



ADAPTIVE CONTROL OF LONGITUDINAL FLIGHT  
OF AIRCRAFT SYSTEM

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ABSTRACT

This paper presents two proposals for adaptive control of supersonic aircraft. In the first proposal, the aircraft complete transfer function is controlled to behave according to a second order chosen model, while in the second proposal the inner loop of the aircraft transfer function is controlled to behave according to a first order model.

A modified gradient technique for a model reference adaptive control system with less order model is employed in the two proposals. Analog computer simulation results are presented and discussed.

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## 1. INTRODUCTION

In the historical development of aircraft systems, controllers were introduced to automatize straight and level flight, to enable flying by instrument without ground visibility and to simply augment the stability properties, and thus improve the handling qualities of the aircraft .

Part of the historical evolution in flight systems is that the region in which the aircraft can operate has been increased significantly, e.g. to high altitudes, high mach numbers, and even to major changes in the geometry of the aircraft, These significant changes in the aircraft dynamics had at least partially to be overcome by means of the control system and, in particular, the controller had to operate satisfactorily for a drastically changing aircraft. Extensive theoretical and experimental studies have been devoted to obtain a fairly accurate model of the dynamic behaviour of the aircraft for all possible flight conditions [1], [2]. Using this knowledge of the possible parameters variations a control system should be designed in a way suitably adopted to the flight condition . Recent progress in the theory of adaptive control makes it applicable and well suited to solve flight control problems in an efficient way.

The available adaptive techniques are of the following type: local parametric optimization theory [3], Lyapunov functions technique [4], and hyperstability and positive concepts [5]. Most of these techniques avoids the identification problem, but stresses that the order of the reference model must be the same as that of the nonadaptive system.

In an attempt to give a perspective for the possible application of adaptive control to aircraft systems, this paper presents, two proposals of adaptive control of a supersonic aircraft. The motivation of this work is based on one of the author earlier studies [6] in the solution of parameter tracking problems. In [6]. A modified gradient method was proposed to solve the problem of adaptive model reference control system when the model reference is less order than the adjustable system. In this paper the method was used to control the longitudinal motion of a supersonic aircraft when the reference model was chosen less order than the aircraft system equation. The organization of the rest of the paper is as follows: in section 2 a modified gradient method for the model reference adaptive control is presented. The system equations for the longitudinal motion of a hypothetical supersonic aircraft is given in section 3. The first proposal where the aircraft complete transfer function is controlled to behave according to a second order chosen reference model is given in section 4. In section 5, the second proposal, where the inner loop of the transfer function of the same aircraft is controlled to behave according to a first order reference model and conclusion appear in section 6.

## 2. PROPOSED ADAPTATION TECHNIQUE

Consider the following single input/single output model reference adaptive system shown in figure (1) and represented by:

a) The adjustable system

$$\sum_{i=0}^n \alpha_i \frac{d^i}{dt^i} c(t) = \sum_{i=0}^{\nu} B_i \frac{d^i}{dt^i} r(t) \quad ; n \geq \nu \quad (1)$$

$$\alpha_i = g_i + h_i \quad ; \quad i = 0, 1, \dots, n \quad (2)$$

Where  $r(t)$  is the input,  $c(t)$  is the output,  $\alpha_i$  and  $B_i$  are functions of the changing coefficients within the operation environments of the system,  $g_i$  are the variable parameters, and  $h_i$  are the controllable parameters.

b) The reference model

$$\sum_{i=0}^m a_i \frac{d^i}{dt^i} y(t) = \sum_{i=0}^{\mu} b_i \frac{d^i}{dt^i} r(t) \quad m \geq \mu \quad (3)$$

Where  $y(t)$  is the output of the reference model,  $a_i$  and  $b_i$  are constant parameters of the system, and  $(m+1)$  equal to the number of controllable parameters of the adjustable system.

c) The generalized output error

$$e(t) = c(t) - y(t) \quad (4)$$

The parameters  $g_i$  are assumed to vary over extreme ranges within the operating environment of the nonadaptive system. Assuming that there exists a values of  $h_i$  for which the system will behave

like the chosen reference model. The objective of the adaptation mechanism is to provide these values. Its input information is the generalized error  $e(t)$  and its output will be the instantaneous values of  $h_i$ . The problem can be formulated as: find the

adaptive mechanism equations which, if no limitations are placed on the values of  $h_i$ , then regardless of what values  $g$  take, the

outputs of both systems will be approximately identical whenever a quadratic function of the generalized error will be minimized. Considering the following error function:

$$f(e) = \sum_{i=0}^m \frac{1}{2} \left[ q_i \frac{d^i}{dt^i} e(t) \right]^2 \quad (5)$$

where  $q_i$  are constant factors, Equation (5) assumed the availab-

ility of the derivatives of the error signal and it depends indirectly on the differences between the parameters of the adjust-

ing system and the reference model which can be defined as:

$$\delta_i = \alpha_i - a_i \quad (6)$$

If the parameters  $g_i$  vary slowly as compared to the basic time constants of the adjustable system and the reference model and the adaptation mechanism is designed to adjust the parameters  $h_i$  at a rate which is much greater than the rate of variation of  $g_i$ , the gradient optimization technique can be used and leads to the basic adaptation rule:

$$\frac{d}{dt} \alpha_i = -K_i \frac{\partial}{\partial \alpha_i} f(e) \quad (7)$$

where  $K_i$  are arbitrary positive constants to be chosen by the designer and depend on the particular system considered.

If the development were to be based upon equation (7), the resulting design would require explicit knowledge of  $\alpha_i$  and consequently  $g_i$ . Now suppose that  $\alpha_i$  are treated as constant, and

$a_i$  are to be adjusted so as to cause the differences  $\delta_i$  to approach the same values then the variations in  $a_i$  becomes

$$\frac{d}{dt} a_i = -K_i \frac{\partial}{\partial a_i} f(e) \quad (8)$$

The objective is not to change  $a_i$ , however, to change  $\alpha_i$ . Since

from equation (6) the same changes in  $\delta_i$  can be obtained by subtracting  $\Delta a_i$  from  $\alpha_i$ , rather than adding them to  $a_i$ . The resulting

instantaneous rates of adjusting  $\alpha_i$  can be written as:

$$\frac{d}{dt} \alpha_i = K_{i\alpha} \frac{\partial}{\partial \alpha_i} f(e) \quad (9)$$

When equation (5) was substituted into equation (9), and the indicated partial derivative was carried out equation (9) becomes:

$$\frac{d}{dt} \alpha_i = -K_{i\alpha} \left[ \sum_{j=0}^m q_j \frac{d^j e}{dt^j} \right] \left[ \sum_{j=0}^m q_j \frac{\partial}{\partial \alpha_i} \frac{d^j}{dt^j} y(t) \right] \quad (10)$$

Introducing the notation:

$$u_{i\alpha} = \frac{\partial y}{\partial \alpha_i} ; i = 0, 1, \dots, m \quad (11)$$

