GC-7	969

MILITARY TECHNICAL COLLEGE

CAIRO - EGYPT

NEAR OPTIMAL GUIDANCE FOR SHORT RANGE HOMING MISSILE,USING THE MINIMAL ORDER OBSERVER

M.O. Tantawy *   M.A. El-Leithy **

ABSTRACT

This paper brought a new trend giving a baseline algorithm which yield an approach for the design of the near optimal controller for the short range homing guided missile. This new trend is based on the minimal order observer as state estimator. The problem is studied based on a linear discrete model taking into consideration the dynamics of the missile motion, the target manuvering capability, errors of measuring sensors and launch initial conditions. Quadratic criterion penalizing the state trajectory as well as the control is used. The near optimal control is derived through the linear quadratic Gaussian technique (L.Q.G.) and the minimal order observer as stato estimator. The derived control accounts for bounded control variable, Limited missile manuvering capability and bounded minimum terminal miss-distance at the intercept point.

* M.T.C., Cairo, Egypt
**Air Defence Forces Research Department.

SUMMARY

This paper brought a new trend giving baseline algorithm which yield an approach for the design of the near optimal controller for the short range homing guided missile. This new trend is based on the minimal order observer as state estimator. The problem is studied based on a Linear discrete model taking into consideration the dynamics of the missile motion, the target manuvering capability,errors of measuring sensors and launch linitial conditions. Quadratic criterion penalizing the state trajectory as well as the control is used. The near optimal control is derived through the linear quadratic Gaussian technique (L.Q.G) and the minimal order observer as state estimator. The derived control accounts for bounded control variable, Limited missile manuvering capability and bounded minimum terminal miss-distance at the intercept point.

1- INTRODUCTION

The task of guiding a missile to a target is affected by a number of factors and constraints;e.g. target manuvering capability, errors of measuring sensors, autopilot dynamics, missil's aerodynamic frame, missil's thrust, bounded control variables, limited missile manuvering capability and launch initial conditions. Terminal guidance process has the function of guiding the missile to the intercept point with some required accuracy in the miss-distance. Through the modern control theories and the Kalman filter as state estimator short range homing guided missile's optimal controller design algorithm has been declared (1). Kalman filter has much troubles in the real time mechanization especiayly in the case of large dimensional systems as it adds complexity in the hardware which increases the error and the time of calculation (2) . The minimal order obsserver is simpler but less optimal estimator (3). The principle contribution of this work is the investigation of the discrete minimal order observer as state estimator to achieve with the optimal feed back;derieved through the linear Quadratic Gaussian technique (L.Q.G);a near optimal Homing terminal stochastic guidance law that accounts for limited state trajectory as well as the bounded missile control variables and minimum terminal missdistance.

2- GUIDANCE PROBLEM MODEL

Homing guidance problem for short range homing guided missile against manuvering target is formulated (4) accounting for:

(1) Cartizian control is used in which the guidance controller produces two controls for the missile motion in pitch and yaw plane independently (lateral control).

(2) "X form" missile is considered so the pitch and yaw motion will be crosscoupled only due to roll rate, so we must have roll rate control besides, and then we have two identical lateral controllers and one roll control.

(3) Vertical plane interception geometry is considered where a nonrotating orthogonal coordinate system is defined with the x-axis chosen along the line of sight between the intercepter and the target at the begining of the engagement. The centre of the coordinate system moves with the target but the coordinate axis donot rotate , if the guidance system works well, then the line of sight rotates very little along the missile's trajectory except near the end where the range to go becomes small. At "t_f" the missile trajectory intersects the y axis almost perpendicularly and the terminal miss

6 distance is approximatly $y\ (t_f)$, fig (2.1)

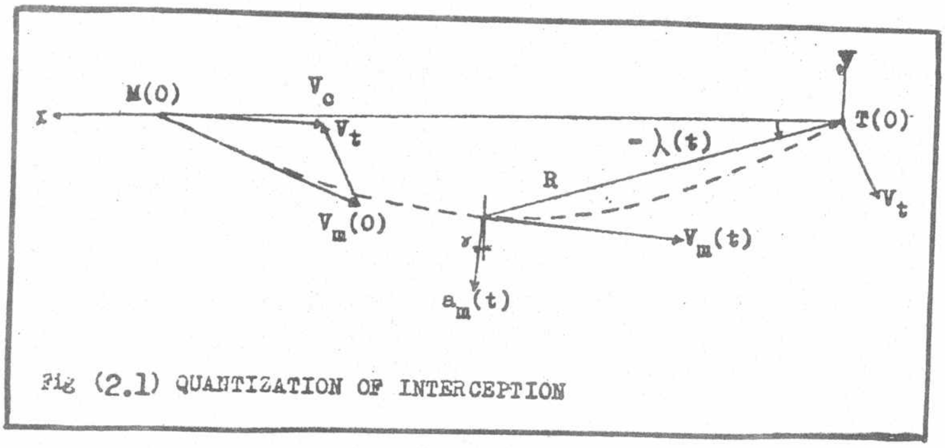


Fig (2.1) QUANTIZATION OF INTERCEPTION

a-let the guidance control variable available is the control surface deflection "δ" considering the autopilot with a constant unite gain. "δ" acts through the dynamics of the missile producing the dynamic motion of the missile expressed in the normal acceleration "$a_m(t)$" that affect the interceptor flight path.

b-let the missile is nonthrousting drag-free vichle and "$a_m(t)$" is approximatly perpendicular to the missile's velocity as shown in (fig 2.1), Hence;

$$a_{my}(t) = - a_m(t) \quad \cos \gamma$$

where $\quad a_m\ (t) = /\ a_m(t)\ /$

If the orintation of V_m is assumed to be slowly varying $\cos\gamma$ can be treated as known scale factor, let us assume $\cos\gamma = 1$.

c-In particular the target acceleration "$a_t(t)$" has an effect on the guidance dynamics as shown in (fig 2.1) as following

$$\ddot{y} = - a_{ty}(t) + a_{my}(t) \qquad (2.1)$$

(4) The airframe of the missile is shown in Fig (2.2)

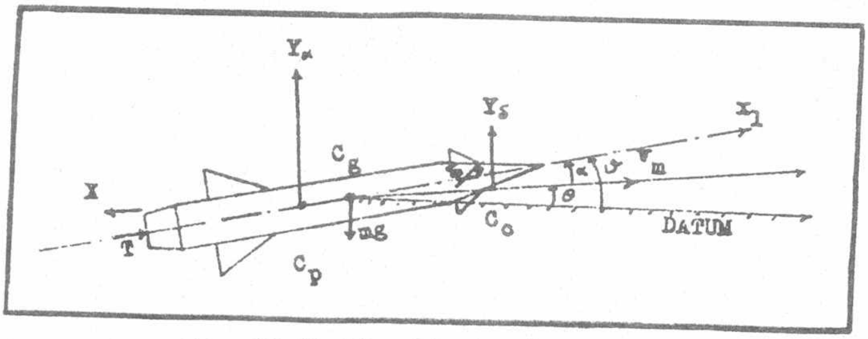


Fig.(2.2) Missliz Atrframe.

where : α= angle of attack
θ= angel of missile flight path direction.

ϑ = direction of missile axis

δ = deflection of control fin

V_m = missile speed.

w_{z1} = pitch rate of the missile.

a'_m = normal acceteration due to body wing lift

L_α, L_δ, M_α, M_δ , M_{wz1} are the stability drevatives with known values for specific missile airframe.

