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ON DISCRETE TIME MODEL REFERENCE ADAPTIVE CONTROL SYSTEM

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ABSTRACT

This article presents a discrete time model reference parameter tracking technique which is applicable for less order reference model. The order of the reference model equal (n-1) where n is the number of the adjustable parameters in the physical plant. The technique utilizes a modified gradient method where the knowledge of the exact order of the nonadaptive system is not required. Computer simulations are provided to demonstrate the feasibility of the proposed technique through different examples.

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1. INTRODUCTION

The model reference adaptive control (MRAC) technique has been a popular approach to the control of systems operating in the presence of parameter and environmental variations. In such a scheme, the desirable dynamic characteristics of the plant are specified in a reference model and the input signal or the controllable parameters of the plant are adjusted, continously or discretely, so that its response will duplicate that of the model as closely as possible. The identification of the plant dynamic performance is not necessary and hence a fast adaptation can be achieved.

A large number of practical control problems can be formulated as discrete time model reference adaptive systems. examples are: process control [1] , power systems control[2], signal processing [3] etc. The available techniques for the solution of such problems are of the following type: local parametric optimization theory [4] , Lyapunov functions technique [5] [6] , and hyperstability and positively concepts [7] . Most of these techniques avoid the identification problem, but stresses that the order of the reference model must be the same as that of the nonadaptive system .

In this paper a modified gradient tracking technique is proposed to solve the discrete time model reference adaptive systems when the exact order of the nonadaptive system is not known. The method requires that the order of the reference model will be equal to (n-1) where n is the number of the controllable parameters of the discrete plant.

The organization of the rest of the paper is as follows: in Section 2 the principle of adaptation is presented followed by the mathematical foundation of the proposed technique illustrated through a second order discrete time linear variant physical system and a first order discrete input-output relationship (reference model). The technique extended to the general case in Section 3. The concept is illustrated in Section 4 through different examples and conclusion appear in Section 5.

2. PRINCIPLE OF ADAPTATION

Consider the following single-input/single output discrete model reference adaptive system shown in Figure (1) and represented by:

(a) The adjustable system Physical Plant .

$$c(k-2) + \alpha_1 c(k-1) + \alpha_0 c(k) = r(k) \tag{1}$$

$$\alpha_1(k) = g_1(k) + h_1(k) \tag{2}$$

$$\alpha_0(k) = g_0(k) + h_0(k) \tag{3}$$

Where $r(k)$ is the input sequence, $c(k)$ is the output of the adjustable system, k is the sample number, $\alpha_0(k)$ and $\alpha_1(k)$ are the adjustable system parameters, and $h_0(k)$ and $h_1(k)$ are the controllable adjustable system parameters.

(b) The desired input-output relationship Reference Model

$$A_1 y(k-1) + A_0 y(k) = r(k) \tag{4}$$

Where $y(k)$ is the desired output of the reference model, and A_1 and A_0 are the constant model reference parameters.

(c) The generalized output error

$$e(k) = c(k) - y(k) \tag{5}$$

The parameters $g_1(k)$ and $g_0(k)$ are assumed to vary over extreme ranges within the operating environment of the nonadaptive system. Assuming also that there exists a value of $h_1(k)$ and $h_0(k)$ for which the system will behave like the chosen model. The input information to the adaptation mechanism is the generalized error $e(k)$ and its output will be the discretely adjusting values of $h_1(k)$ and $h_0(k)$. The objective is to formulate the adaptation mechanism equations. If no limitations are placed on the values which may be assumed by $h_1(k)$ and $h_0(k)$, then regardless of what values $g_1(k)$ and $g_0(k)$ take on, the output of both systems will be approximately identical whenever some chosen function of the error will be minimized. The error function which will be used in this work is

$$f(e) = \frac{1}{2}\left[q_0 e(k) + q_1 e(k-1)\right]^2 \tag{6}$$

Where q_0 and q_1 are constant factors.

