $\varepsilon(k)$ which is independent on both the excitation and response signals. Thus, the identification algorithm is given by the following recursive adaptation law [23]:

$$\hat{\beta}(k+1) = \hat{\beta}_I(k+1) + \hat{\beta}_p(k+1) \tag{18}$$

$$\hat{\beta}_I(k+1) = \hat{\beta}_I(k) + \frac{G(k)\ \Gamma(k)\ E(k+1)}{1 + \Gamma(k)^T\ [G(k) + R(k)]\ \Gamma(k)} \tag{19}$$

$$\hat{\beta}_p(k+1) = \frac{R(k)\ \Gamma(k)\ E(k+1)}{1 + \Gamma(k)^T\ [G(k) + R(k)]\ \Gamma(k)} \tag{20}$$

Artificial Intelligence Unit

Fig. 2. shows the basic structure of MRAIC system proposed and investigated in the present study. The mechanical manipulator, the identified mathematical model, and the tracking model to enhance the dynamic characteristics of the system are arranged as shown. An artificial intelligent unit is assumed to exist and be able to tune the parameters of the identified model in an optimum manner to obtain the best tracking model, this part is still under-investigation.

Tracking

The task of the adaptive controller shown in Figs.2,3 is to adjust the feedback gains of the manipulator so that the closed-loop performance closely match the set of desired performance embodied in the behaviour of the tracking model. The discrete time schemes of such controller are based on the technique developed by Dubowsky [20]. Assuming that the reference model for each degree of freedom is defined by a linear, second-order, and time-invariant differential equation of the form

$$\ddot{q}_i + 2\,\xi_i\,\omega_i \dot{q}_i + \omega_i^2\, q_i = \omega_i^2\, u_i(t);\quad i = 1,..,N \tag{21}$$

By introducing the state space representation and following same steps as for eq.12 ,eq.20 can be put in difference form.

Performance Indices

Three performances were used to compare the performance of different schemes. They are given by

$$\zeta_1 = \|\hat{\beta}-\beta\|/\|\beta\| \;;\; \zeta_2 = \|\beta-\beta_m\|/\|\beta_m\|;$$
$$\zeta_3 = \sum_{l=0}^{n} \left[\gamma(l)\varepsilon^2(l) + \nu(k)\dot{\varepsilon}^2(l)\right] \qquad (22)$$

SIMULATION AND RESULTS

The recursive algorithm required for identification has been designed by the proper choice of three scalar quantities, they are: the starting adaptation gain g, the rate of decreasing the gain α, and the ratio of the proportional to integral gains μ[24]. The adaptation gain factors γ's and ν's for the tracking algorithm were selected with the aid of the stability analysis approach[20]. The same rate of decreasing the gain α is used for both the identification and tracking algorithms. The advantages of the proposed schemes were demonstrated by simulating a two degrees of freedom manipulator with two inputs and two outputs. The study was carried out on a PDP11/45 computer and the manipulator dynamics were simulated by numerically integrating eq.16 using fourth order Runge-Kutta method.

The performance of the overall system was studied as a function of the sampling time T. This result shown in Fig. 4. It is clear that the performance index ζ_3 is uniformly good for small sampling times. Then the performance begins to degrade rapidly as the sampling period approaches the maximum allowable sampling time of 0.15 seconds predicted by the stability analysis. Fig.5 shows the performance indices of the MRAIC and they indicate that the proposed scheme achieved the exact results in almost 2N steps in the absence of measurement noise. Similar results are obtained by employing measurement data sets which are contaminated with Gaussian noise and have a noise level up to 0.1 , 10% noise-to-signal ratio. These results also reveal that the rate of convergence is almost independent of the starting gain g. Increasing either the rate of decreasing the gain, α , or the ratio of the proportional to integral gains, μ , yield a slower rate of convergence.

CONCLUSIONS

In this paper the problem of manipulator's dynamics has been considered and an explicit discrete-time form, MRAIC scheme based on the positivity concept for stability has been proposed and investigated for its control. The proposed scheme achieves both identification and tracking objectives using a bounded input and maintains high performance over a wide range of system motions and payloads. Such scheme assumes a quasi-linear mathematical model of the systems' dynamics and does not require the knowledge of the environment. This model is extracted from input-response data sets, in the presence of appreciable amount of measurement noise, gathered on-line while the manipulator performs its tasks.

The effectiveness of the proposed scheme and its noise rejection properties have been verified by a computer simulation study using the defined performance indices and the best results are obtained when a time variable gains are used. The memory requirements and computational speed of the proposed scheme are well within the capabilities of microprocessor control computers likely to be in actual use today.