Again, assuming that the parameters  $\delta_i$  are changing with slow

rates, the order of differentiation in the right-hand side of equation (10) can be interchanged, yielding to

$$\frac{d}{dt} \alpha_i = -K_{i\alpha} \left[ \sum_{j=0}^m q_j \frac{d^j e}{dt^j} \right] \left[ \sum_{j=0}^m q_j \frac{d^j}{dt^j} u_{j\alpha} \right] \quad (12)$$

Since it is assumed that the adaptation mechanism will be designed so as to adjust the controllable parameters  $h_i$  in a rate greater than the changes in  $g_i$ . The rates of  $g_i$  can be neglected

and from equation (12) the rates of  $h_i$  can be written as:

$$\frac{dh_i}{dt} \approx \frac{d\alpha_i}{dt} = K_{i\alpha} \left[ \sum_{j=0}^m q_j \frac{d^j e}{dt^j} \right] \left[ \sum_{j=0}^m q_j \frac{d^j}{dt^j} u_{j\alpha} \right] \quad (13)$$

The only unknown quantities in equations (13) are  $u_{j\alpha}$ . Consid-

ering the differential equation of the reference model (3), and taken the partial derivative of both sides with respect to the parameters  $a_i$ , treating the derivatives of  $y$  with respect to time

as trivial partial derivative, and interchanging the order of differentiation and employing the notation introduced in equation (11), the following set of equations for  $u_{j\alpha}$  can be written as:

$$\sum_{i=0}^m a_i \frac{d^i u_{j\alpha}}{dt^i} = - \frac{d^j y}{dt^j} \quad , \quad j=0, \dots, m \quad (14)$$

Equation (14) represent a set of differential equations with available forcing functions and their solutions provides the values of  $u$  for equations (13).

Similarly for the case of the parameters  $B_i$ , the following adaptive mechanism equations are required.

$$\frac{dB_i}{dt} = -K_{iB} \left[ \sum_{j=0}^m q_j \frac{d^j e}{dt^j} \right] \left[ \sum_{j=0}^m q_j \frac{d^j}{dt^j} u_{jB} \right] \quad i=0, \dots, \nu \quad (15)$$

$$\sum_{i=0}^m a_i \frac{d^i u_{jB}}{dt^i} = \frac{d^j}{dt^j} r(t) \quad ; \quad j=0, 1, \dots, \nu \quad (16)$$

$$\text{where } u_{jB} = \frac{\partial y}{\partial b_j} \quad i=0, 1, \dots, \nu \quad (17)$$

The four sets of equations (13), (14), (15), and (16) represent the adaptive mechanism algorithm required for adaptation of the adjusting system to follow the reference model. The complete set of differential equations which describes the adaptive system operation can be written as:

$$\sum_{i=0}^n \alpha_i \frac{d^i c(t)}{dt^i} = \sum_{i=0}^{\nu} B_i \frac{d^i}{dt^i} r(t)$$

$$\sum_{i=0}^m a_i \frac{d^i}{dt^i} y(t) = \sum_{i=0}^{\mu} b_i \frac{d^i}{dt^i} r(t)$$

$$\sum_{i=0}^m a_i \frac{d^i}{dt^i} u_{j\alpha} = - \frac{d^j}{dt^j} y(t) \quad ; \quad j=0, \dots, m$$



$$\sum_{i=0}^m a_i \frac{d^i}{dt^i} u_{jB} = \frac{d^j}{dt^j} r(t); \quad j=0, 1, \dots, \nu$$

$$\frac{dh_i}{dt} = -K_{i\alpha} \left[ \sum_{j=0}^m q_j \frac{d^j e(t)}{dt^j} \right] \left[ \sum_{j=0}^m q_j \frac{d^j}{dt^j} u_{i\alpha} \right]; \quad i=0, \dots, m$$

$$\frac{d^B i}{dt} = -K_{iB} \left[ \sum_{j=0}^m q_j \frac{d^j e(t)}{dt^j} \right] \left[ \sum_{j=0}^m q_j \frac{d^j}{dt^j} u_{iB} \right]; \quad i=0, \dots, \nu \quad (18)$$

## 2 - MATHEMATICAL MODEL OF AIRCRAFT SYSTEM

It is well known that the short period mode can be used to represent the longitudinal dynamics of supersonic aircraft with sufficient accuracy due to its simplicity [1]. The fundamental transfer functions that describe the dynamical motions of an aircraft in the longitudinal-vertical plane of motion are given by:

$$\frac{\alpha}{\delta_e} = \frac{n_{35}}{p^2 + 2d\omega_0 p + \omega_0^2} \quad (19)$$

$$\frac{\theta}{\delta_e} = \frac{n_{35}(p + n_{22})}{p(p^2 + 2d\omega_0 p + \omega_0^2)} \quad (20)$$

$$\omega_0 = \sqrt{n_{22}n_{23} + n_{32}}, \quad \& \quad d = \frac{n_{22} + 3n_{33} + n_{30}}{2\omega_0} \quad (21)$$

Where:

$\alpha$  is the change in angle of attack  
 $\theta$  is the change in pitch angle  
 $\delta_e$  is the change in elevator deflection  
 $n_{ij}$  are the coefficients for the short period mode

For the purpose of illustration, consider a hypothetical aircraft with the following basic parameters:

Mass of aircraft	$m = 8100 \text{ kg}$
Wing area	$s = 32.4 \text{ m}^2$
Wing span	$b = 9.52 \text{ m}$
length of mean aerodynamic chord	$c = 3.4 \text{ m}$

For such aircraft, the numerical values of the coefficients for the short period mode  $n_{ij}$  were calculated for initial horizontal flight also the damping factor  $d$  and the natural frequency  $\omega_0$ ,

the calculations were performed for different altitudes and given in table (1). For the supersonic aircraft described abo-

ve a typical classical autopilot can be represented by the following equation [7];[8]:

$$\delta e = \frac{K_1}{p(Tp+1)} [K_2 (\theta - \theta_r) + (\varepsilon + \eta p) p \theta] \quad (22)$$

Where:

- $\theta_r$  the input reference signal
- $\eta$  the accelerometer gain
- $\varepsilon$  the rate gyro gain
- $T$  time constant of servomechanism
- $K_1$  the amplifier gain
- $K_2$  the vertical gyro gain.

The block diagram of the complete aircraft system is given in figure (2). From which the transfer function representing the total behaviour of the system can be written as:

$$\frac{\theta(p)}{\theta_r(p)} = \frac{-n_{35} K_1 K_2 (p + n_{22})}{a_0 p^5 + a_1 p^4 + a_2 p^3 + a_3 p^2 + a_4 p + a_5} \quad (23)$$

Where  $a_0 = T$

$$a_1 = 1 + T (n_{22} + n_{33} + n_{30})$$

$$a_2 = n_{22} + n_{33} + n_{30} - n_{35} K_1 \eta + T (n_{32} + n_{22} n_{33})$$

$$a_3 = n_{22} n_{33} + n_{32} - n_{22} n_{35} K_1 \eta - n_{35} K_1 \varepsilon$$

$$a_4 = -n_{22} n_{35} K_1 \varepsilon - n_{35} K_1 K_2$$

$$a_5 = -n_{35} n_{22} K_1 K_2$$

It is clear that the behaviour of the aircraft system will vary according to the flight altitude because of the variation of the coefficients  $n_{ij}$ . For this reason the need for adaptive

control for such aircraft is required. Sensitivity analysis and analog computer simulation for the above aircraft shows that the aircraft system parameters which have the main effect on the transient response are the accelerometer gain  $\eta$  and the rate gyro gain  $\varepsilon$ . In the following sections two proposals for the adaptation of the aircraft are given.