Hence;the dynamics of the missile motion is formulated as following:

$$\dot{w}_{z1} = M_{w_{z1}} \cdot w_{z1} + \frac{M_\alpha}{V_m L_\alpha} \grave{a}_m + M_\delta \cdot \delta \tag{2.2}$$

$$\dot{\grave{a}}_m = V_m L_\alpha W_{z1} - L_\delta \grave{a}_m - V_m L_\alpha L_\delta \cdot \delta$$

$$a_m = \grave{a}_m + V_m L_\delta \cdot \delta$$

(5) The manuvering terget motion has random structure and the target acceleration is assumed to be a markove process of first order with the following mathematical formulation

$$\dot{a}_{ty} = f_t \cdot a_{ty} + \mu(t) \tag{2.3}$$

where $\mu(t)$ is zero mean white Gaussian noise process having the following statistical parameters:

$$E\ (\ \mu(t)\,\mu(\tau) = f_t^2 \ \sigma^2 \ \delta\ (t-\tau) \tag{2.4}$$

$$E\ (\ a_{ty}^2\ (\tau)) = \sigma^2 \tag{2.5}$$

(6) Considering the states describing the guidance problem of a short range homing guided missile against manuvering target in the pitch plane to be:

y = miss - distance measure

W_{z1} = pitch rate

$\grave{a}_m$ = normal acceleration of the missile due to body wing lift

a_{ty} = target acceleration

$\dot{y}$ = derivative of the miss distance

let us define:

$$X = \text{the state vector} = (\ y\ w_{z1}\ a'_m a_{ty}\ \dot{y})^T \tag{2.6}$$

A_c = dynamics matrix =

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & M_{wz1} & \frac{M_\alpha}{V_m L_\alpha} & 0 & 0 \\ 0 & V_m L_\alpha & -L_\alpha & 0 & 0 \\ 0 & 0 & 0 & f_t & 0 \\ 0 & 0 & -1 & -1 & 0 \end{bmatrix} \tag{2.7}$$

B_c = control transion vector = $(0\ M_\delta\ -V_m L_\alpha\ L_\delta\ 0$

6

$- V_m L_s)^T$ (2.8)

$U(t)$ = control variable = $\delta(t)$ (2.9)

$W(t)$ = disturbance noise vector = $(0\ 0\ 0\ \mu(t)\ 0)^T$ (2.10)

$$E(w(t).\ W(\tau) = Q_w^{(t)}\ \delta(t - \tau) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & f_t^2\ \sigma^2 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (2.11)$$

Hence;the state space model of the problem is :

$$\dot{X}(t) = A_c\ X(t) + B_c\ U(t) + W(t) \quad (2.12$$

(a) let us assume the following given data for the problem :

$M_{wzl} = - 0.455$ Sec^{-1}

$M\ \alpha = -8.4$ Sec^{-2}

$M\ \delta = -71.2$ Sec^{-2}

$L\alpha = -0.315$ Sec^{-1}

$L_\delta = 0.058$ Sec^{-1} (2.13)

$V_m = 1800$ ft/Sec = 1.6 (Mach)

$V_c = 2000$ ft/Sec = 1.8 (Mach)

$f_t = -0.3$ Sec^{-1}

$\sigma^2 = 9 \times 10^3\ (ft/Sec^2)^2$

(b) The initial condition of the problem X(0) is assumed to be random Gaussian vetor with the following statistical data:

$$x(0) = X_o\ ;\ X_0^T = (0\ \ 0\ \ 0\ \ 90\ \ 0\ ) \quad (2.14)$$

$$E(x_o) = \bar{X}_o\ ;\ \bar{X}_o^T = (0\ \ 0\ \ 0\ \ 0\ \ \ 0\ ) \quad (2.15)$$

$$E(x_o x_o^T) = M_o; M_o = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 10^{-4} & 0 & 0 & 0 \\ 0 & 0 & 10^3 & 0 & 0 \\ 0 & 0 & 0 & 9x10^3 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (2.16)$$

(7) The measurements avaliable are :

$\lambda(t)$ = line of sight angle by a homing rader sensor (4) , which has a constant unite gain and measurement is corrupted with zero mean and variance

$$\sigma_a^2 = \frac{1.9 x 10^2}{2}\ (rad)^2 \quad (2.17)$$

r_{tg}

where r_{tg} is the range to go.

W_{z1} = pitch rate measured by a rate gyro (4) , which has a constant unite gain and measurement is corrupted with white noise with zero mean and variance σW^2

$$\sigma W^2 = 0.5 \times 10^{-6} \ (\text{rad/sec}^2)^2 \qquad (2.18)$$

Hence;let us define:

$$Y(t) = \text{the measurement vector} = (\ \lambda(t) \quad W_{z1})^T \qquad (2.19)$$

$$H(t) = \text{measurement matrix} = \begin{bmatrix} \frac{1}{V_c(t_f-t)} & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix} \qquad (2.20)$$

V(t) = noise measurement vector where :

$$E(v(t)\ .\ V(\tau) = R\ (t).\ \delta(t-\tau)$$

$$= \begin{bmatrix} \frac{190}{r_{tg}^2} & 0 \\ 0 & 0.5 \times 10^{-6} \end{bmatrix} (t-\tau) \qquad (2.21)$$

$$\text{Hence };Y\ (t) = H\ (t)\ .\ X\ (t)\ +\ V(t) \qquad (2.22)$$

3- PROBLEM FORMULATION:

Our task is to design a controller to guide the missile to the target modeled in section 2;(2.12,2.22) with the following main objects :

(1) Bounded control variable $\delta(t) \leqslant 0.3$ rad.

(2) Limited missile manuvering capability $a_m \leqslant 10$ g

(3) Terminal miss distance at tf is minimum $y(t_f) \leqslant 50$ ft

Through the linear quadratic Gaussian regulation problem we can achieve such mentioned requirements, using the minimal order observer as state estimator.

Hence;the desired object is formulated mathematically using the quadratic perfomrance index:

$$J = \tfrac{1}{2} X'(t_f).\ S.\ x(t_f) + \tfrac{1}{2} \int_0^{t_f} X'(t).Q_c.\ X(t)+U'(t).R_c.U(t)dt \qquad (3.1)$$

i.e we wish to bring the guidance system from an initial state x(o) to a terminal state $x(t_f)$ using acceptable level of control and not exceeding acceptable dispersion of states x(t) during the flight trajectory and realize the minimum miss distance at the terminal time. This task is accomplished by minimizing the performance index which is of quadratic form in states and control, where:

S, Q_c are positive semidefinite matrices

R_c is a positive definite matrix.

An appropriate choice of these matrices must be made to obtain acceptable levels of $(x(t_f), x(t)$ respectively. Such a choice have no well predetermined

6

way, so we will depend on the computer simulation.

4- DISCRITIZATION OF THE SYSTEM

1- Through suitable choice of sampling period "T" the model of the guidance problem is discretized such that:

(1) The controllability of the continous system must be preserved in the discretized model (5).

(2) The optimal cost must not be missed as function of the sempling period (5).

(3) The uncertainty in the states of the system as a function of the sampling period must not exceed some required covariance. (6).

(4) The time of the system's response must be less than the sampling period in the wide sence of control. But if it is not possible as in our case ((Proportional + integral) control), controllability must be preserved and the states of the system must be constraind during flight to the acceptable levels by the optimal gaindesign (suitable choice of S, Q_c, R_c).

(5) The time of calculation of the control command must not exceed the sampling period by any way, and as we have to control the error in the trajectory in a very short time, the time of calculation must be as small as possible, (7).

Hence; the sampling period is to be chosen to take values lie between two extremes:

a- T_{max} = first time at which controllability is lost

(5) . Hence; $T_{max} = 1.08$ sec. (4.1)

b- t_c = possible time of calculation, (7).

available $t_c = 4.5$ m sec. (4.2)

Hence; from (4.1), (4.2) we can say :

$$4.5 \text{ m. sec} < T < 1.08 \text{ sec}$$

let us choose T = 50 (m sec) as a first estimate for T and through computer runs for simulation we can choose the most suitable T.

