

MILITARY TECHNICAL COLLEGE
CAIRO - EGYPT

$r$<br>AN INTEGFATED FLIGHT CONTROL SYSTEM PERFORMING TERRAIN<br>FOLLOWING MISSIONS<br>Lt. Col.Dip.Eng. Mohammed Fouad M. Abdel Moniem (M.T.C.)<br>Col.Dr. Wahid M. Kassim (Military Technical College) Prof. Dr. Abdel Moniem Y. Bilal (Cairo University)

## ABSTRACT

An integrated digital flight control system has been designed for the terrain following task. The control system based on the use of cubic spline paths and energy management techniques, controlling flight path and engine thrust. A nonlinear reference model based on the desired path, complete reference state and command were computed. Deviations from the reference model form a linear regulator problem, solved by discrete optimal control theory.

This system consider the operational constraints (speed, acceleration), the speed adjusts the desired energy level for the vehicle, which allows the interchange between kinetic and potential energy improving fuel consumption.

This flight control system can be used also in remotely piloted vehicles in performing similar tasks.

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## 1. INTRODUCTION

As radar technology in enemy defensive systems improves, however, future terrain - following controllers must be improved to produce lower flight paths, that is closest to the terrain while satisfying all of the physical and operational constraints on the flying vehicle. James E. Funk has proposed an optimal path processing system (Ref. 3) , uses a cubic - spline curve (a series of cubic polynomial segments with continuous first and second derivatives) to provide a very smooth reference path for the vehicle to follow from a known altitude, slope and range.

By using this flight path generator, the desired path and its derivatives have been computed. A simple form of constant energy control is applied to determine the desired forwand speed. In conjunction with aircraft eqneiions of motion, a reference command generator computes a compatible set of reference states and reference commands. Variations of estimated (actual) states from desired one were controlled by an optimal linear control regulator. Quadratic performance matrices were adjusted by suitable closed loop Eigen values, with linear aircraft model, all were discretized by a suitable sampling period. Optimal gain, G, was computed after solving the matrix Riccati equation, to regulate the variations.

Considering recent advances in digital computer, the required processing for the programing problem is feasible for a real time airborne controller, while providing great flexibility to a variety of missions and vehicles by straight forward software changes.

The flight control system described here is a good candidate for an onboard control system in many types of advanced vehicles.
2. OVERALL GUIDANCE SYSTEM.

The system incorporates nonlinearities into the structure of the reference model and control commands, andalinear part in regulating the variations in the aircraft states about the required one, as Fig. 1.

| GC-4 | 919 |
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Fig. 1.

The control variables composed of two parts :
u - Optimal control command regulating variations in aircraft state variables about reference one, computed by the optimal linear control theory.
$u_{r}$ - reference control command to elevator and thrust level, generated by reference command generator, which receives its input from a path generator.

The total control variable $U$ is the sum of $u$ and $u_{r}$ as illustrated in the block diagram of hall guidance system in Fig. 2 .


Fig. 2.
2.1. Flight Path Generator

It generates the optimal path that makes maximum use of the terrain maSk ing for the maneuvers in the longitudinal plane using an optimal cubic

| GC-4 | 920 |
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spline algorithm (Ref. 3). The algorithm stores the desired altitude and the first derivatire of altitude w.r.t. horizontal range indexed at prescribed range interials, or formulate these data currently from terrain data and navigational i:formation through forward looking radar or GPS.

The optimization of the flight path is described in detail in(Ref. 3.) but the basic concept is summarized as in Fig. 3.


Fig. 3.
The object is to minimize the magnitudes of the excess clearance distances at specified sample intervals along the path. The minimization is performed as a mathematical programming problem, in which the following inequalities constraints are satisfied :

- The path will not be below the minimum clearance curve.
- The path slope do not exceed specified limits.
- The kurvatures (second derivative of path hight w.r.t. range), will not exceed specified minimum or maximum values according to normal acceleration limits imposed by vehicle man-uverability.

The rate of change of kurvatures (kink) is bounded and the slope of the path can be limited at specified intervals, if required.

The path altitude $h_{r}$, and its successive derivatives w.r.t. range (slope $S_{r}$, kurvature $k_{r}$, and kink $P_{r}$ ) are computed from the path optimization problem.

### 2.2. Reference State \& Command Generator.

Computing of reference state model is based on the optimal spline path (mentioned previously), and a desired velocity.