#### 4- FIRST ADAPTIVE PROPOSAL FOR THE AIRCRAFT SYSTEM:

The first proposal for the adaptation of the aircraft system



consist of controlling the feedback parameters  $\eta$  and  $\xi$  by the adaptive mechanism so that the aircraft system response behave according to a second order reference model. The block diagram of the proposed adaptation scheme is given in Figure (3).

According to the proposed principle of adaptation given in section 2 let the reference model and the error function equations be:

$$\alpha_0 \frac{d^2}{dt^2} y(t) + \alpha_1 \frac{d}{dt} y(t) + \alpha_2 y(t) = \theta_r(t) \quad (24)$$

$$f(e) = \frac{1}{2} [q_0 e(t) + q_1 \dot{e}(t) + q_2 \ddot{e}(t)]^2 \quad (25)$$

The equations representing the dynamics of the adaptation mechanism are:

$$\begin{aligned} \dot{a}_4 &= -c_1 [q_0 e + q_1 \dot{e} + q_2 \ddot{e}] [q_0 u_1 + q_1 \dot{u}_1 + q_2 \ddot{u}_1] \\ \dot{a}_3 &= -c_2 [q_0 e + q_1 \dot{e} + q_2 \ddot{e}] [q_0 u_2 + q_1 \dot{u}_2 + q_2 \ddot{u}_2] \end{aligned} \quad (26)$$

and

$$\begin{aligned} \alpha_0 \ddot{u}_1 + \alpha_1 \dot{u}_1 + \alpha_2 u_1 &= -\dot{y}(t) \\ \alpha_0 \ddot{u}_2 + \alpha_1 \dot{u}_2 + \alpha_2 u_2 &= -\ddot{y}(t) \end{aligned} \quad (27)$$

From Equations (23), differentiating  $a_4$  and  $a_3$  with respect to time, and considering that the time derivatives of the aircraft parameters  $n_{ij}$  are neglected, the time rates at which the accelerometer gain  $\eta$  and the rate gyro gain  $\xi$  will be

$$\dot{\xi} = - \frac{\dot{a}_4}{n_{22} n_{35} K_1} \quad (28)$$

$$\dot{\eta} = - \frac{\dot{a}_3}{n_{22} n_{35} K_1} + \frac{\dot{a}_4}{n_{22}^2 n_{35} K_1} \quad (29)$$

The rate of changing the accelerometer  $\eta$  in equation (28) depends on both  $\dot{a}_4$  and  $\dot{a}_3$ , but to secure the stability of the

adaptive process, it will be considered that  $\dot{\eta}$  will be controlled only by the changes in  $\dot{a}_3$ , also the coefficient  $[- \frac{1}{n_{22} n_{35} K_1}]$

will be absorbed into the coefficients  $c_1$  and  $c_2$ .

The complete adaptive system were simulated on analog computer with the reference model equation as:

$$0.0625 \ddot{y} + 0.35 \dot{y} + y = \theta_r \quad (30)$$

The numerical values of the autopilot's coefficients  $K_1$  and  $K_2$



and the time constant of the servosystem were chosen, as to get the best transient response, to be:

$$K_1 = 20 \quad \text{and} \quad K_2 = 16 \quad \text{and} \quad T = 0.05 \text{ sec.} \quad (31)$$

The best function of the adaptive mechanism, where obtained for the following values of the proportionality factors:

$$\begin{aligned} q_0 &= q_1 = q_2 = 1 \\ c_1 &= 19.81 \quad c_2 = 0.589 \end{aligned} \quad (32)$$

The aircraft autopilot system was excited by input pulses of amplitude  $0.09^\circ$  and with 5 seconds duration in one polarity, then the convergence of autopilot's coefficients  $\eta$  and  $\xi$  to an optimum value, from extreme initial values, was checked. In Figure (4), the aircraft was set to the parameters of altitude 5 km. It was found that the parameters  $\eta$  and  $\xi$  converged nearly to the same optimum value after 5 changes of the input level, from extremely large initial values. Figure (5) represents the same trajectories of  $\eta$  and  $\xi$  when the aircraft parameters set to the values of altitude 25 km. The system was simulated for a step input, for altitudes (5, 10, 15, 20, and 25 km), with the adaptive autopilot parameters initially are  $\eta_0 = 1.12$  and  $\xi_0 = 5.7$ .

Figure (6) represent the responses of the aircraft system  $\theta$ ; rate of pitch  $\dot{\theta}$ , the changes in the angle of attack  $\alpha$ , and the generalized error  $e$ .

## 5. SECOND ADAPTIVE PROPOSAL.

In this proposal the same aircraft controlled with the same autopilot is considered. The difference is that the adaptive mechanism will be designed to control the two parameters and as for the inner loop, to behave according to a first order reference model. The block diagram of the adaptation scheme is shown in figure (7).

The transfer function of the inner loop was derived to be:

$$\frac{\dot{\theta}(p)}{\mu(p)} = \frac{-n_{35}K_1(p+n_{22})}{b_0p^4 + b_1p^3 + b_2p^2 + b_3p + b_4} \quad (33)$$

Where

$$b_0 = T$$

$$b_1 = 1 + T(n_{22} + n_{33} + n_{30})$$

$$b_2 = n_{22} + n_{33} + n_{30} - n_{35}K_1\eta + T(n_{32} + n_{22}n_{33})$$

$$b_3 = n_{22}n_{33} + n_{32} - n_{22}n_{35}K_1\eta - n_{35}K_1\xi$$

$$b_4 = -n_{22}n_{35}K_1\xi$$

The reference model and the error function was chosen to be:

$$a_1 \dot{z} + a_0 z = \mu \quad (34)$$

$$f(e) = \frac{1}{2} [q_0 e + q_1 \dot{e}]^2 \quad (35)$$

The equations representing the dynamics of the adaptive mechanism in this case are:

$$\begin{aligned} \dot{b}_4 &= -c_0 (q_0 e + q_1 \dot{e}) (q_0 u_0 + q_1 \dot{u}_0) \\ \dot{b}_3 &= -c_1 (q_0 e + q_1 \dot{e}) (q_0 u_1 + q_1 \dot{u}_1) \end{aligned} \quad (36)$$

and

$$\begin{aligned} a_1 \dot{u}_0 + a_0 u_0 &= -z \\ a_1 \dot{u}_1 + a_0 u_1 &= -\dot{z} \end{aligned} \quad (37)$$

From equations (33), differentiating  $b_4$  and  $b_3$  with respect to time, considering the same assumptions stated before, the time rates at which the accelerometer gain  $\eta$  and the rate gyro gain  $\xi$  will be:

$$\dot{\xi} = -\dot{b}_4 / n_{22} n_{35} K_1 \quad (38)$$

$$\dot{\eta} = -\dot{b}_3 / n_{22} n_{35} K_1 + \dot{b}_4 / n_{22}^2 n_{35} K_1 \quad (39)$$

Taking into account the same assumptions as in the first proposal the rate of change of the accelerometer gain  $\dot{\eta}$  will depend only on  $\dot{b}_3$ .

