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ROBUST CAD APPROACH TO MULTIVARIABLE CONTROL SYSTEM

S.Batesha* , D. Viault**

ABSTRACT

The requirement to provide satisfactory performance in the face of variation in the system parameters and the uncertainty in the design model, was the original motivation for the development of the CAD approach represented in this paper. Such an approach embraces system geometric theory together with modern multivariable frequency-response theory to establish a design main framework that would be compatible with a large class of practical control problems.

The application of the proposed approach is undertaken to design an automatic flight control system for a real aircraft. The adequancy of the resulting feedback configuration is confirmed via simulation.

* It. Colonel Dr., Dpt. of aircraft special equipment, MTC, Cairo , Egypt.
** Head of Control Dpt., Ecole Superieure d'Electricite', Gif sur Yvette, France.

I. INTRODUCTION

The classical approaches to SISO feedback system analysis/design involving
1) study of open-loop gain as a function of imposed frequency :Nyquist/Bode approach, or
2) study of closed -loop characteristic frequency (equivalently referred to as poles or modes) as a function of imposed gain:Evan's root-locus approaches,are extented to the multivariable case.The key to Nyquist criterion generalization lies in the association of a set of algebraic functions with the eigenvalues of the transfer-function matrix (t_fm) [11].

Based upon this criterion, a powerful multivariable design technique, commonly referred to as characteristic-locus method (CLM),has been developed [9,11]. The characteristic-loci (CL) emerge as the images of the eigenvalues $g_j(s)$; $1 \leqslant j \leqslant m$ of the t_fm G(s) when s sweeps the standard Nyquist contour. On the other hand, in MIMO case the multivariable root-locus (MRL) starts, as well as SISO case, from the open-loop poles and when loop-gains become infinitely large some loci terminate at finite cluster points given by the system finite-zeros.Remaining branches migrate to the point at infinity. The behavior of these unbounded loci is known as the asymptotic root-locus behavior (ARLB) and is investigated by appropriately expanding G(s) by a Taylor series about the point at infinity [10,11]. Unlike SISO case, a MIMO system has more than one asymptotic Butterwarth pattern, each corresponds to a certain order of divergence dictated by the eigen-structure of same-order projected-Markov-parameter (PMP). In particular, if the first-order PMP is of full-rank, the MRL will have ARLB of first-order only consists of a number of asymptotes equal to the rank of this PMP, and making with the negative-real axis angles equal to the phases of its eigenvalues [10]. The layout of this paper is as follows:after this introductory account there remain 4 other sections and the references. In section II, the problem of uncertainty in design-model is discussed in some detail. Furthermore, the way of achievement the benefits of feedback in the face of uncertainty and ill-definition of model is briefly reviewed. Section III concerns the development of the robust CAD approach . The application of this approach to a practical 11-th order ill-defined model is undertaken is section IV, followed by the conclusion and references.

II. MODELLING AND UNCERTAINTY

Uncertainty in some form or other will alaways be present in finite-dimensional linear time-invariant (FDLTI) model of on engineering process, no matter how it is derived, whether from test signal or physical equations. Therefore if a model is to be used for design purpose, it should not be regarded as complete unless it is accompanied by some specification of the uncertainty involved.

Problems Created by Modelling Uncertainty

(i) High frequency unmodelled-interaction:when modelling a complex plant, the only alternative to going to ever larger system-model is to model, at first, its subsystems. This is followed by assuming almost rigid coupling. Consequently the resulting model is ill-defined in a band of high-frequency due to unmodelled dynamic interaction.
(ii) Spill-over problem:Due to limits on model-order or/and frequency-domain bounds on transfer functions to describe a model set, the system high-frequency modes are usually either ill-defined or completely missed in the model.

This problem is of great significance when modelling aircraft flight-dynamics wherein high-frequency modes are generated by the structure elasticity.
(iii) Approximations:A mathematical model provides a map from inputs to responses. The quality of a model depends on how closely its responses match those of the true plant. Since no single fixed model can respond exactly like the true plant, we neet, at very least, a set of maps. However, a good model should be simple so as to facilitate design , yet complex enough to give us confidence that design based on model will work on the true plant. Therefore to achieve a better compromise, model approximation becomes inevitable.