The desired velocity and its derivatives can be computed to incorporate L

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operational constraints. An energy management scheme in Fig. 4.


Fig. 4.
Shows that an allowable energy corridor can be defined using $\mathrm{V}_{\max }, \mathrm{V}_{\min }$ and $h_{r}$. Thus the selection of energy level could be based on fuel consumption, engine life, mission timing or some other operational constraints. By maintaining constant energy results in an interchange of potential and kinetic energy. After selecting total energy level $E_{d}$, which introduce a desired altitude $h_{d}$, the vehicle velocity can be computed at any range on the interval under consideration, at altitude $h_{r}$.

$$
\begin{equation*}
v_{d}=\left[2 g\left(h_{d}-h_{r}\right)\right]^{\frac{1 / 2}{2}} \tag{1}
\end{equation*}
$$

from which its time derivatives are,

$$
\begin{align*}
& \dot{\mathrm{V}}_{\mathrm{d}}=\frac{d \mathrm{~V}_{\mathrm{d}}}{d \mathrm{R}} \cdot \frac{d R}{d t}=-\frac{g s_{r}}{\mathrm{~V}_{\mathrm{d}}}\left(\mathrm{~V}_{\mathrm{d}} \cos \gamma_{r}\right)=-g \sin \gamma_{r}  \tag{2}\\
& \ddot{\mathrm{~V}}_{\mathrm{d}}=\frac{d \dot{V}_{d}}{d \gamma_{r}} \cdot \frac{d \gamma_{r}}{d R} \cdot \frac{d R}{d t}=-g V_{d} K \cos \gamma_{r} \tag{3}
\end{align*}
$$

The kinematic relationships describing the motion along the reference path are ,

$$
\begin{align*}
& \mathrm{K}=\frac{1}{r_{C}}=k_{r} \cos ^{3} \gamma_{r}  \tag{4}\\
& \gamma_{r}=\tan ^{-1} S_{r} \tag{5}
\end{align*}
$$

$\gamma_{r}$ derivatives w.r.t. to range and time are :

| $\mathrm{GC}-4$ | 922 |
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$$
\begin{align*}
& \gamma_{r}^{\prime}=\frac{d \gamma_{r}}{d R}=k_{r} \cos ^{2} \gamma_{r}  \tag{6}\\
& \dot{\gamma}_{r}=\frac{d \gamma_{r}}{d t}=\gamma_{r}^{\prime} \cdot \dot{R}=v k_{r} \cos ^{3} \gamma_{r}=v K \tag{7}
\end{align*}
$$

Since

$$
\begin{equation*}
\dot{R}=\frac{d R}{d t}=V \cos \gamma_{r} \tag{8}
\end{equation*}
$$

The time derivatives of the path curvature are given by :

$$
\begin{align*}
& \dot{K}=V\left(P_{r} \cos ^{4} \gamma_{r}-3 K^{2} S_{r}\right)  \tag{9}\\
& \ddot{K}=\dot{K}\left(\frac{\dot{V}}{V}-10 K V S_{r}\right)+V^{2}\left[P_{r}^{\prime} \cos ^{5} \gamma_{r}-3 K^{3}\left(5 S_{r}^{2}+1\right)\right] \tag{10}
\end{align*}
$$

For the cubic spline path $P_{r}^{\prime}=h_{r}^{(4)}$ is zero along the path and, the normal , acceleration $a_{n}$ is ;

$$
\begin{equation*}
a_{n}=\frac{v^{2}}{r_{c}}=k v^{2} \tag{11}
\end{equation*}
$$

To derive the reference state variables, the dynamic equation, describing vehicle motion in vertical plane are required, besides the previous kinematic equations. Considering Fig. 5.


The lift end moment equations are ;

$$
\begin{align*}
& L=m\left(a_{n}+g \cos \gamma\right)  \tag{12}\\
& M=I_{Y} \dot{q} \tag{13}
\end{align*}
$$