The objective of the adaptation mechanism will be the minimization of the error function which is a quadratic form of the error and depends indirectly on the differences:

$$\delta_1(k) = \alpha_1(k) - A_1 \tag{7}$$

$$\delta_0(k) = \alpha_0(k) - A_0 \tag{8}$$

Assuming that $g_1(k)$ and $g_0(k)$ vary slowly as compared to the basic time constants of the adjustable system and the reference model, and the adaptation mechanism will be designed to adjust the parameters $h_1(k)$ and $h_0(k)$ at a rate which is much greater than the rate of variation of $g_1(k)$ and $g_0(k)$, one can use a gradient optimization technique which leads to the basic adaptation rule:

$$\Delta\alpha_1 = -k_1 \,\text{grad}\,[f(e)] = -k_1 \frac{\partial f(e)}{\partial \alpha_1} \quad (9)$$

$$\Delta\alpha_0 = -k_0 \,\text{grad}\,[f(e)] = -k_0 \frac{\partial f(e)}{\partial \alpha_0} \quad (10)$$

where k_1 and k_0 are arbitrary positive constants.

If the development were to continue to be based upon equations (9) and (10). The resulting design would require explicit knowknowledge of $\alpha_1(k)$ and $\alpha_0(k)$ and consequently $g_1(k)$ and $g_0(k)$.

Now suppose that $\alpha_1(k)$ and $\alpha_0(k)$ are treated as constant, and A_1 and A_0 are to be adjusted so as to cause δ_0 and δ_1 to approach the same value , then the variations in A_1 and A_0 becomes:

$$\Delta A_1 = -k_1 \frac{\partial f(e)}{\partial A_1} \quad (11)$$

$$\Delta A_0 = -k_0 \frac{\partial f(e)}{\partial A_0} \quad (12)$$

The objective is not to change A_1 and A_0 . However, to change $\alpha_1(k)$ and $\alpha_0(k)$. Since the same change in δ_0 and δ_1 can be obtained by substracting ΔA_0 and ΔA_1 and from α_0 and α_1 rather than adding them to A_0 and A_1. Now, the error function f(e) is driven to minimum by determining the appropriate increments for A_1 and A_0 and applying the negative of these increments to α_1 and α_0. The resulting equations for the variation in α_1 and α_0 are

$$\Delta\alpha_1 = k_1 \frac{\partial f(e)}{\partial A_1} \quad (13)$$

$$\Delta\alpha_0 = k_0 \frac{\partial f(e)}{\partial A_0} \quad (14)$$

Since it is assumed that the adaptation mechanism will be designed so as to adjust the parameters h_1 and h_0 in a rate greater than the changes in g_1 and g_0 (i.e. $\Delta h_i \gg \Delta g_i$).

Equations (13) and (14) can be approximated as:

$$\Delta h_1 = h_1(k) - h_1(k-1) = k_1 \frac{\partial f(e)}{\partial A_1} \tag{15}$$

$$\Delta h_0 = h_0(k) - h_0(k-1) = k_0 \frac{\partial f(e)}{\partial A_0} \tag{16}$$

When equation (6) was substituted into (15) and (16) and the partial derivative was carried out equations (15) and (16) yield to:

$$\Delta h_1 = -k_1 \left[q_0\ e(k) + q_1\ e(k-1)\right] \left[q_0 \frac{\partial y(k)}{\partial A_1} + q_1 \frac{\partial y(k-1)}{\partial A_1}\right] \tag{17}$$

$$\Delta h_0 = -k_0 \left[q_0\ e(k) + q_1\ e(k-1)\right] \left[q_0 \frac{\partial y(k)}{\partial A_0} + q_1 \frac{\partial y(k-1)}{\partial A_0}\right] \tag{18}$$

The terms $\frac{\partial y(k-1)}{\partial A_i}$ in the above equations can be represented by:

$$\frac{\partial y(k-1)}{\partial A_i} = \frac{\partial}{\partial A_i} D\left[y(k)\right] \tag{19}$$

Where D represent the delay operation. Again assuming slow adaptation, the order of the two linear operators in the righthand side of equation (19) can be interchanged, yielding to:

$$\frac{\partial y(k-1)}{\partial A_i} = D \frac{\partial}{\partial A_i} y(k) \tag{20}$$

Now introducing the notation

$$u_i = \frac{\partial}{\partial A_i} y(k) \tag{21}$$

The adaptive mechanism equations could be written as:

$$h_1(k) = h_1(k-1) - k_1\left[q_0 e(k) + q_1 e(k-1)\right]\left[q_0 u_1(k) + q_1 u_1(k-1)\right] \tag{22}$$

$$h_0(k) = h_0(k-1) - k_0\left[q_0 e(k) + q_1 e(k-1)\right]\left[q_0 u_0(k) + q_1 u_0(k-1)\right] \tag{23}$$