The complete adaptive system was simulated on the analog computer with the reference model equation as:

$$0.5 \dot{z} + z = \mu \quad (40)$$

The numerical values of the autopilot's coefficients  $K_1$  and  $K_2$  and the time constant of the servosystem were chosen to be:

$$K_1 = 20, \quad K_2 = 2.2 \text{ and } T = 0.05 \text{ sec.} \quad (41)$$

The adaptive mechanism parameters was taken as:

$$\begin{aligned} c_0 &= 31.25 & c_1 &= 3.9 \\ q_0 &= q_1 & &= 1 \end{aligned} \quad (42)$$

The aircraft autopilot system was excited by input pulses of

amplitude  $0.15^\circ$  and 5 seconds duration in one polarity, then the convergence of the same parameters  $\eta$  and  $\xi$  was checked. In figure (8), the aircraft parameters were set to the values of altitude 5 km, the parameters  $\eta$  and  $\xi$  converged nearly to the optimum values after 5 changes of the input level from extremely large initial values. The system were simulated with aircraft parameters at altitude 25 km and the convergence result as shown in Figure (9). The system was simulated for a step input for altitudes 5, 10, 15, 20, 25 km, with the adaptive autopilot parameters initially are  $\eta_0 = 0.51$  and  $\xi_0 = 0.94$ . Figure (10) represents the responses of the aircraft system  $\theta$ , the rate of pitch  $\dot{\theta}$ , the changes in angle of attack  $\alpha$  and the generalized error  $e$ .

#### 6. CONCLUSION:

The concept of a proposed gradient method is used to seek out solution of the problem of adaptive control for aircraft system where the reference model used is less order than the aircraft system. Two proposals for adaptation of the longitudinal flight of aircraft system were presented. In both cases, the suggested technique proved to secure the needs of aircraft stability and control. The adaptive mechanism is quite reasonable from the stand point of actual physical hardware. In the second proposal the control of the inner loop of the aircraft system leads to a considerable simplification of the mechanization of the adaptive mechanism. Analog computer simulation for the two cases shows that the stability of the adaptive system depend on the choice of the adaptive mechanism parameters.

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#### FIGURE CAPTIONS

- Figure (1) Proposed adaptation scheme
- Figure (2) Block diagram of the aircraft autopilot system
- Figure (3) First adaptive proposal scheme for the aircraft system
- Figure (4) Convergence of the aircraft parameters at altitude of 5 Km
- Figure (5) Convergence of the aircraft parameters at altitude of 25 Km
- Figure (6) Responses of the aircraft adaptive system at different altitudes for the first adaptation proposal
- Figure (7) Second adaptive proposal scheme for the aircraft system
- Figure (8) Convergence of the aircraft parameters at altitude of 5 Km for the second proposal
- Figure (9) Convergence of the aircraft parameters at altitude of 25 Km for the second proposal
- Figure (10) Responses of the aircraft adaptive system at different altitudes for the second adaptation proposal
- Table (1) The aircraft short period mode coefficients  $n_{ij}$  for different altitudes.

H(km)	5	10	15	20	25
$n_{30}$	0.68	0.507	0.28	0.1175	0.047
$n_{22}$	1.29	1.06	0.672	0.34	0.168
$n_{32}$	4.85	24.7	25.3	18.6	13.3
$n_{33}$	1.5	1.23	0.782	0.391	0.184
$n_{35}$	-15.9	-17.0	-13.15	-7.17	-4.67
d	0.665	0.274	0.169	0.098	0.0548
$\omega_0$	2.61	5.1	5.08	4.32	3.642

Table (1)

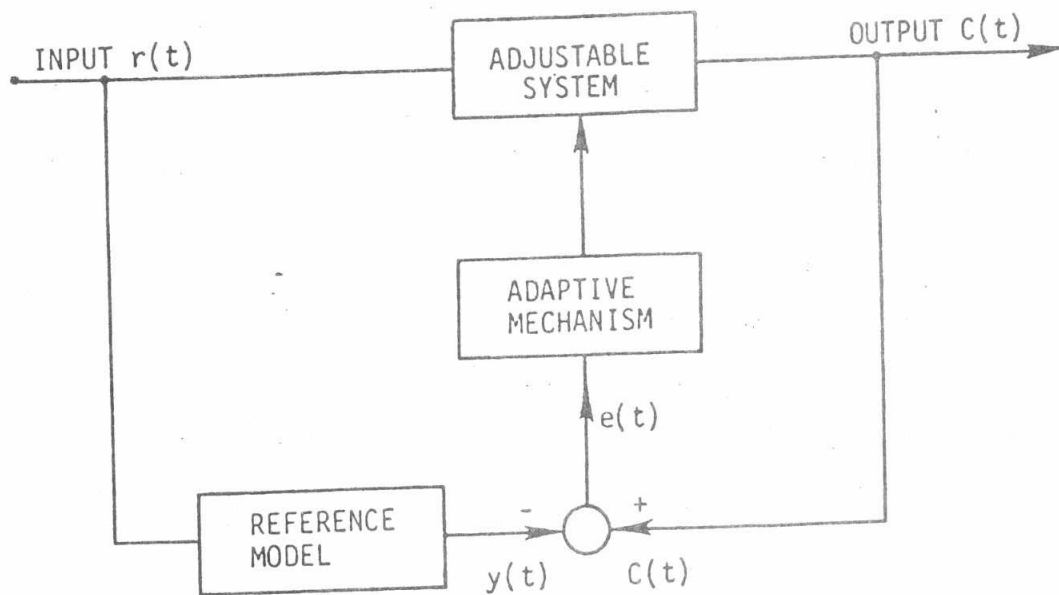


Figure 1.