2- Refering to (8) the discretized guidance problem of the continus guidance given by (2.12), (2.22), (3.1) is the following:

t_f = NT , N = number of the sampling periods

$$x_{k+1} = A.X_k + BU_k + \xi_k ; x(0) = X_o \quad (4.3)$$

$$y_k = H_k.X_k + v_k \quad (4.4)$$

$$J = \tfrac{1}{2} X_N'.S.\ X_N + \tfrac{1}{2} \sum_{k=0}^{N-1} x_k' Q x_k + 2 X_k M U_k + U_k'.R.U_k \quad (4.5)$$

where :(1) $A = \phi(T) = e^{A_c.T}$

(2) $B = \Gamma(T-S) = \int_o^T e^{A_c(T-S)} . B_c . ds$

(3) $Q_w = \int_o^T \phi(T-S).\ Q_w.\ \phi'(T-S)\ ds$

where : Q_w is the coveriance matrix of the white noise sequence ξ_k disturbing the discrete model. Q_w is the covariance matrix of the white noise w(t) disturbing the continuous model.

(4) $Q = \int_0^T \phi'(S) \cdot Q_c \cdot \phi(S)\, ds$

(5) $M = \int_0^T \phi'(S) \cdot Q_c\, \Gamma(S)\, ds$

(6) $R = T.R_c + \int_0^T \Gamma'(S) \cdot Q_c \cdot \Gamma(S)\, ds$

5- Near Opitmal Control Law (Guidance Law) Calculation

$$U_i = G(i) \cdot \hat{X}_i \tag{5.1}$$

where :1-G_i is the feed back optimal gain (8)

$$G(i) = -(R + B' \cdot \eta(i+1) \cdot B)^{-1} (M' + B \cdot \eta(i+1) \cdot A) \tag{5.2}$$

$$\eta(i) = A' \cdot \eta(i+1) \cdot A + Q - G'(i) \cdot (R+B' \cdot \eta(i+1) \cdot B) \cdot G(i) \tag{5.3}$$

with final condition $\eta(N) = S$ (5.4)

2- X_i is the estimated state vector using the minimal order observer with the following algorithm (3) :

Given the linear discrete stochastic system.

$$X_{i+1} = A_i X_i + B_i W_i \;, X(0) = X_o \tag{5.5}$$

$$Y_i = H_i\, X_i + vi$$

where:

X_i = "n" dimensional state vecotr at time instant "i"

U_i = "P" dimensional control vector at time instant "i"

Y_i = "m" dimensional measurement vector at time instant "i"

H_i = measurement matrix of the form $(I_m/0)$ and if not it must be normilized (3).

X_o = is a random vecotr with known mean and covariance

$E(X_o) = \bar{X}_o$, $E((X_o-\bar{X}_o)(X_o-\bar{X}_o)') = M_o$

w_i, vi are assumed to be random vectors with known means and covariances:

$E(w_i) = o$, $E(vi) = 0$ for all i

$E(w_i w_j') = Q_i\, \delta_{ij}$, $E(v_i v_j') = R_i\, \delta_{ij}$ for all i,j

where :

δ_{ij} is the kronecker delta.

R_i is positive definite ($R_i > 0$)

Q_i is positive semidefinite ($Q_i \geqslant 0$)

The various random vectors are assumed to be mutually uncorrelated, that is to say:

$E(X_o w_i') = 0$ for all i

$E(X_o v_i') = 0$ for all i

$E(w_i v_j') = 0$ for all i,j

6

and w_i, v_i are time wise uncorrelated sequances which shall be reffered as white sequences. To construct the minimal order observer's recursive algorithm we proceed as following :

$$\hat{X}_{i+1} = A_i . \hat{X}_i + P_{i+1} . T_{i+1} . B_i . U_i + V_{i+1} (Y_{i+1} - H_{i+1} . A_i . \hat{X}_i)$$
$$, \hat{X}_o = \bar{X}_o \qquad (5.7)$$

where:

$$P_{i+1} = \begin{bmatrix} 0 \\ \hline I_{n-m} \end{bmatrix} n$$

$$V_{i+1} = \begin{bmatrix} I_m \\ \hline K_{i+1} \end{bmatrix} n$$

$$T_{i+1} = \begin{bmatrix} -K_{i+1} & I_{n-m} \end{bmatrix}$$
$$\quad n$$

$$K_{i+1} = \Omega^i_{21} (\Omega^i_{11} + R_{i+1})^{-1}$$

$$\Omega_i = A_i . P_i . \overline{\varepsilon_i \varepsilon_i'} . P_i' . A_i' + A_i . V_i . R_i . V_i' . A_i' + Q_i$$

$$\begin{array}{cc} \rightarrow n \leftarrow & \\ = \left[\begin{array}{c|c} \Omega_{11} & \Omega_{12} \\ \hline \Omega_{21} & \Omega_{22} \end{array}\right] & \updownarrow m \\ \rightarrow m \leftarrow & \end{array}$$

$$; \Omega_o = A_o . M_o . A_o + Q_o$$

$$\varepsilon_{i+1} \varepsilon_{i+1} = T_{i+1} . \Omega_i . T_{i+1}$$

6- Results, Discussions and Conclusions

The design objectives are achieved through the computer calculations for specified preknown flight time in the interval $t_f = 1 \div 5$ sec.

1- The weighting matrix of the states during flight trajectories presented in the cost functional (3.1) Q_c is :

$$Q_c = \begin{bmatrix} 0.05 & 0. & 0. & 0. & 0. \\ 0. & 1. & 0. & 0. & 0. \\ 0. & 0. & 1. & -1. & 0. \\ 0. & 0. & -1. & 1. & 0. \\ 0. & 0. & 0. & 0. & 1. \end{bmatrix}$$

The obtained Q_c is not a diagonal one but it has out-of-diagonal elements translating our requirement (minimization of the difference between the target manuverability and the missile manuverability/$(a_m - a_t)$/). Or generally speaking minimization of the linear combination of states as following? we can choose our P.I as a quadratic performance of the form:

$$J = \int_o^{t_f} q_1 y^2 + q_2 W_{z1}^2 + q_3 (a_m - a_t)^2 + a_4 \dot{y}^2 + r_1 U_1^2 \, dt$$

i.e. we demand to minimize the difference between missile and targets accelerations.

$$(a_m - a_t)^2 = a_m^2 - 2\,a_m a_t + a_t^2$$

$$\text{then } J = \int_o^{t_f} (q_1 y^2 + q_2 W_{21}^2 + q_3 a_m^2 + q_3 a_t^2 - 2q_3 a_m a_t + q_4 \dot{y}^2 + r_1 U_1^2)\, dt$$

$$= \int_o^{t_f} (X\, Q_c X + U\, R_c\, U)\, dt$$

$$\text{then } Q_c = \begin{bmatrix} q_1 & 0 & 0 & 0 & 0 \\ 0 & q_2 & 0 & 0 & 0 \\ 0 & 0 & q_3 & -q_3 & 0 \\ 0 & 0 & -q_3 & q_3 & 0 \\ 0 & 0 & 0 & 0 & q_4 \end{bmatrix}$$

To properly choose the weighting factor upon y, (q_1 in the Q_c), we can consider the output of the missile as a dynamic system in $V_{my} \equiv \dot{y}$. So the kinematic part of our problem is (y) which is the integration of the missile output ($\dot{y}$) . The controller is designed by the L-Q technique, takes a feed, back from all states (both dynamics and kinematics) through the optimal gain matrix calculated. So our controller is in fact, a proportional-integral one. In the design of the cost functional of such a controller by the L.Q. technique, we have to separte the effect of proprtional part and the intrg-ral action (9). Such separtion allows the integral part to operate only during the steady state period to eliminate the steady state error. This decoupling of both effects can nearly be achieved by choosing the weighting factor q_1 upon the integral part to be much less than that upon the propor-tional part (9). Doing this, we will be sure that the behavuoir of the system during the transiant period will be mainly given by the proportional part which have a good acceptable performance. While the integral action will interfere only at the final time to minimize the final miss-distance (y(tf). This is achieved by putting a weighting factor on the integral part (y) at the final time and then the cost functional will be :

$$J = \tfrac{1}{2} X^{`}(t_f\, .S.X\,(t_f + \tfrac{1}{2} \int_o^{t_f} X^{`} Q_c\, X + U^{`} R_c\, U\, dt$$

2- $$S = \begin{bmatrix} 50 & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. \end{bmatrix}$$

such a form shows that the integral action interferes at the final time, and hence the final time is still in the transiant period an error still exist and it is minimized by suitable choise of (S).