The only unknown quantities in the above equations are $u_1(k)$ and $u_0(k)$. Considering the difference equation of the reference model, and taking the partial derivative of both sides with respect to the parameter A_0, and interchanging the two linear operators as in equation (20), employing the notation introduced in equation (21), we get:

$$A_1 \; u_0(k-1) + A_0 \; u_0(k) \;\; = -y(k) \tag{24}$$

Similarly, but differentiating with respect to A , yield to:

$$A_1 \; u_1(k-1) + A_0 \; u_1(k) \;\; = -y(k-1) \tag{25}$$

Equations (24) and (25) represent two difference equations with available forcing functions and their solutions provides the values of $u_0(k)$ and $u_1(k)$ for equations (22) and (23).

The complete set of difference equations which describes the adaptive system operation can be written as:

$$\begin{aligned}
c(k-2) + \alpha_1 c(k-1) + \alpha_0 c(k) &= r(k) \\
A_1 \; y(k-1) + A_0 \; y(k) &= r(k) \\
A_1 \; u_1(k-1) + A_0 \; u_1(k) &= -y(k-1) \\
A_1 \; u_0(k-1) + A_0 \; u_0(k) &= -y(k) \\
h_1(k) = h_1(k-1) - K_1\left[q_0 e(k) + q_1 e(k-1)\right]&\left[q_0 u_1(k) + q_1 u_1(k-1)\right] \\
h_0(k) = h_0(k-1) - K_0\left[q_0 e(k) + q_1 e(k-1)\right]&\left[q_0 u_0(k) + q_1 u_0(k-1)\right]
\end{aligned} \tag{26}$$

3. GENERAL PROCEDURE OF ADAPTATION

In this section, the principle of adaptation described in section 2 will be extended to the general sampled data time variant case represented by the following equation:

$$\sum_{i=0}^{n} \alpha_i(k) c(k-i) = \sum_{i=0}^{m} \beta_i(K) \; r(k-i), \quad m \leqslant n \tag{27}$$

Where α_i and β_i are functions of the changing coefficients within the operating environments of the physical process. Define h_i as the compensating parameters which will be changed by the adaptive mechanism in a rate which is relatively greater than that of the changing coefficients g_i, and their relationships, will depend on the particular system being considered. The generalized equations to be derived will yield the incremental values at which α_i and β_i must be adjusted to effect the adaptation.

The.required input-output relationship is described by:

$$\sum_{i=0}^{\delta} A_i\, y(k-1) = \sum_{i=0}^{\gamma} B_i\, r(k-1) \quad ; \; \delta \geqslant \gamma \tag{28}$$

Assumption for using a less order reference model are:

1- The desired output could be obtained by the variation of the coefficient α_i and/or β_i .

2- The order of the reference model is chosen arbitrarily according to the required behaviour of the system to be less or equal to the order of the nonadaptive portion n .

3- The number of parameters to be adjusted can be $\leqslant (\delta+1)$.

The generalized quadratic function of the error e(k) = c(k) - y(k), is chosen in the following forms:

$$f(e) = \frac{1}{2}\left[\sum_{i=0}^{\delta} q_i\, e(k-i)\right]^2 \tag{29}$$

or

$$f_1(e) = \frac{1}{2}\left[\sum_{i=0}^{\delta} q_i\, e(k-i)^2\right] \tag{30}$$

Keeping the same procedure explained in section 2, the increments by which α_j and/or β_j should be adjusted are:

1) for f(e)

$$\Delta\alpha_j = -K_{j\alpha}\left(\sum_{i=0}^{\delta} q_i\, e(k-i)\right)\left(\sum_{i=0}^{\delta} q_i\, u_{j\alpha}(k-i)\right) \; ; \; j = 0,\dots,\delta$$

$$\Delta\beta_j = -K_{j\beta}\left(\sum_{i=0}^{\delta} q_i\, e(k-i)\right)\left(\sum_{i=0}^{\delta} q_i\, u_{j\beta}(k-i)\right) \; ; \; j = 0,\dots,\gamma \tag{31}$$