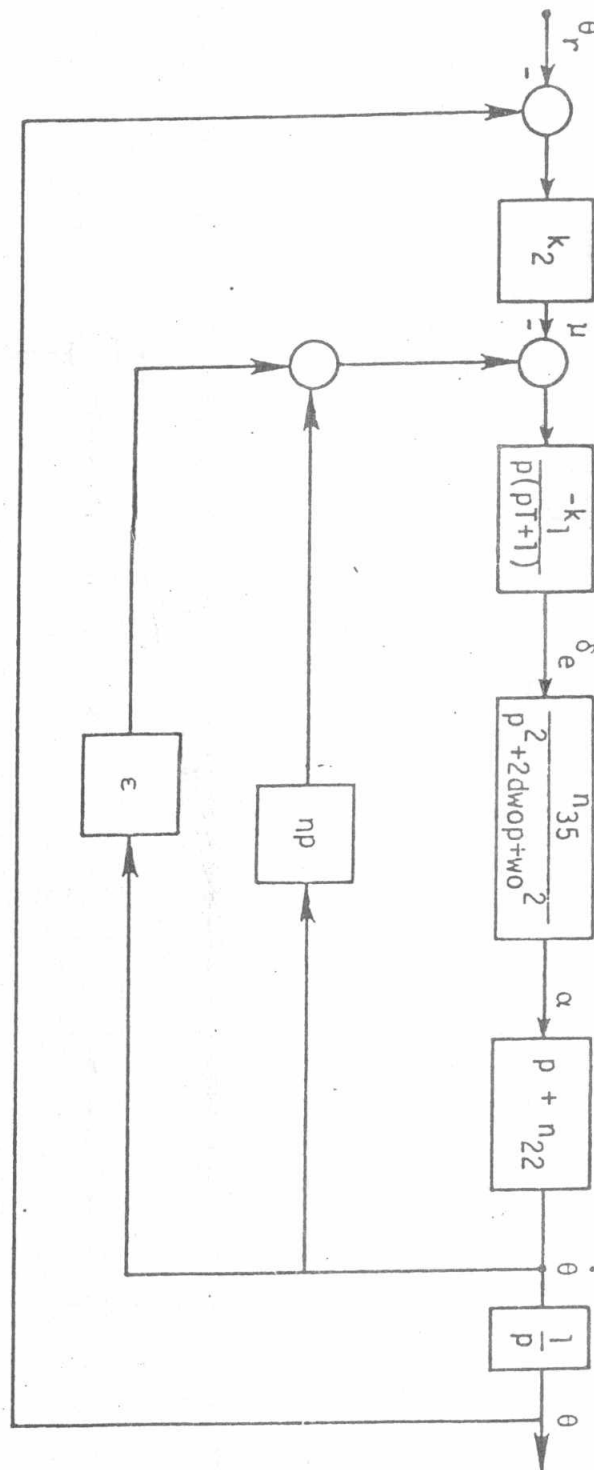


Figure 2.





$\eta_0 = 0.27$   
 $\epsilon_0 = 1.71$   
 $\eta_5 = 1.084$   
 $\epsilon_5 = 5.77$

$\eta_0 = 0.438$   
 $\epsilon_0 = 7.88$   
 $\eta_4 = 1.074$   
 $\epsilon_4 = 5.7$

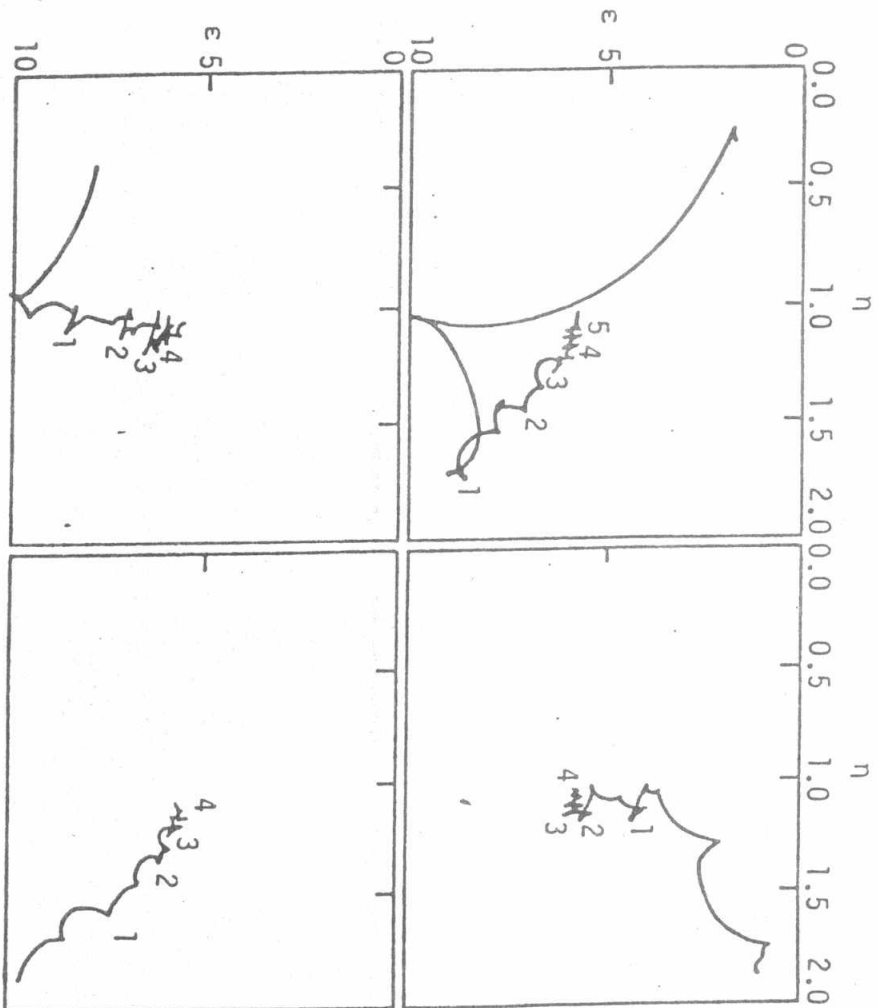


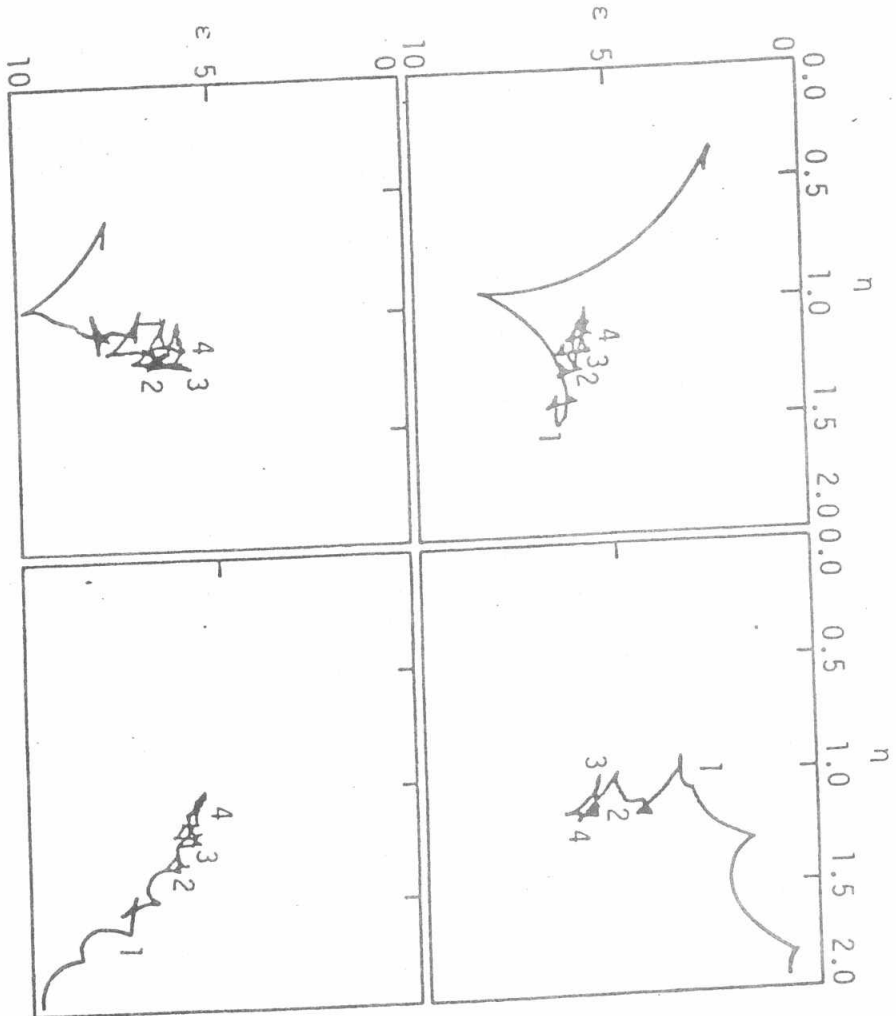
Figure 4.