The lift and moment coefficients can be closely approximated along the reference path by a linear combination of variations about a nominal flight 」 L

| GC-4 | 923 |
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Condition (subscript "O") :

$$
\begin{equation*}
C_{L_{r}}=C_{L_{O}}+C_{L_{\alpha}} \Delta \alpha_{r}+C_{L_{\delta}} \Delta \delta_{r}+C_{L_{V}} \frac{\Delta v}{V_{0}} \tag{14}
\end{equation*}
$$

and

$$
\begin{align*}
C_{r}=C_{m_{0}}+C_{m_{\alpha}} \Delta \alpha_{r} & +C_{m_{\delta}} \Delta \delta_{r}+C_{m_{v}} \frac{\Delta v}{V_{o}} \\
& +\frac{\bar{C}}{2 \mathrm{~V}_{o}}\left(m_{q} \Delta q_{r}+C_{m_{\dot{\alpha}}} \Delta \dot{\alpha}_{r}\right) \tag{15}
\end{align*}
$$

where the $\Delta \alpha_{r}$ and other variations are the change from the nominal flight condition to the reference flight condition along the spline path. The coefficients of the variations are stability derivatives which are evaluated at the nominal condition. For convenience, the usual choice of straight and level flight is made for the nominal condition.

This choice results in

$$
\begin{equation*}
\alpha_{0}=\delta_{0}=0 \tag{.16}
\end{equation*}
$$

and

$$
\begin{equation*}
C_{L_{0}}=C_{m_{0}}=0 \tag{17}
\end{equation*}
$$

Thus

$$
\begin{align*}
& \alpha_{r}=\Delta \alpha_{r}  \tag{18}\\
& \delta_{r}=\Delta \delta_{r} \tag{19}
\end{align*}
$$

and the reference pitch rate is

$$
\begin{equation*}
q_{r}=\dot{\alpha}_{r}+\dot{\gamma}_{r}=\dot{\alpha}_{r}+K v_{d} \tag{20}
\end{equation*}
$$

From dynamic equations (Eqs. 12 and 13) evaluated at the reference condition and use Eqs. 14 through 20 we obtain Eqs. 21 and 22

$$
\begin{align*}
& \mathrm{C}_{\mathrm{L}_{\delta}} \quad \delta_{r}+\mathrm{C}_{\mathrm{L}_{\alpha}} \alpha_{r}+\frac{T}{\bar{q} S} \sin \alpha_{r}=\frac{m}{\bar{q} S}\left(-\mathrm{K} \mathrm{~V}_{\mathrm{d}}^{2}+\mathrm{g} \cos \gamma_{r}\right) \\
& -C_{L_{V}} \frac{V_{d}}{V_{0}}  \tag{21}\\
& C_{m_{\delta}} \delta_{r}+C_{m_{\alpha}} \alpha_{r}+\frac{\bar{C}}{2 V_{o}}\left(C_{m_{\dot{\alpha}}}+C_{m_{q}}\right) \dot{\alpha}_{r}-I_{y} \frac{\ddot{\alpha}_{r}}{\overline{q S} \bar{C}}= \\
& \frac{I_{y} \ddot{\gamma}_{r}}{\bar{q} S \bar{c}}-\frac{\bar{c}}{2 V_{o}} c_{m_{q}} K V_{d}-C_{m_{v}} \frac{V_{d}}{V_{o}} \tag{22}
\end{align*}
$$

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To solve Eqs. 21 and 22 avoiding solution of iterative nonlinear differential equation with unknown $\alpha_{r}$ and thrust, a direct algebraic method is required to obtain the estimated $\dot{\alpha}_{r e}$ and $\ddot{\alpha}_{r e}$, using the approximation $\sin \alpha_{r}$ as $\alpha_{r}$ in Eq. 21 we can get the estimated angle of attack values $\alpha_{\text {re }}$ as :

$$
\begin{equation*}
\alpha_{r e}=C_{a}\left[\frac{2 m}{\rho S}\left(-K+\frac{g}{V_{d}^{2}} \cos \gamma_{r}\right)-C_{L_{V}} \frac{V_{d}}{V_{o}}-C_{L_{\delta}} \delta_{r e}\right] \tag{23}
\end{equation*}
$$

where

$$
\begin{equation*}
c_{a} \stackrel{\Delta}{\triangleq}\left(C_{L_{\alpha}}+C_{T}\right)^{-1} \tag{24}
\end{equation*}
$$

and

$$
\begin{equation*}
C_{T}=\frac{T}{\bar{q} S} \tag{25}
\end{equation*}
$$

differentiating Eq. 23, assuming that the quantities
$\mathrm{g}, \rho, \mathrm{C}_{\mathrm{L}_{\alpha}}$ and $\mathrm{C}_{\mathrm{T}}$ are, constant we get :