2) for $f_1(e)$

$$\Delta\alpha_j = -K_{j\alpha}\left[\sum_{i=0}^{\delta} q_i\, e(k-i).\,u_{j\alpha}(k-i)\right]; \; j=0,1,\dots,\delta$$

$$\Delta\beta_j = -K_{j\beta}\left[\sum_{i=0}^{\delta} q_i\, e(k-i)\, u_{j\beta}(k-i)\right]; \; j=0,1,\dots\gamma \tag{32}$$

Where the generalized error signal are assumed to be available and the k's and q's are constant parameters and would be chosen by the designer to secure the stability of the adaptive process. The equations providing the auxiliary variables $U_{j\alpha}$ and $U_{j\beta}$ are:

$$\sum_{i=0}^{\delta} A_i \, u_{j\alpha}(k-i) = -y(k-j) \qquad j=0,\ldots,\delta \tag{33}$$

$$\sum_{i=0}^{\delta} A_i \, u_{j\beta}(k-i) = r(k-j) \qquad j=0,\ldots,\gamma \tag{34}$$

If an error function of the form given in equation (29) is used, the appropriate equations which describe the adaptive system operation are:

$$\sum_{i=0}^{n} \alpha_i(k) c(k-i) = \sum_{i=0}^{m} \beta_i(k) r(k-i)$$

$$\sum_{i=0}^{\delta} A_i y(k-i) = \sum_{i=0}^{\gamma} B_i r(k-i)$$

$$\Delta\alpha_j = -K_{j\alpha} \Big[\sum_{i=0}^{\delta} q_i e(k-i)\Big]\Big[\sum_{i=0}^{\delta} q_i U_{j\alpha}(k-i)\Big] ; \; j=0,\ldots,\delta$$

$$\Delta\beta_j = -K_{j\beta} \Big[\sum_{i=0}^{\delta} q_i e(k-i)\Big]\Big[\sum_{i=0}^{\delta} q_i U_{j\beta}(k-i)\Big] ; \; j=0,\ldots,\gamma$$

$$\sum_{i=0}^{\delta} A_i \, U_{j\alpha}(k-i) = -Y(k-j) ; \; j=0,\ldots,\delta$$

$$\sum_{i=0}^{\delta} A_i \, U_{j\beta}(k-i) = r(k-j) ; \; j=0,\ldots,\gamma \tag{35}$$

If the error function is in the form of equation (30), the appropriate equations which describe the adaptive system operation are:

$$\sum_{i=0}^{n} \alpha_i(k) c(k-i) = \sum_{i=0}^{m} \beta_i(k) r(k-i)$$

$$\sum_{i=0}^{\delta} A_i y(k-i) = \sum_{i=0}^{\delta} B_i r(k-i)$$

$$\Delta\alpha_j = -K_{j\alpha} \Big[\sum_{i=0}^{\delta} q_i e(k-i) U_{j\alpha}(k-i)\Big] ; \; J=0,1,\ldots,\delta$$

$$\Delta\beta_j = -K_{j\beta} \Big[\sum_{i=0}^{\gamma} q_i e(k-i) U_{j\beta}(k-i)\Big] ; \; J=0,1,\ldots,\gamma$$

$$\sum_{i=0}^{\delta} A_i \, U_{j\alpha}(k-i) = -y(k-j) ; \quad J=0,1,\ldots,\delta$$

$$\sum_{i=0}^{\delta} A_i \, U_{j\beta}(k-i) = r(k-j) ; \quad J=0,1,\ldots,\gamma$$

(36)

4. EXAMPLES

Using the proposed adaptation technique, computer simulation were carried out to adapt the following examples:

1) Example 1.

a) The adjustable system

$$c(k-2) + (g_1+h_1)\ c(k-1) + (g_0+h_0)\ c(k) = r(k)$$

Where the parameters g_0 and g_1 are assumed to vary

and the numerical values of the system parameters were chosen initially to be:

$$(g_1 + h_1) = -4.0 \quad , \quad (g_0 + h_0) = 4.0$$

b) The reference model

$$-2.0\ y(k-1) + 3.0\ y(k) = r(k)$$

c) The Input: The input signal r(k) was chosen to be sampled square pulses of magnitude equal to ± 0.2 and a duration of 100 sec.