$\eta_0 = 1.872$   
 $\epsilon_0 = 0.5$   
 $\eta_4 = 1.074$   
 $\epsilon_4 = 5.67$

$\eta_0 = 1.874$   
 $\epsilon_0 = 9.73$   
 $\eta_4 = 1.094$   
 $\epsilon_4 = 5.7$

$$\begin{aligned}\eta_0 &= 0.436 \\ \epsilon_0 &= 2.34 \\ \eta_5 &= 1.138 \\ \epsilon_4 &= 5.78\end{aligned}$$

$$\begin{aligned}\eta_0 &= 0.67 \\ \epsilon_0 &= 7.7 \\ \eta_4 &= 1.14 \\ \epsilon_4 &= 5.81\end{aligned}$$



$$\begin{aligned}\eta_0 &= 1.984 \\ \epsilon_0 &= 0.73 \\ \eta_4 &= 1.128 \\ \epsilon_4 &= 5.78\end{aligned}$$

$$\begin{aligned}\eta_0 &= 1.96 \\ \epsilon_0 &= 9.8 \\ \eta_4 &= 1.152 \\ \epsilon_4 &= 5.77\end{aligned}$$

Figure 5.

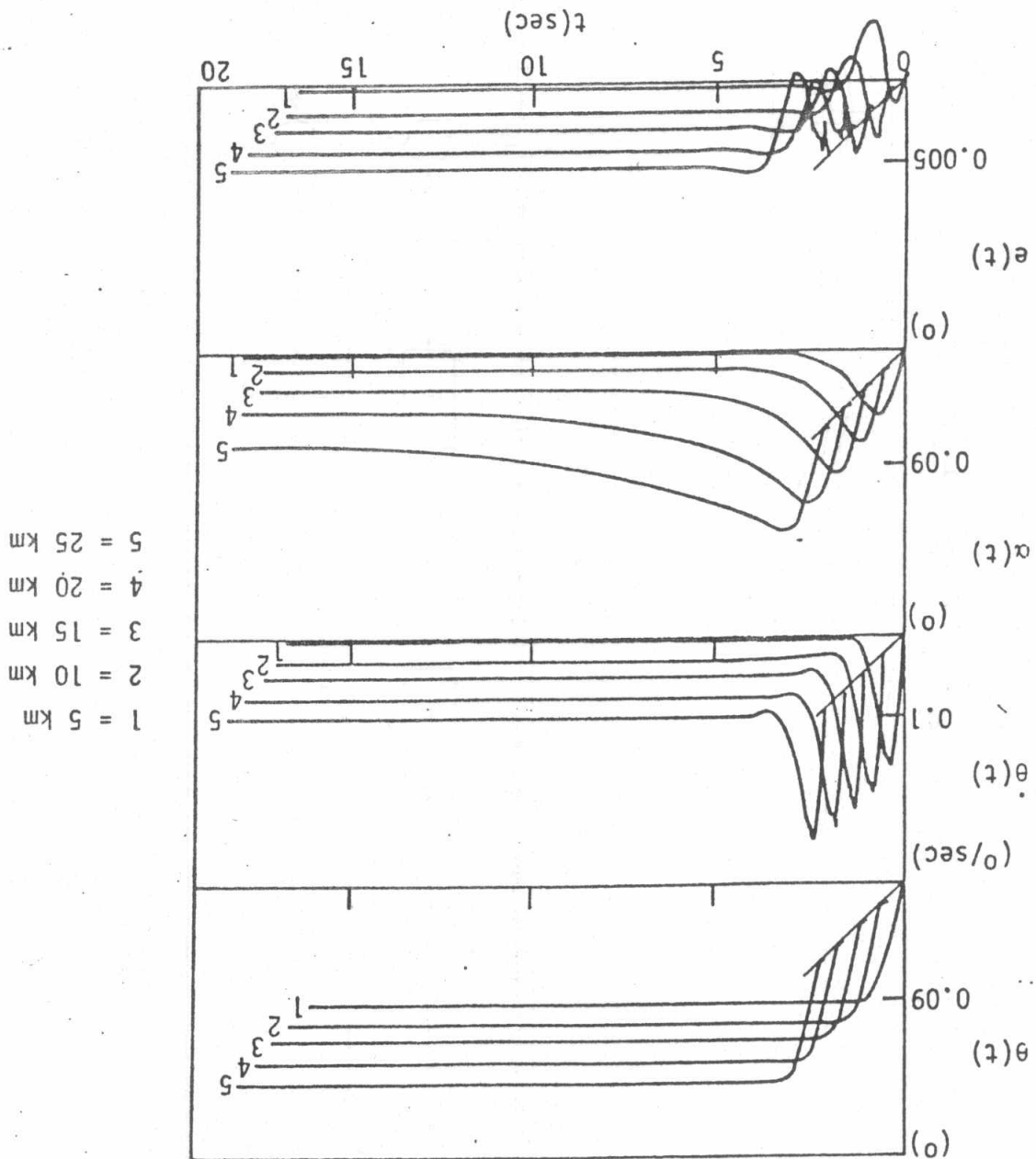


Figure 6.