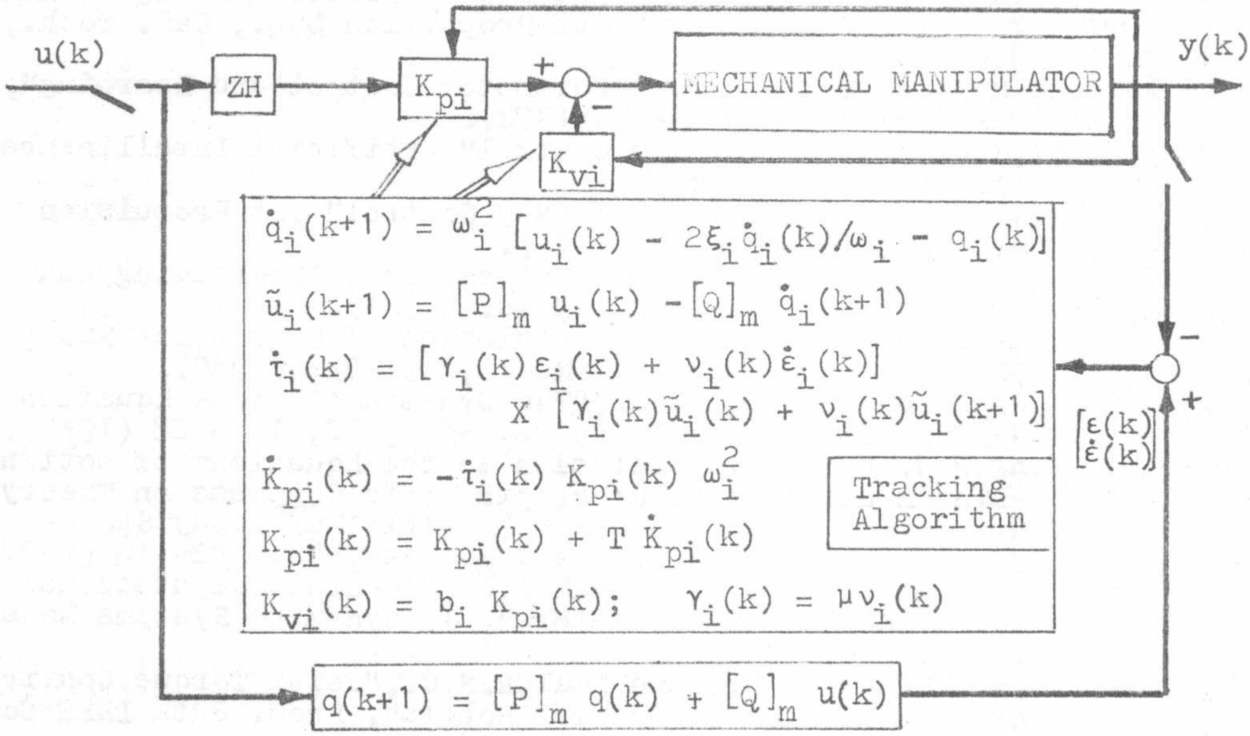

Fig. 3 Adaptive controller algorithm for tracking the manipulator.

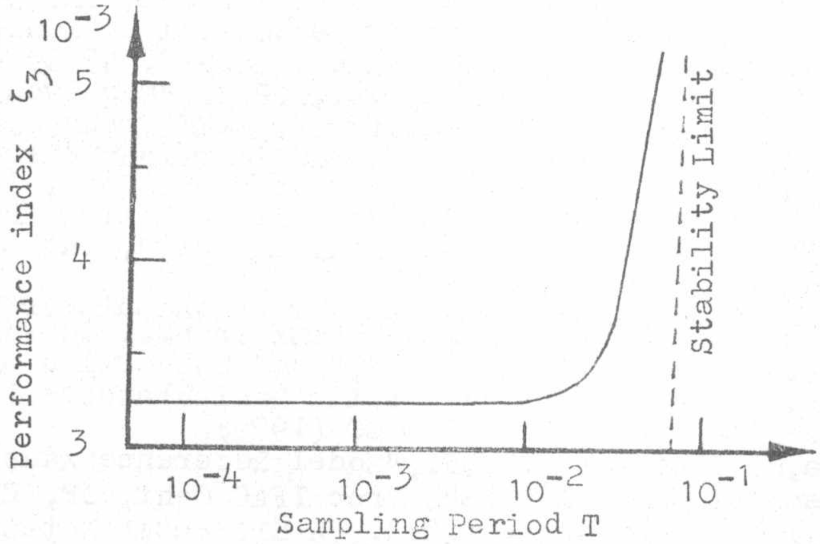

Fig. 4 Effect of the Sampling Period on the proposed scheme.