Simulation Results: Simulation were carried out with the system parameters g and g varying linearly according to the following

equations:

$$g_1(k) = -4.0 - 0.01\ k$$

$$g_0(k) = 4.0 - 0.04\ k \quad ; \ 0 \leq k \leq 100$$
$$= 0.0 \qquad\qquad k > 100$$

The initial values of the parameters h_0 and h_1 are taken to be

zeroes. According to the adaptation technique described in section 3, the set of difference equations describing the dynamics of the adaptive mechanism are:

$$h_1(k) = h_1(k-1) - K_1\left[q_0 e(k) + q_1 e(k-1)\right]\left[q_0 u_1(k) + q_1 u_1(k-1)\right]$$

$$h_0(k) = h_0(k-1) - K_0\left[q_0 e(k) + q_1 e(k-1)\right]\left[q_0 u_0(k) + q_1 u_0(k-1)\right]$$

$$-2\ u_0(k-1) + 3\ u_0(k) = -y(k)$$

$$-2\ u_1(k-1) + 3\ u_1(k) = -y(k-1)$$

The constant parameters of the adaptive mechanism were obtained by trial and error technique to be:

$$K_o = 22.0, \quad K_1 = 5.5, \quad q_o = 2.0, \quad \text{and} \quad q_1 = -1.$$

The results of the simulation of this system are shown in Figure (2).

2) Example 2

a) The adjustable system:

$$g_3\ c(k-3) + (g_2 + h_2)\ c(k-2) + (g_1 + h_1)\ c(k-1)$$
$$+ (g_o + h_o)\ c(k) = r(k)$$

Where the system parameters g_o, g_1, and g_2 are assumed to vary and g_3 are constant. The numerical values of the system parameters initially are:

$$(g_o + h_o) = 7.96 \qquad (g_1 + h_1) = -11.82$$
$$(g_2 + h_2) = 5.86 \qquad g_3 = -1$$

b) The reference model

$$2.25\ y(k-2) - 7.5\ y(k-1) + 6.25\ y(k) = r(k)$$

c) Input: The input signal $r(k)$ was chosen to be as in the previous example.

Simulation Results: Simulation were carried out with the adjustable system parameters vary linearly as follows:

$$g_2(k) \doteq 5.86 - 0.01\ k$$
$$g_1(k) = -11.82 - 0.01\ k$$
$$g_o(k) = 7.96 - 0.01\ k$$

Initially h_o, h_1, and h_2 were taken to be zeroes, and the adaptive mechanism equations for this system are:

$$h_o(k) = h_o(k-1) - K_o\left[q_o e(k) + q_1 e(k-1) + q_2 e(k-2)\right]$$
$$\left[q_o u_o(k) + q_1 u_o(k-1) + q_2 u_o(k-2)\right]$$

$$h_1(k) = h_1(k-1) - K_1\left[q_o e(k) + q_1 e(k-1) + q_2 e(k-2)\right]$$
$$\left[q_o u_1(k) + q_1 u_1(k-1) + q_2 u_1(k-2)\right]$$

$$h_2(k) = h_2(k-1) - K_2\left[q_o e(k) + q_1 e(k-1) + q_2 e(k-2)\right]$$
$$\left[q_o u_2(k) + q_1 u_2(k-1) + q_2 u_2(k-2)\right]$$

$$2.25\, u_0(k-2) - 7.5\, u_0(k-1) + 6.25\, u_0(k) = -y(k)$$

$$2.25\, u_1(k-2) - 7.5\, u_1(k-1) + 6.25\, u_1(k) = -y(k-1)$$

$$2.25\, u_2(k-2) - 7.5\, u_2(k-1) + 6.25\, u_2(k) = -y(k-2)$$

The constant parameters of the adaptive mechanism were obtained by trial and error technique to be:

$$K_0 = 15 \quad ; \quad K_1 = 30 \quad ; \quad K_2 = 30$$

$$q_0 = 3 \quad ; \quad q_1 = -3 \quad ; \quad q_2 = 1.$$

The results of the simulation of this example are given in Figure (3).

Comments:

1) The response of the models chosen in the above examples are initially very near to the response of the adjustable systems, but this is not a condition, the only condition is that the desired output response could be obtained by the variations of the compensating parameters h_i .

2) The aim of the introduced examples were to prove the applicability of the proposed adaptation method only, our interest were not to find the shortest time of adaptation or the exact values of the adaptive mechanism-parameters which could be obtained by trial and error technique.