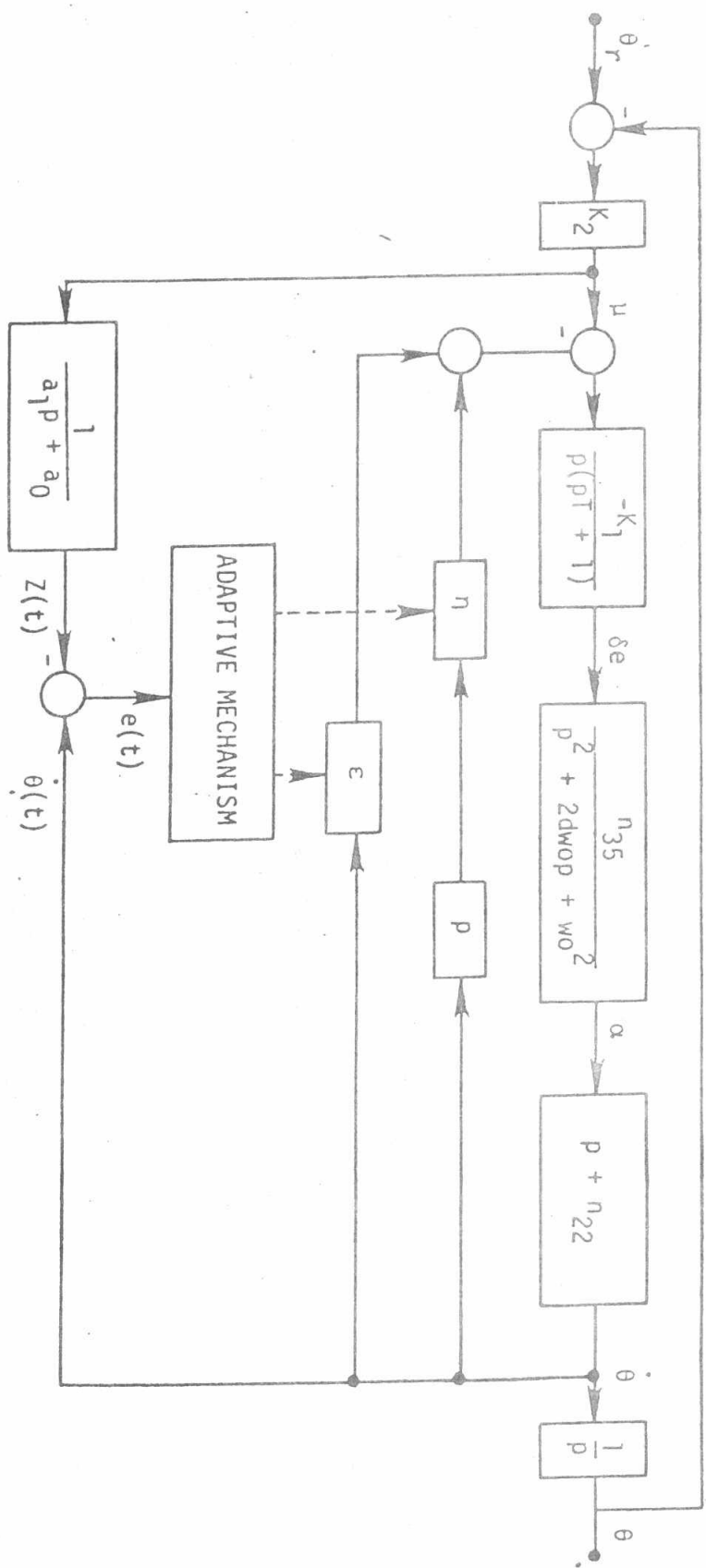


Figure 7.

$$\begin{aligned}\eta_0 &= 0.0 \\ \epsilon_0 &= 0.0 \\ \eta_3 &= 0.52 \\ \epsilon_3 &= 1.015\end{aligned}$$

$$\begin{aligned}\eta_0 &= 0.2 \\ \epsilon_0 &= 4.5 \\ \eta_3 &= 0.516 \\ \epsilon_3 &= 1.02\end{aligned}$$

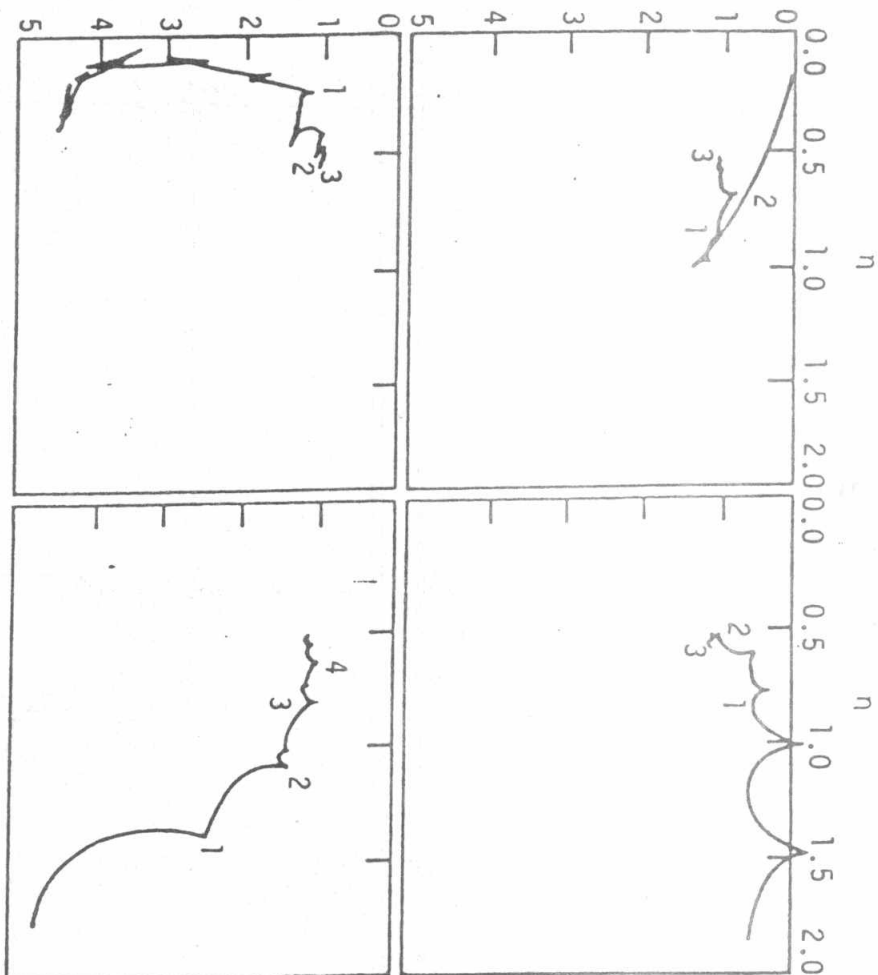


Figure 8.

$$\begin{aligned}\eta_0 &= 1.8 \\ \epsilon_0 &= 0.5 \\ \eta_3 &= 0.546 \\ \epsilon_3 &= 1.0\end{aligned}$$

$$\begin{aligned}\eta_0 &= 1.8 \\ \epsilon_0 &= 4.5 \\ \eta_5 &= 0.528 \\ \epsilon_5 &= 1.02\end{aligned}$$

$$\begin{aligned}\eta_0 &= 0.0 \\ \epsilon_0 &= 0.0 \\ \eta_3 &= 0.526 \\ \epsilon_3 &= 0.93\end{aligned}$$

$$\begin{aligned}\eta_0 &= 0.4 \\ \epsilon_0 &= 4.5 \\ \eta_4 &= 0.52 \\ \epsilon_4 &= 0.935\end{aligned}$$