$$
\begin{array}{r}
\dot{\alpha}_{r e}=c_{a e}\left\{\frac{2 m}{\rho S}\left[-\dot{K}^{-}-\frac{g}{v_{d}^{3}} \cos \gamma_{r}\left(2 \dot{\mathrm{~V}}_{d}+\mathrm{V}_{d}^{2} S_{r} K\right)\right]\right. \\
\left.-\dot{C}_{L_{V}} \frac{\dot{V}_{d}}{\mathrm{~V}_{o}}-C_{L_{\delta}} \dot{\delta}_{r e}\right\} \tag{26}
\end{array}
$$

and

$$
\begin{align*}
\ddot{\alpha}_{r e}= & c_{a}\left\{\frac { 2 m } { \rho s } \left[-\ddot{\mathrm{K}}+\frac{g}{\mathrm{~V}_{d}^{4}} \cos \gamma_{r}\left(6 \dot{\mathrm{~V}}_{\mathrm{d}}^{2}-2 \ddot{\mathrm{~V}}_{\mathrm{d}} \mathrm{~V}_{\mathrm{d}}-\mathrm{K}^{2} \mathrm{v}_{\mathrm{d}}^{4}\right.\right.\right. \\
& \left.\left.\left.+3 \mathrm{~K} \dot{\mathrm{~V}}_{\mathrm{d}} \mathrm{~V}_{\mathrm{d}}^{2} \mathrm{~S}_{r}-\dot{\mathrm{K}}_{\mathrm{V}}^{3} \mathrm{~V}_{\mathrm{d}}^{3}\right)\right]-\mathrm{C}_{L_{\mathrm{V}}} \frac{\ddot{\mathrm{~V}}_{\mathrm{d}}}{\mathrm{~V}_{0}}-\mathrm{C}_{L^{\prime}} \quad \ddot{\delta}_{r e}\right\} \tag{27}
\end{align*}
$$

With all values $\mathrm{K}, \dot{K}_{,} \mathrm{V}_{\mathrm{d}}, \dot{\mathrm{V}}_{\mathrm{d}}, \ddot{\mathrm{V}}_{\mathrm{d}}$ are previously defined.
We estimate $\dot{\delta}_{r e}$ as

$$
\begin{align*}
\dot{\delta}(t) & \cong-\frac{1}{\tau_{e}}\left[\delta_{r e}(t)-\delta_{r C}(t-\Delta t)\right] \\
& =-\frac{1}{\tau_{e}}\left[\delta_{r}(t-\Delta t) e^{-\Delta t / \tau_{e}}\right] \tag{28}
\end{align*}
$$

where $\delta_{r C}(t-\Delta t)$ is the previous command elevator angle, and $\ddot{\delta}_{r e}$ as

$$
\begin{equation*}
\ddot{\delta}_{r e}(t)=-\frac{1}{\tau_{e}}\left[\dot{\delta}_{r}(t-\Delta t) e^{\left.-\Delta t / \tau_{e}\right]}\right. \tag{29}
\end{equation*}
$$

An estimated thrust value can be obtained from the estimated values via the longitudinal force equation.

| GC-4 | 925 |
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$$
\begin{equation*}
T_{r e}=\sec \alpha_{r e}\left(m \dot{V}_{d}+\bar{q} S C_{D_{r e}}+m g \sin \gamma_{r}\right) \tag{30}
\end{equation*}
$$

The drag coefficient estimate is based on the lift coefficient estimate.

$$
\begin{align*}
& C_{L_{r e}}=C_{I_{\alpha}} \alpha_{r e}+C_{L_{\delta}} \delta_{r e}+C_{L_{v}} \frac{V_{d}}{V_{o}}  \tag{31}\\
& C_{D_{r e}}=C_{D_{o}}+C_{L_{r e}}^{2} C_{D_{i}} \tag{32}
\end{align*}
$$

Thus the estimated thrust coefficient is

$$
\begin{equation*}
C_{T_{r e}}=\frac{T_{r e}}{\bar{q} S}=\sec \alpha_{r e}\left[C_{D_{r e}}+\frac{m}{\bar{q}_{d} S}\left(\dot{V}_{d}+g \sin \gamma_{r}\right)\right] \tag{33}
\end{equation*}
$$

which in turn is used for

$$
\begin{equation*}
C_{a e}^{-1}=C_{L_{\alpha}}+C_{T_{r e}} \tag{34}
\end{equation*}
$$