3- R_c (0.2×10^5)

R_c is the weighting factor on the control level.
So the ratios (Q_c/R_c), (S/R_c) are the compromize between acceptable level of states and acceptable level of control

4- Using the minimal order observer for state estimation the calculated and estimated states are presented in a group of figurs as following.

a- Fig (6.1) shows the state "y" and the estimated state "$\hat{y}$" with respect to time. It is clear that they coincide to each other as the line of sight angle " λ" is measurable, then y is a measured state.

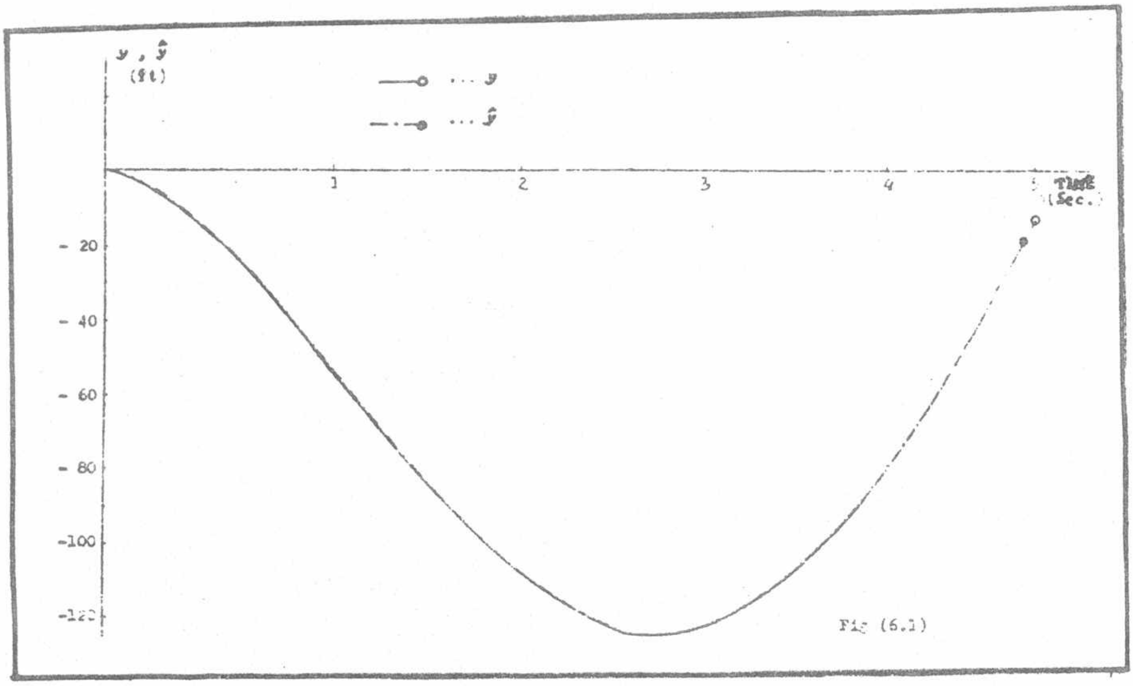


Fig (6.1)

b- Fig (6.2) shows the state "w_{z1}" and the estimated state "$\hat{w}_{z1}$" with respect to time. It is clear that they coincide to each other as w_{z1} is measurable.

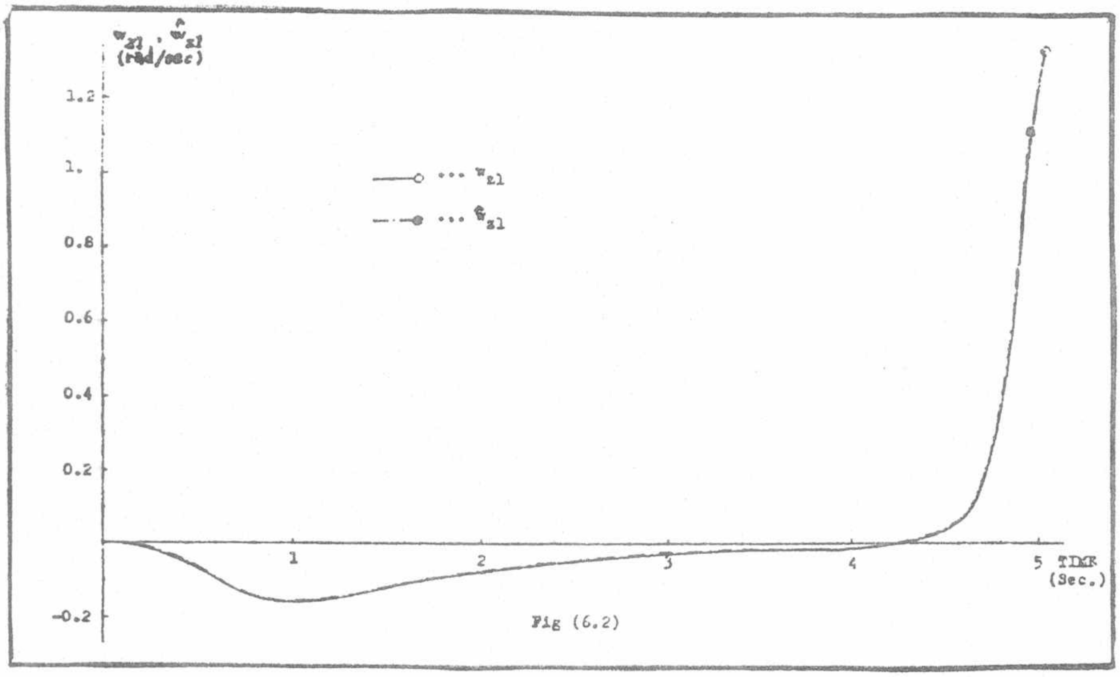
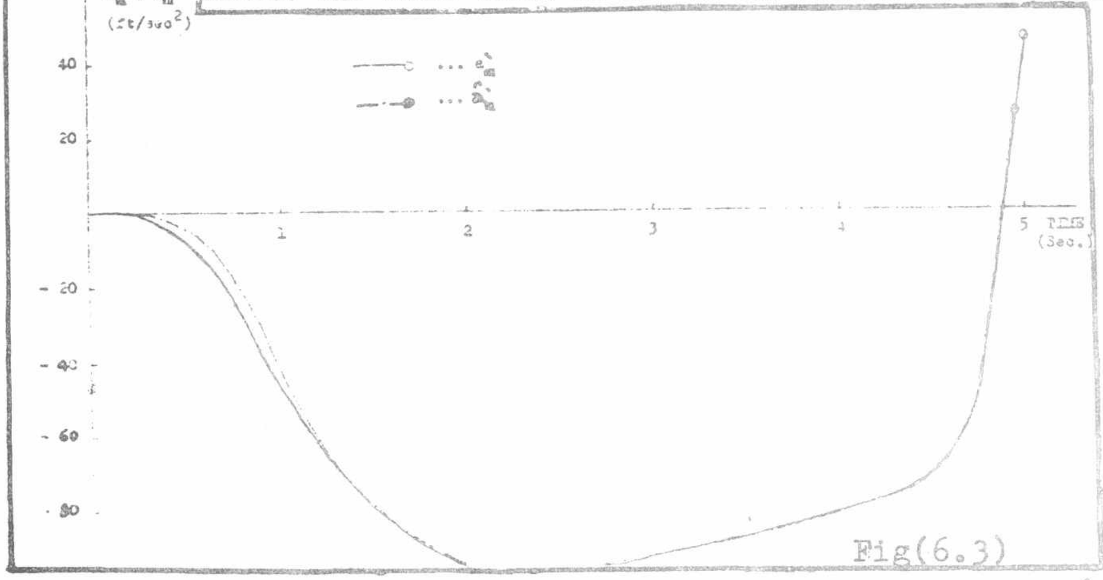