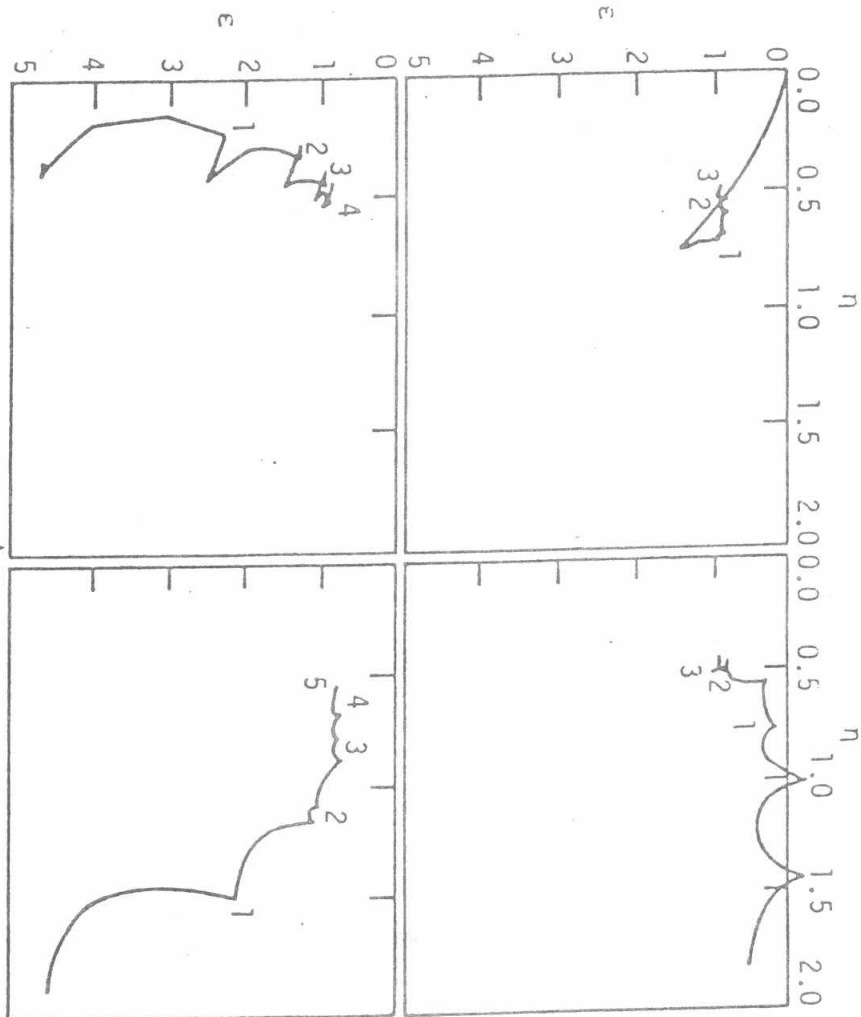


Figure 9.



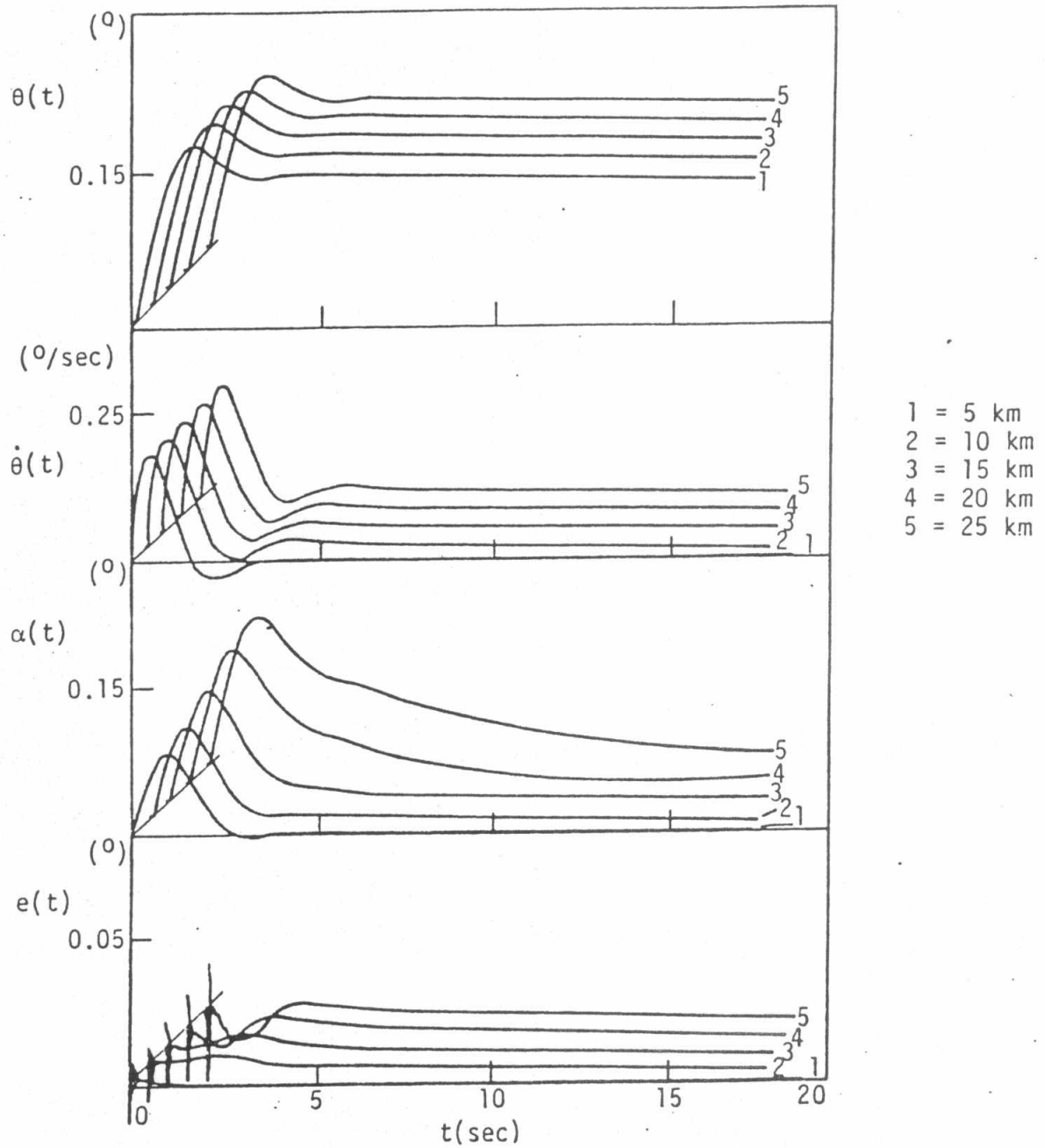


Figure 10.