Finally the estimated value of $\mathrm{c}_{\mathrm{ae}}^{-1}, \dot{\alpha}_{r e}$ and $\ddot{\alpha}_{r e}$ can be used for calculation of the, reference values $\delta_{r}$ and $\alpha_{r}$ as

$$
\begin{align*}
\delta_{r} & =\frac{C_{a e}^{-1} \Delta C_{m}-C_{m \alpha}}{C_{a e}^{-1}} \frac{\Delta C_{L}}{C_{m}-C_{m_{\alpha}}} \frac{C_{L_{\delta}}}{C_{\delta}}  \tag{35}\\
\alpha_{r} & =\frac{C_{m}}{C_{a e}^{-1}} \frac{\Delta C_{L}-C_{L_{\delta}} C_{\delta}-C_{m_{\alpha}}}{} \frac{C_{m}}{C_{L_{\delta}}} \tag{36}
\end{align*}
$$

where

$$
\begin{equation*}
\Delta C_{L} \equiv \frac{m}{\bar{q}_{d} S}\left(-K V_{d}^{2}+g \cos \gamma_{r}\right)-C_{L_{V}} \frac{V_{\mathrm{d}}}{V_{\circ}} \tag{37}
\end{equation*}
$$

and

$$
\begin{align*}
\Delta C_{m} \Delta \frac{I_{y}}{\bar{q} S \bar{c}}\left(\ddot{\alpha}_{r e}+\ddot{\gamma}_{r e}\right) & -\frac{\bar{c}}{2 v_{o}}\left[\left(c_{m_{\dot{\alpha}}}+c_{m_{q}}\right) \dot{\alpha}_{r e}+c_{m_{q}} K v_{d}\right] \\
& -c_{m_{v}} \frac{v_{d}}{v_{o}} \tag{38}
\end{align*}
$$

### 2.2.1. Reference State Variables :

Now a complete state reference model can be obtained; seven state variables are used, two velocity components, pitch angle and its rate, the elevator deflection, the thrust coefficient, and the path hight.

| GC-4 | 926 |
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$$
\begin{align*}
& \mathrm{v}_{r}=\mathrm{v}_{\mathrm{d}} \cos \alpha_{r}  \tag{39}\\
& \omega_{r}=\mathrm{v}_{\mathrm{d}} \sin \alpha_{r}  \tag{40}\\
& \theta_{r}=\alpha_{r}+\gamma_{r}  \tag{41}\\
& q_{r}=\dot{\alpha}_{r}+\dot{\gamma}_{r}  \tag{42}\\
& \delta_{r}=\text { from Eq. } 35  \tag{43}\\
& C_{\mathrm{T}_{r}}=\sec \alpha_{r}\left[C_{D_{r}}+\frac{m}{q_{d}}{ }^{s}\left(v_{d}+g \sin \gamma_{r}\right)\right]  \tag{44}\\
& h_{r}=\text { from the optimal spline routine }
\end{align*}
$$

The $\dot{\alpha}_{r}$ and $C_{D}$ values must be computed from equations of the same form as Eq. 26 and $3 \kappa$ where the $\alpha_{r e}$ and $\delta_{r e}$ replaced by $\alpha_{r}$ and $\delta_{r}$. Note that $C_{L_{r}}$ and $C_{a}$ obtained based on $\alpha_{r}$ and $\delta_{r}$ from equations similar to Eqs. $r_{31}$ and 24, the latter requires the value of $C_{T_{r}}$ from Eq. 44 .

### 2.2.2. Refexence Control Variables :

To complete the model, the reference control variables are needed in add ition to the reference state variables. The two control variables are the commanded elevator deflection $\delta_{r c}$ and the commanded thrust coefficient $C_{T}$. It is assumed that the actuator responses for the elevator and thrust ${ }^{r C}$ Controller are satisfactorily modeled as first order linear differential equations. The time response of the doflection angle over a control sample interval, $\Delta t$, is

$$
\begin{equation*}
\delta_{r}(t)=\delta_{r}(t-\Delta t) e^{-\Delta t / T} e+\delta_{r c} \tag{45}
\end{equation*}
$$

where $\delta_{r C}$ is the control value used over the interval.
This value is used for the command at time $t$, since it can be computed from previously known values.

$$
\begin{equation*}
\delta_{r c}(t)=\delta_{r}(t)-\delta_{r}(t-\Delta t) e^{-\Delta t / T_{e}} \tag{46}
\end{equation*}
$$