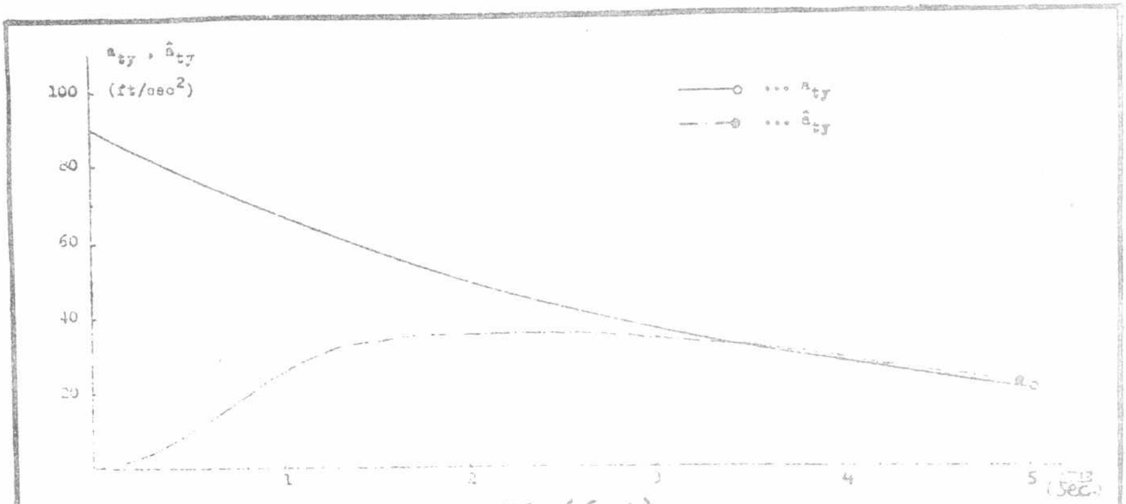
Fig (6.2)

c- Fig (6.3) shows the state "$\grave{a}_m$" and the estimated state "$\hat{\grave{a}}_m$" with respect to time. It is clear that both are very near to each other. They are rapidly approaching the same value. This is due to the zero vriance of the state as one state of the plant from point of view of plant disturbing noise. The small error is due to the random initial condition.



Fig(6.3)

d- Fig (6.4) shown the state "a_{ty}" and the estimated "$\hat{a}_{ty}$" with respect to time. It is clear that the minimal order observer is an asymptotic estimator with speed of response dependent on the sampling period.



Fig(6.4)

e- Fig (6.5) which shows that the sampling period affects the estimation error and the speed of response.

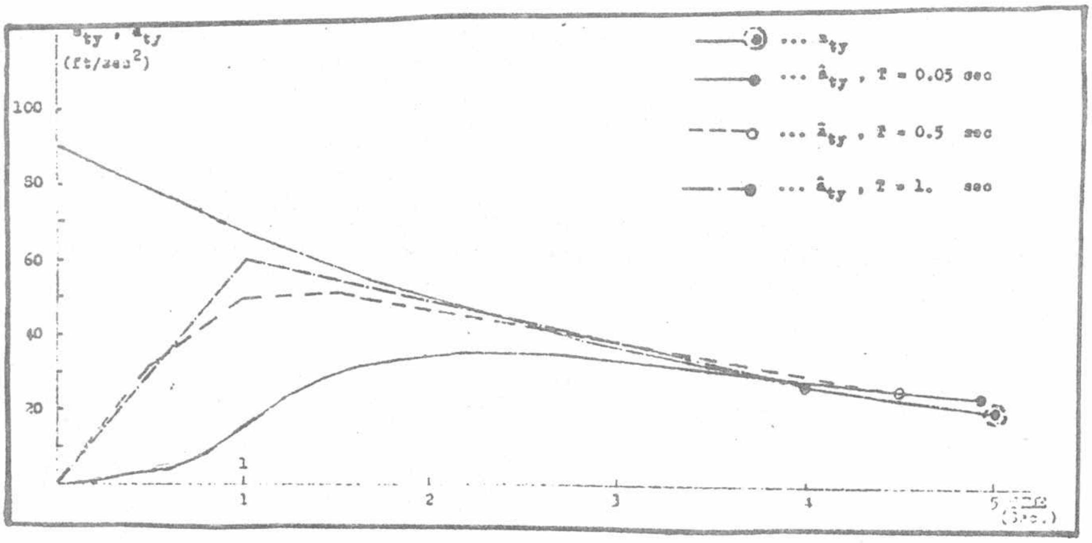
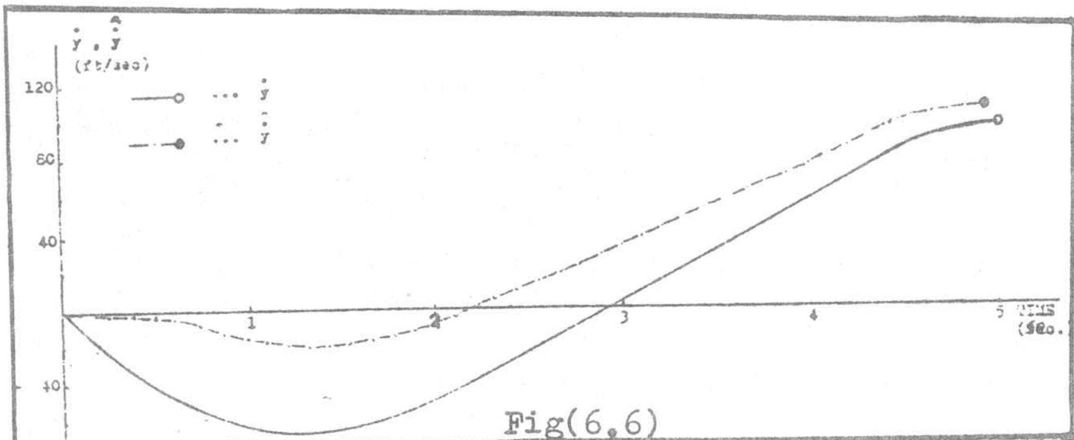


Fig. (6.5)

f- Fig. (6.6) shows the state "y" and the estimated state "$\hat{y}$" with respect to time. It is clear that the minimal order observer is an estimator for "y" and the error of estimation decreases with increasing the sampling period but the cost increases. Fig. (6.7).