3) In the above examples, the behaviour of the adaptive mechanism were studied when the rate of variation of the parameters g_i were doubled, also the system were simulated for great initial error. In both cases, the adaptive mechanism proved to be working satisfactory.

5. CONCLUSION

The concept of gradient technique is used to seek out solutions of discrete time model reference adaptive system. In these systems the knowledge of the exact order of the nonadaptive system is not required. The only restriction is that the number of controllable parameters, to be adjusted by the adaptive mechanism can not be greater than the number of coefficients that appear in the reference model equation. The order of the generalized error function should be chosen equal to the order of the characteristic equation of the reference model.

Computational experience from the solved examples shows that the stability of the adaptive system depend on the choice of the adaptive mechanism parameters.

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FIGURE CAPTIONS

Figure (1) Adaptive model-following control system.

Figure (2) Digital Computer Simulation for the first adaptive Example.

Figure (3) Digital Computer Simulation for the Second adaptive Example.

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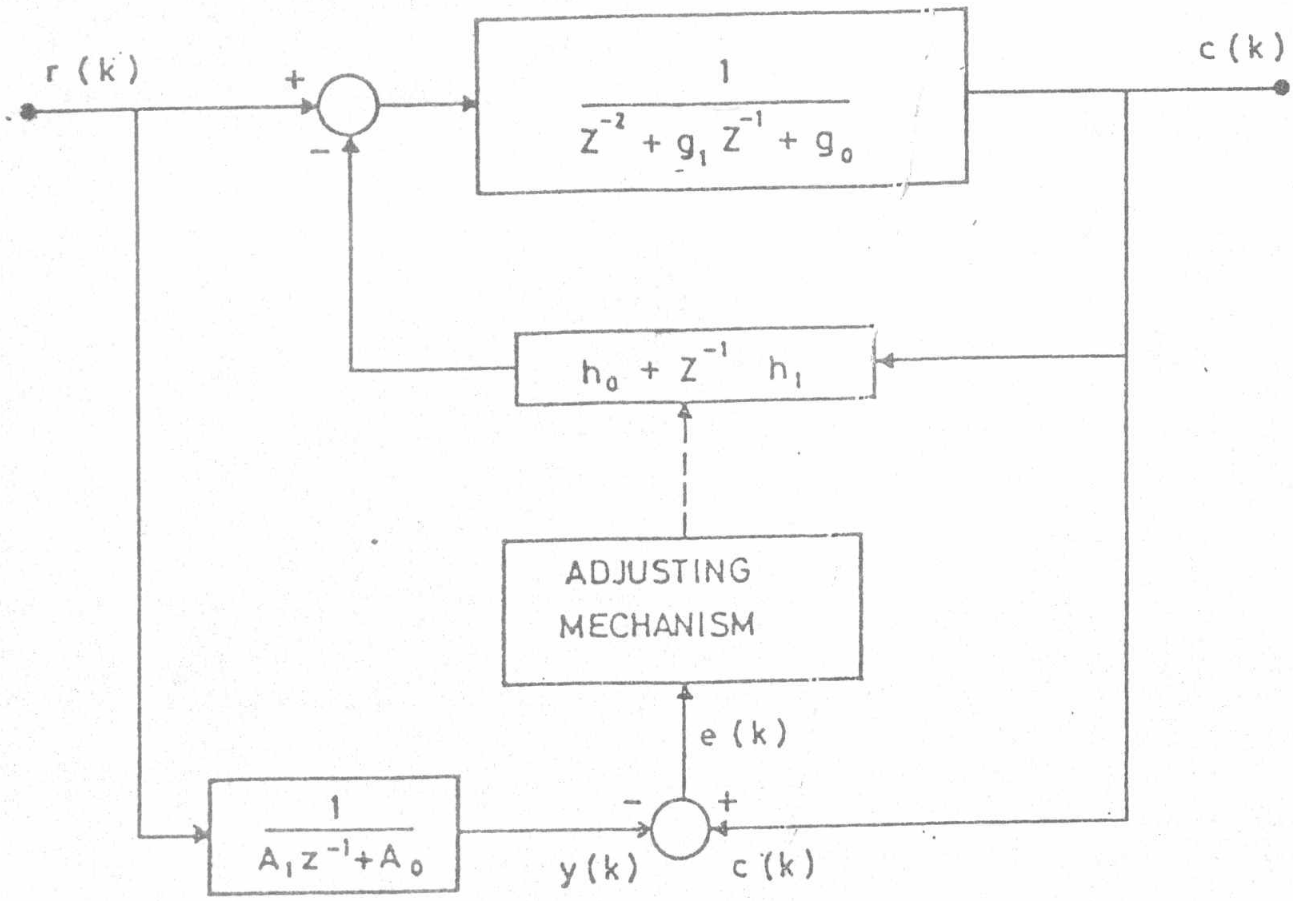