Similarly, ${ }^{\circ}$

$$
\begin{equation*}
C_{T_{X C}}(t)=C_{T_{X}}(t)-C_{T_{X}}(t-\Delta t) e^{-\Delta t / r_{T}} \tag{4}
\end{equation*}
$$

For circular cross-sections, the quantity in square brackets equals the section area. Accordingly, for cross-sections of general form, this quantity can be interpreted as the area of an equivalent circle of raduis $\operatorname{Re}$ given by

$$
\begin{equation*}
S_{e}=\left[2 \pi\left(c^{2}-R_{e s}\right)-S\right]=\pi R_{e}^{2} \tag{2.15}
\end{equation*}
$$

This circular cross-section has the same virtual momentum as that of the given body section of general form.
By virtue of (2.14) and (2.10), the body formed by these equivalent crosssections has the same aerodynamic loading (excluding viscous effects ) as the original body. Using this argument, the integral in the expression for the additional potential of the presented extention is evaluated. For calculating this potential near the body surface at small deflections, the equivalent radius $\operatorname{Re}_{e}$, function of $x$ only, substitutes $r$ in these integrals, enabling their easy solution.
Aerodynamic forces and moments are determined within this extension using an iterational procedure with the first step given by the original Slender Body theory. The coefficient $\boldsymbol{a}_{\boldsymbol{\prime}}$ is firstly calculated from relation (2.12) using the cross flow velocity component $v=-D w_{g} / D_{t}$.The additional transvere velocity given by (2.8) is then evaluated and added to the cross flow velocity- $D \operatorname{wg} / \Delta_{t}$ - The resulting value of $v$ is used for calculating the viretual momentum $\mathbb{Q}$ of the quasi-slender body from relation (2.14).

## 3. THE APPLICATION TO WING-BODY COMBINATIONS

The presented extended theory is applied for calculating the slopes of lift and moment curves as function of angle of attack of quasi-slender wing body combinations in steady supersonic flow. The chosen configuration is a low aspect ratio wing of local span $2 b(x)$, mounted on a pointed body of revolution of radius $\boldsymbol{a}(x)$, see Fig.(3.1).


Fig.(3.1) Selected wing-s body combination

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Fig.(3.2) Conformal mapping of body cross-sections

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ponse behaviour $W+T h \frac{a^{2}}{W}$ i $a \Delta=\left\{1+\frac{c^{2}}{\xi}\right.$ from the calculated open loop Eigen values which where are

$$
c(x)=\frac{1}{2} b(x)\left[1+\frac{a^{2}(x)}{b^{2}(x)}\right]
$$

$-2.326 \pm j 7.063,-0.00035 \pm j 0.0567$,
 mapps the domain exterior to the winged body cross-seetions in the wiplane tolathe, domfain Qxterior to the flat plate $-2 c<\lambda\langle 2 c$ in the $\lambda$ plane and hence to the exterior of the circle $/ \xi /=c$, as shown in Fig. X. 2. Constidering the case of rigid cross-sections, the equivalent ondssesfecional areas of子ht wititeband CombrintiostaMatrices:

through the solution of the continuous
Solving for simplicity, the case of delta wing mounted /offar conical body,
 obesin flowt velpfity derivatioswithopespect to angle orrattack is found to be independent of $x$ eigen values as a function of the ele-
 Fig. 8 and 9 show the root loci of the İBseqnidiensignal lift curve and moment curve sfopes withriespect to angle of attack related to wing planform area $A=$
 ustment, other elements of matrix $\mathcal{Z}$
 and

Fig. 7.






 with lift coefficient and a position of detodynamlc center at two thirds of wing root chédd, hich agrees with the known value ifbr similar wing-

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Using the cornell Aeronautical Laboratory "thumbprint" (R:£. 6), as a guide, a damping ratio of approximately. 7 and a natural frequeriy of 12 radians per-second was chosen as the desired operating point for $\mathrm{t}^{\text {n }} \mathrm{e}$ short period.

### 3.2. Sampling Period Adjustment :

After choice of the matrices Q and $\mathbb{R}$, a suitable sampling period was chosen to complete the discrete regulator parameters, the chcice of the sampling period is done by calculating and plotting the eigen values as a function of sampling period in the S-plane and Z -plane.


Im
Fig. 11

Fig. 10 shows the discrete closed loop root movement as a function of sampling periode in the z-plane, while Fig. ll illustrate this relation in the S-plane.