Fig(6.6)

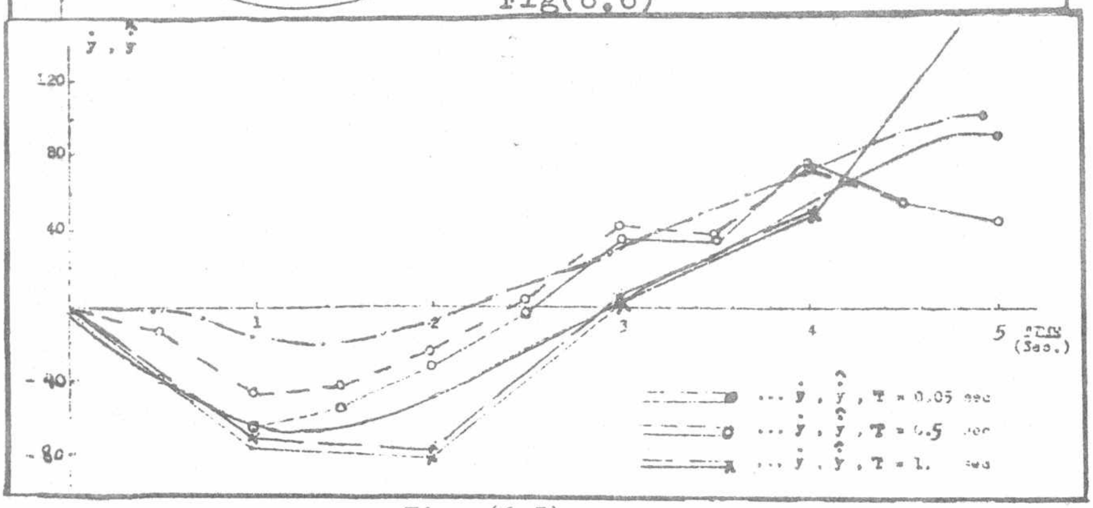


Fig. (6.7)

Hence;we conclude that minimal order observer is an asymptotic convergent estimator.

5- SYSTEM PERFORMANCE

The system performances are measured by the states $(y, \grave{a}_m)$ with the control "δ" which are presented in a group of figures as follows:

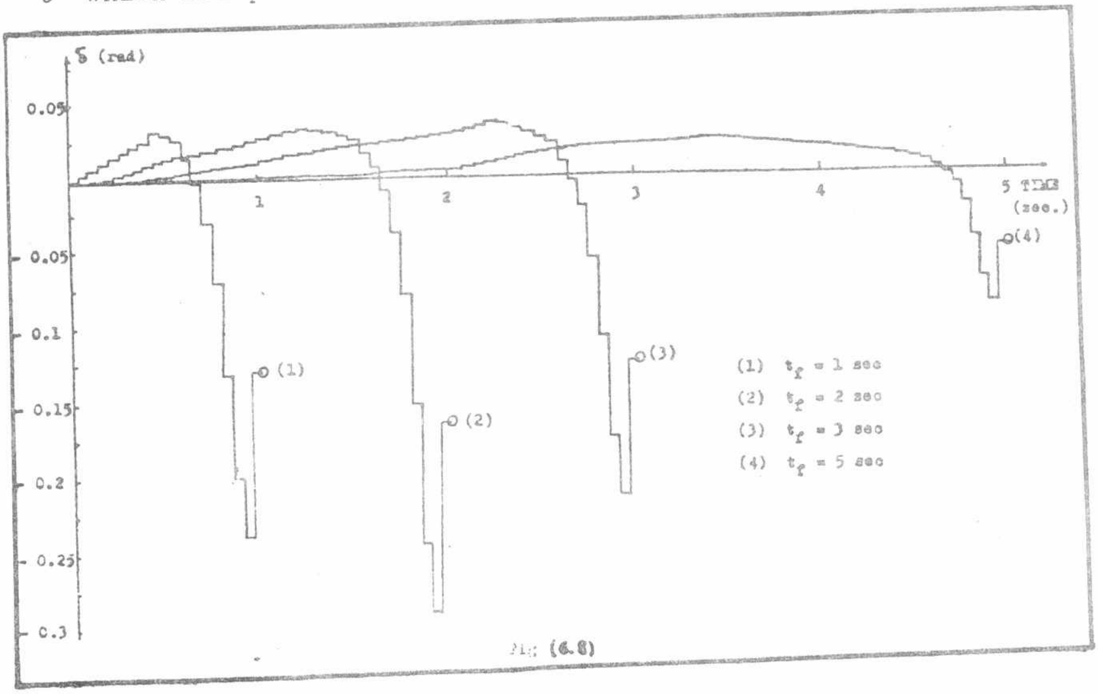
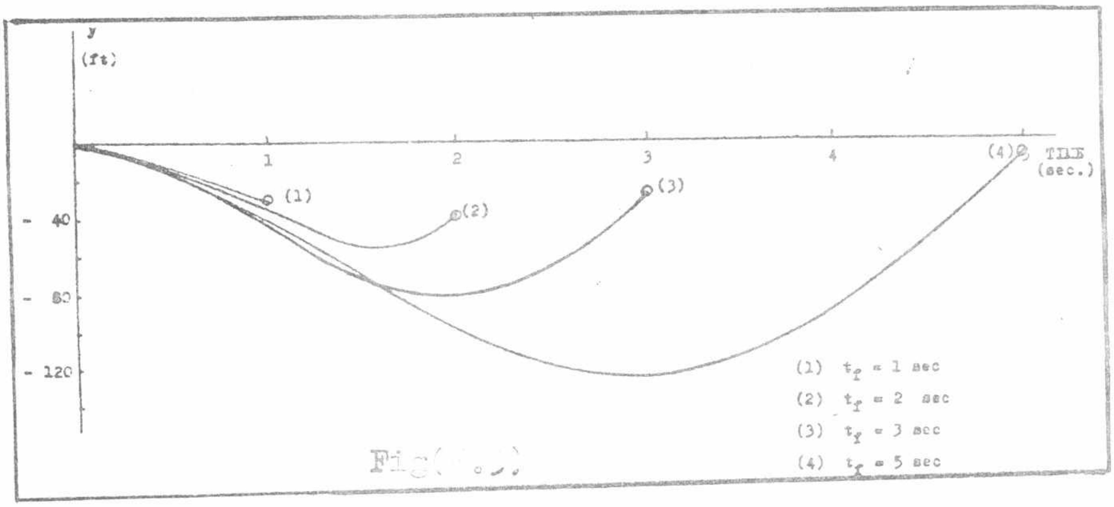


Fig (6.8)

a- Fig. (6.8) shows the control "δ" with respect to time for different "t_f".



Fig(6.9)

b- Fig. (6.9) shows the state "y" as a function of time for different "t_f" It is clear that "y" during flight gives a line of sight "λ" which is always.

vissible to the homing seeker. (practically homing sseker maximum capability for measurement is about 60^{o} (10)).

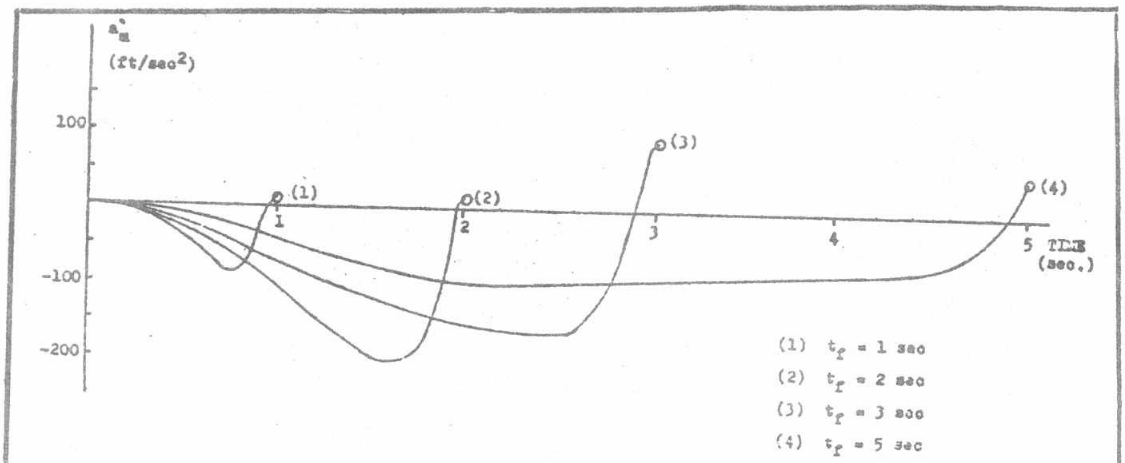


Fig (6.10)

c- Fig. (6.10) shows the normal acceleration "$a_m^{`}$" as a function of time for different "t_f".

d- Fig. (6.11) shows maximum control $/\delta_{max}/$ as a function of "t_f". It is clear that till t_f= 6 sec, $/\delta_{max}/ < 0.3$ rad.

e- Fig.(6.12) shows the final miss-distance/$y(t_f)$/as a function of "t_f". It is clear that till t_f=6 sec, $/y(t_f)/_{max} < 50$ ft.