Figure (1)

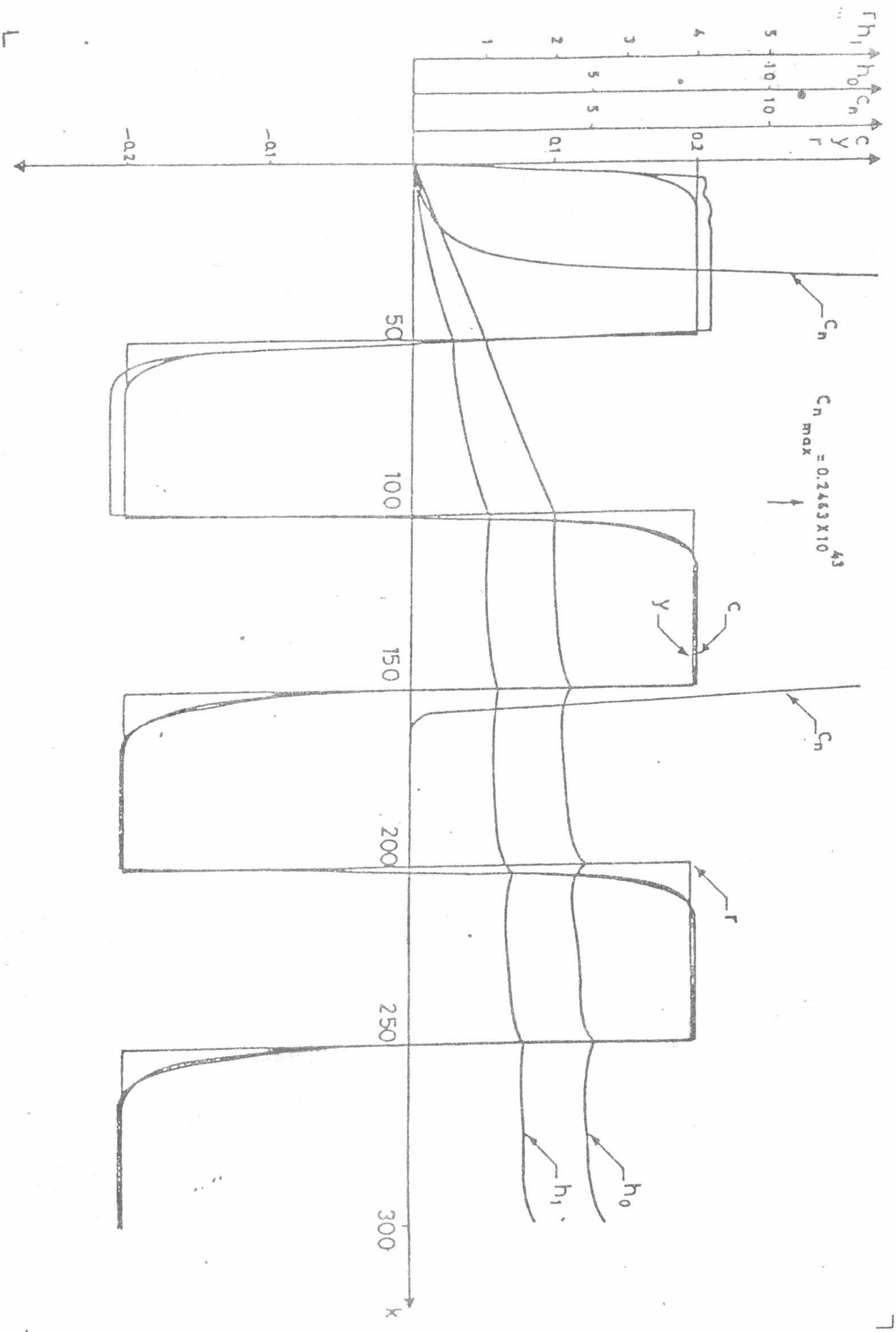
$C_{n_{max}} = 0.2463 X 10^{43}$
h_1
h_0
C_n
c
y
r
x
50
100
150
200
250
300
0.1
0.2
-0.1
-0.2