A value of . 1 sampling time is enough to have the first folded root to be far enough from the dominant roots to have minimal effect on the system ( $Z$-plane), in the same time to keep the short period damping ratio near to 0.7 (S- plane). And higher sampling rate does not give a considerable improvement on model response.

| $G C-4$ | 930 |
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FIRST A.S.A.T. CONFERENCE
14-16 May 1985 , CAIRO
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3.3. Discrete regulator Simulation:

The continuous linear system with performance index was discretized at sampling frequency 10 Hz , then the discrete regulator problem was solved to get the optimal gain, G, (Ref, 4) Considering that output states [y] are all measured.

The discrete optimal regulator was tested, and Fig. 12 illustrates the closed loop time response of all the model states, to an initial error 20 feet on the state variable $h$.

And with the following closed loop Eigen values

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```
0.29286790E+00
```



```
U.29286790E+J0
```



```
0.60709187E+00
```



```
0.t7222529E+00
```



```
0.87071470E+00
```



```
0.90283999E+し0
```



```
0.90283999E+00
```



Because of long time spent in construction of a complatible sets of reference states and commandes, only the deterministic regulator was solved, lefting the wind effects on aircraft states (stochastique regulator) for future work.

It's clear that the system overcomes its states deviations during a short time, about 4 seconds.

Closed loop state Response


Fig. 12.

| GC-4 | 931 |
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4. INTEGRATED FLIGHT CONTROL SYSTEM SIMULATION.

The performance of the whole system was tested in digital computer. Assuming that all the output states are known, linear aircraft model given in the Appendix its appropriate matrices, the incremental acceleration was constrained between $-1,5$ and $+3 g$, which denotes "hard ride" quality.


Fig. 13.

Fig. 13 shows that the aircraft follows the reference trajectory with a good manner, with maximum error 20 feet.

## 5. CONCLUSIONS.

This flight control system is a flexible technique that can be adapted to variety of sensor (GPS,...)/ aircraft configurations (fighters, RPV's), with varying operational considerations (check point times, maximum and minimum speed and acceleration limits). The control system incorporates both flight path control and engine control. Using the optimal spline path and adjusting the desired energy level for the vehicle improves engine life, fuel consumption and engine noise. The overall performance of the system depend on the accuracy of the terrain data and navigational information that define the true state of the vehicle relative to the terrain, considering recent advances in using satelites in these tasks, the」

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required accuracy can be accomplished.
Finally, this flight control system is very useful for the special tasks of remotely controlled vehicles.

## Appendix:

The aircraft linearized model using the stability axes system assuming straight and level flight as nominal state is given by:
$\underline{x}=\underline{A} \underline{x}+\underline{B} \underline{u}$
$\underline{y}=\underline{C}$
where

$$
\begin{align*}
& x=\left[\begin{array}{lllllll}
v & w & q & \Theta & \delta_{2} & C_{T} & h
\end{array}\right]^{T}  \tag{A.3}\\
& u=\left[\begin{array}{ll}
\delta_{\text {ec }} & C_{T C}
\end{array}\right]^{\mathrm{T}}  \tag{A.4}\\
& y=\left[\begin{array}{llll}
h & \stackrel{\dot{h}}{V_{0}} & \frac{\ddot{h}}{V_{0}^{2}} & u
\end{array}\right] \tag{A.5}
\end{align*}
$$

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 $-U .778 d U 1-U 1 *-U .27,51 E+U 1 * 0.8924+U E+U 3 * 0 . U U U O U E+U 0 *-U .2027 G L+U 3 *-U .14551 t+01 * 0.0 J U 00 t+U 0$





> ATHIXC

$B=\left[\begin{array}{lllllll}0 & 0 & 0 & 0 & 10 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 5 & 0\end{array}\right]$

| GC-4 | 933 |
| :---: | :---: |

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## NOMENCLATURE

m aircraft mass
Iy moment of inertia arround axis $y$
R horizontal range
$\mathrm{V}_{\mathrm{o}} \quad$ nominal speed
$h_{r} \quad$ reference altitude
$S_{r} \quad r e f e r e n c e ~ s l o p e ~$
$k_{r} \quad$ reference kurvature
$P_{r} \quad$ reference kink
$r_{c} \quad$ radius of terrain curvature
$a_{n}$ normal acceleration
L,M lift force and moment
T,D thrust and drag forces
Q,R state and control cost matrices,

$\gamma_{r}$ reference flight path angle.
$L$