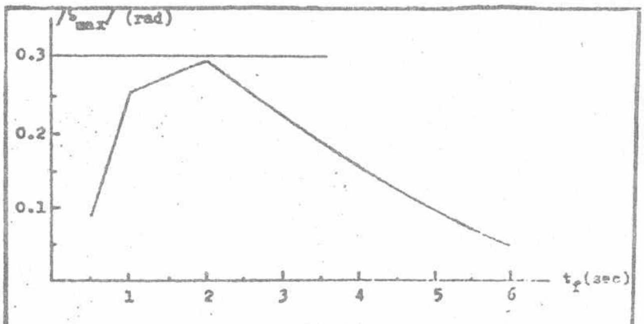


Fig (6.11)

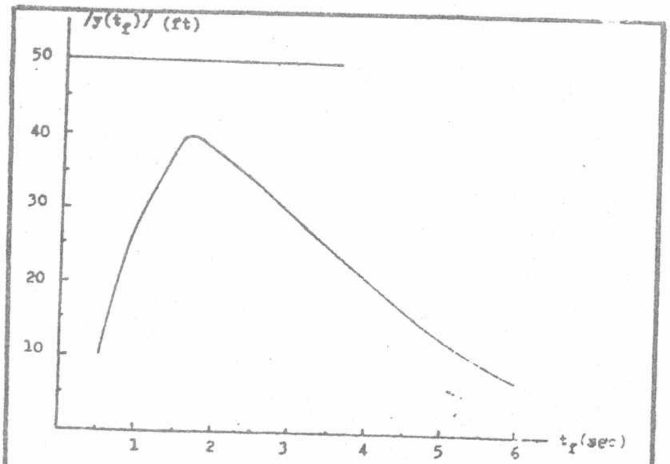


Fig (6.12)

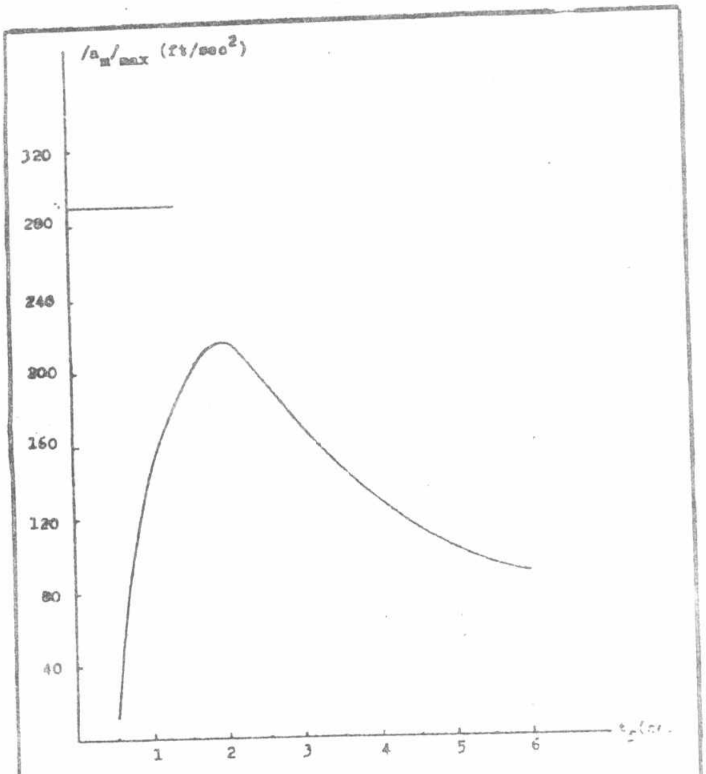


Fig (6.13)

f- Fig. (6.13) shows the maximum normal acceleration $/a_m/_{max}$ as a function of "t_f". It is clear that $/a_m/_{max} < 10$ g.

g- Fig.(6.14) shows the performance of the system ($/\delta/_{max}$, $/y(t_f)/_{max}$, $/a_m/_{max}$) with respect to the sampling period at t_f=5 sec. and and for fixed initial condition (X_o). It is clear that the sampling period ("T" =0.05 sec) is a suitable value to achieve all objectives of the design.

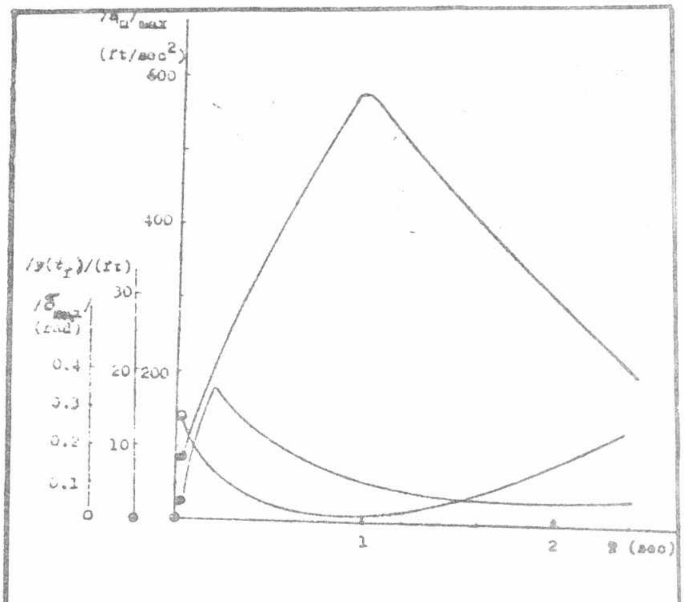


Fig (6.14)

Hence;we conclude that the designed controller with the minimal order observer achieves the design objectives till t_f= 5 with sampling period (T = 0.05 sec).

6- TARGET MANUVERABILITY IS INCREASED

The performances of the system when target manuverability is increased are presented in a group of figures as follows:

a- Fig. (6.15) shows the state "a_{ty}" and the corresponding estimated state "$\hat{a}_{ty}$" .It is clear that the minimal order observer is still a convergent asymptotic estimator.

b- Fig. (6.16) shows the normal acceleration "$a_m^{`}$" and the estimated "$\hat{a}_m^{`}$". It is clear that they cooncide to each other rapidly.

c- Fig. (6.17) shows the "$\dot{y}$" and the estimated state $\hat{\dot{y}}$.