Figure (2)

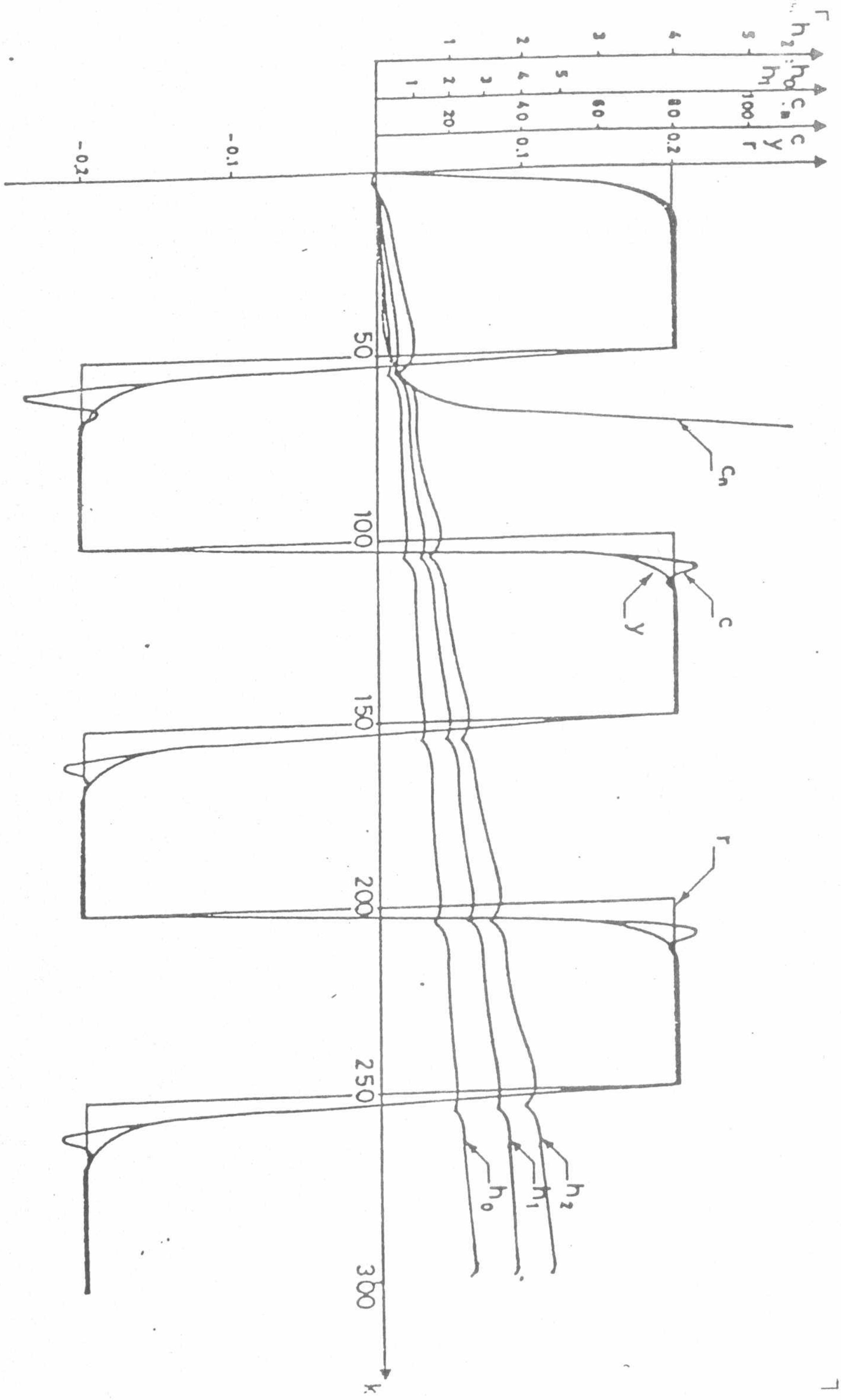

Figure (3)