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NOMENCLATURE

C	NXN viscous friction matrix.
D	NX1 vector defining Coriolis and centrifugal forces.
E	NXN loading matrix.
E(k+1)	error of estimation measured at time (k+1)T.
G(k)	integral gain matrix at time kT.
$G(\theta)$	NX1 vector defining the gravity forces.
$J(\theta)$	NXN inertia tensor.
Q	NX1 vector of input generalized forces associated with generalized coordinates.
R(k)	proportional gain matrix at time kT.
v(k)	excitation measurement noise.
w(k)	response measurement noise.
β^T	manipulator system parameter vector $= \lfloor -p_1 \;..\; -p_n \; q_0 \;..\; q_m \rfloor$
$\hat{\beta}^T(k)$	esimated parameter vector at time kT.
β_m	tracking model parameter vector.
$\Gamma(k-1)$	observation vector at time (k-1)T $= \lfloor y(k-1) \;..\; y(k-n) \; u(k) \;..\; u(k-m) \rfloor$
ξ_i	damping ratio of the i-th mode of the tracking model.
ω	angular velocity.
ω_i	natural frequency of the i-th mode.

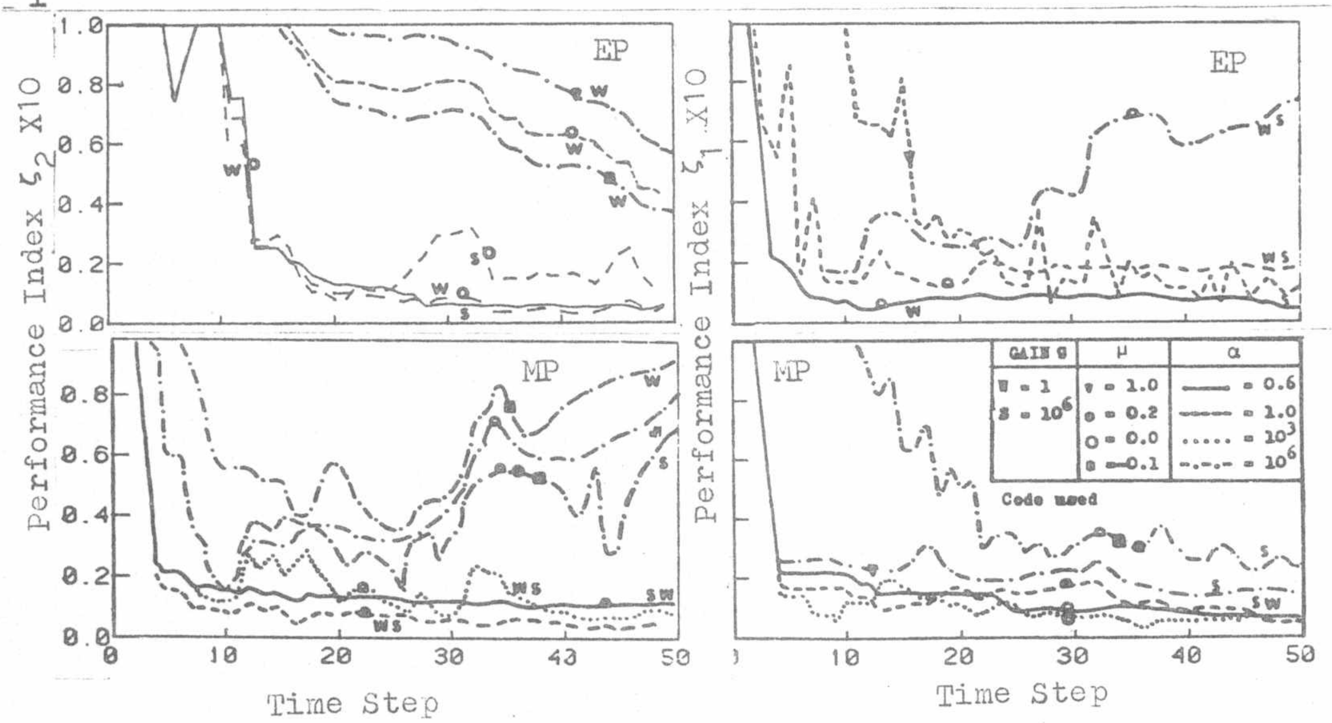

Fig. 5 Typical performance indices obtained for proposed schemes.