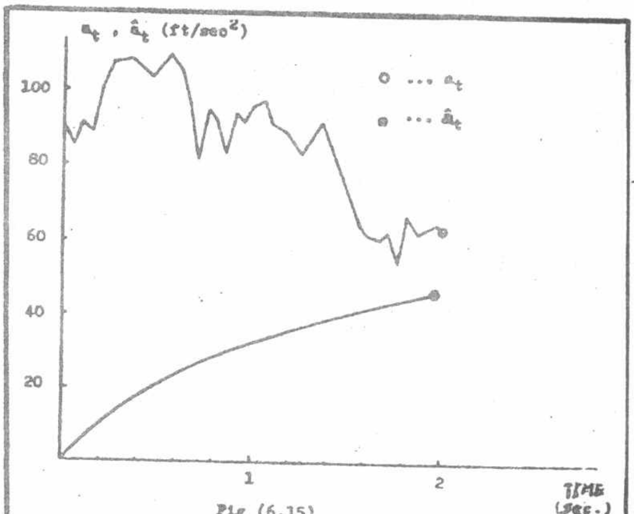


Fig (6.15)

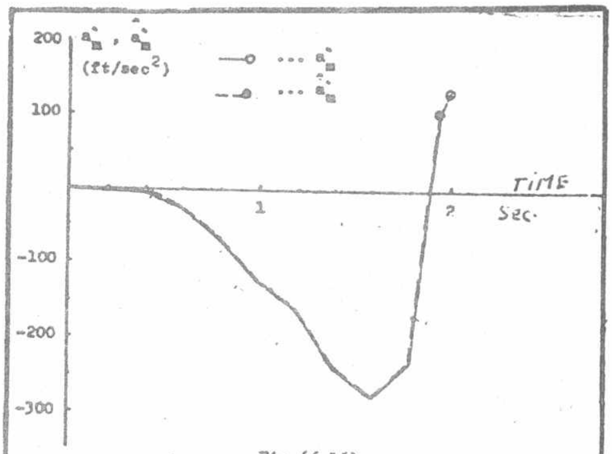


Fig (6.16)

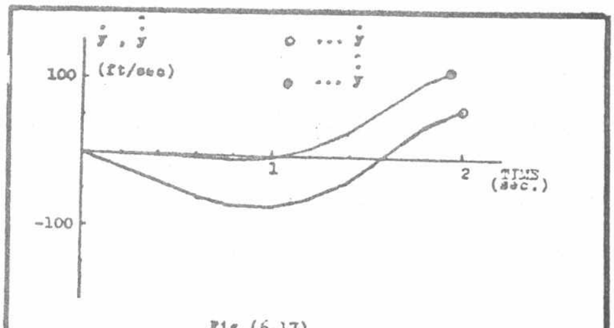


Fig (6.17)

The target manuverability increases the cost but to the acceptable limited when the manuverability is expressed as shown in fig. (6.15). And this is clear from the following figurs which shows the performances of the system (II) compared with that of the system when target manuverability is expressed as in fig.(6.4) (I), for t_f=2 sec, T=0.05 sec.

a- Fig. (6.18) shows the control (δ_I, δ_{II})

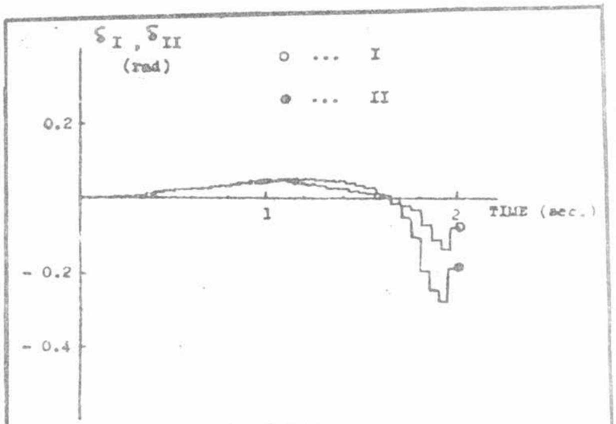


Fig (6.18)

b- Fig.(6.19) shows the state of the missdistance (y_I, y_{II}).

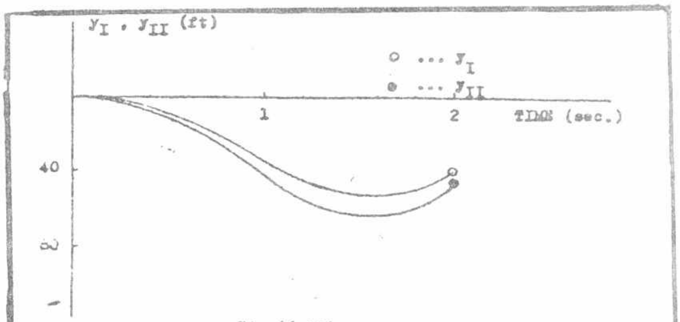


Fig (6.19)

c- Fig (6.20) shows the state of the normal acceleration ($\grave{a}_{m_I}$, $\grave{a}_{m_{II}}$).

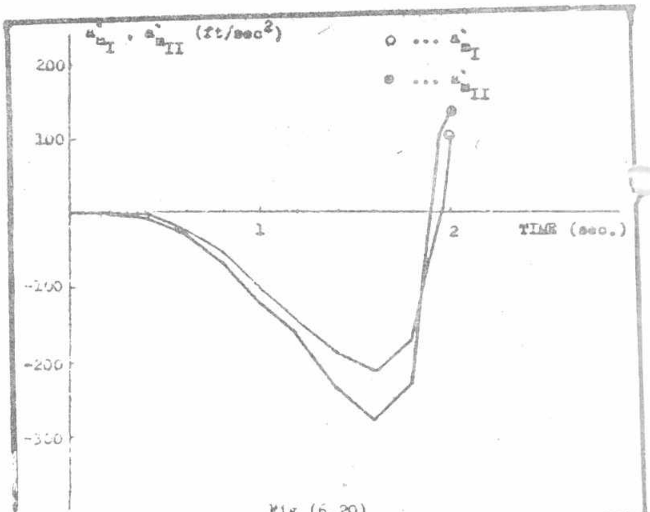


Fig (6.20)

7- CONCLUSION

(1) The controller for the short range homing guided missile against manuvering target is desinged by designing two separate components:

a- Feedback optimal gain matrix using the minimum discrete principle.

b- State estimation using the minimal order observer.

(2) The target manuverability is simulated in two cases:

a- Normal acceleration is simulated mathematically as exponential course with time.

b- NOrmal acceleration is simulated mathematically as exponential pluse random white Gaussian sequence.

(3) With the designed controller and the two cases of simulation the following objectives of the design are achieved:

a- Bounded control variable $/\delta(t)/ < 0.3$ (rad)

b- Limited missile manuvering capability $/a_m(t)/ < 10$ g.

c- Minimum terminal miss-distance $/y(t_f)/ < 50$ (ft).

with the note that in the second case of target simulation the cost is increased more than the first case i.e. increasing the target manuverability causes the increase of the cost. Hence;the main conclusion is that the controller designed on the base of the linear quadratic Gaussian theory (L.Q.G.) with the minimal order observer as state estimator provides a theoritical base line yielding an approach which can be applied to the design of a near optimal controller for the short range homing guided missile against manuvering target.

REFERENCES

1 Larry A. Stockum " Optimal and Supoptimal Guidance for a short Range missiles" IEEE Transactions on Aerspace and Electronic System, May 1976.

2 J. Gonzalez A Report "New Methods in the Terminal Guidance and Control of Tactical Missiles" . Air Force Armament Laboratory, U.S.A. 1979.

3 Leslie M.Novak."Optimal observer Techniques for LInear Discrete Time Systems" Hughes Air Craft Company , Ground System Group, U.S.A 1973.

4 John J. Deyst JR. - Charles, F. Price " Optimal Stochastic Guidance Laws for Tactical Missiles". J. Spacecraft, May 1973.

5 Alexander H. Levis and Rober t A. Schlueter Michael Athans. " On the Behaviour of Optimal Linear Sampled Data Regulators" Int. J. Control 1971.

6 Howard Berman-Richard Cran. "Design principles for Digital Autopilot Synthesis" Journal of Aircraft, Jul 1974.

7 Dr. Sol. W. Gully-Dr. Norm Coleman "Microcomputer Control Algorithms for weapon pointing and Stabilization". Control System, September 1981.

8 Nesim Halyo-Alper K. Caglyan. " A Separation theorem for the Stochastic Sampled Data L.Q.G. Problem". Int. J. Control 1976.

9 G.L. Slater. " Analysis of Integral Control in Linear Regulator Design" AIAA paper 79 1744, 1979.

10 P. Garnell- D. J. East" Guided Weapon Control Systems" Headington Hill, Oxford OX 3 OBW, England 1977.