Representation of Uncertainty

In state-space, unstructured uncertainty is often represented as a perturbation in the transition matrix A about a certain nominal value:

$$\{A,B,C\} \triangleq \{ (A_o + \delta A), B,C\} \quad :\sigma\, [\delta A] < \delta_1 \tag{1}$$

whilst, in terms of t.f.m there are commonly used:

$$G(s) = G_o(s) + \Delta G(s) \quad : \text{additive perturbation} \tag{2,1}$$

$$G(s) = G_o(s)\{ I + \Delta G(s)\} ;\text{multiplicative perturbation} \tag{2,b}$$

$$\sigma[\Delta G(s)] < \delta(s) \quad \nabla\, s\, \varepsilon\, D \tag{3}$$

where $\sigma[.]$ denotes the maximum sigular value of X and D is the standard Nyquist contour. Each of the above equations defines a set of perturbed plant model in the neighbourhood of $G_o(s)$.

The Benefits of Feedback

Consider the standard feedback configuration illustrated in Fig.1:It consists of the interconnected plant G and a controller K forced by commands r , measurement noise η and disturbances α. The dashed precompensator P is an optional element used either to conduct deliberate command shaping or to represent a non-unity feedback system in equaivalent unity feedback form. Then it is well known that the configuration, if it is stable, has the following major properties:

(1) Input-output nominal behaviour :

$$y = G_oK (1 + G_oK)^{-1}(r-\eta) + (I + G_oK)^{-1}d \tag{4}$$

$$e = r-y \triangleq (I + G_oK)^{-1}(r-d) + G_oK(I+ G_oK)^{-1}\eta \tag{5}$$

(2) System sensitivity

$$\Delta H_c = (1+GK)^{-1}\; \Delta H_o \tag{6}$$

where ΔH_c and ΔH_o denote changes in the closed-loop system and changes in a nominally equivalent open-loop system, respectively, caused by changes in the plant G :G = G_o+ΔG.
The above three equations summarize the fundametal benefits and design objectives inherent in feedback loops. Specifically, eqn. (5) shows that the loop's errors in the presence of commands and disturbances can be made small by making the sensitivity operator, or inverse return difference matrix $(I+G_oK)^{-1}$ small, in the sense that $\sigma[(I+G_oK)^{-1}]$ small. Eqn.(6) shows that loop sensitivity is improved under the same condition, provided G does not, practically,

stray too far from G_o within the frequency range of interest. This result can be interpreted as merely a restatement of the common intution that large loop gain, or " tight" loops, yield robust performance. The realisation of the above condition is straightforward in low-frequency range by employing a PI controller, however, this is not the case in high-frequency band as increasing loop gains may violate the closed-loop stability.

One of the main objectives of the our developed approach is to circumvent this conflict by devising an inner-loop compensator that allows for large gain injection over high-frequency band.

III. A ROBUST HYBRID CAD APPROACH

The aim of this section is to develop a CAD framework that fairly distributes the burden of design between computer and engineer. This stipulates an appealing medium for man/machine communication which places high premium on techniques using graphics and those based on the geometric concepts.

The main framework concerned here is a natural development of that proposed by Batesha and Viault [2-3] and further enhanced [5] to cope with the requirement of controller robustness.

The configuration considered is shown Fig.2. The given plant is assumed to be of n-dimensions, completely controllable, observable with m-inputs and m-outputs. The design process is carried out in two stages:inner-loop and outer-loop.

Inner-Loop Design

The discussion deployed in the second section underlines the importance of controller robustness and enables us to, straight forwardly, state the ideas behind the inner-loop design as follows:

(1) Satisfying requirements on high-frequency behaviour:
To satisfy such requirements it is necessary to, directly, manipulate the ARLB in order to achieve;

a) inifinte gain-reserve over high-frequency band by trying to align all MRL along the negative real-axis.
b) elimination of high-frequency interaction by forcing the closed-loop transmittance to have symmetrical eigenstructure [2-3], .
c) moreover, the above achievements should by secured under unstructured model-uncertainty.

Now let us consider a FDLTI model A,B where A is generally ill-defined such that.

$$A = A_o + \delta A \qquad , \quad 6 \left[\delta A \right] < \delta_1 , \tag{7}$$

it is required to design a state-feedback F such that the ARLB is aligned with the negative real-axis independantly of δA, moreover any high-frequency open-loop interactions are eliminated.

A state unimodular projection operator P_1 exists [3] such that

$$\{ A,B,F \} \xrightarrow{P_1} \{ A^{(1)},B^{(1)},F^{(1)} \} : \tag{8}$$
$$P_1 = [N \quad B^{\dagger}]^T$$

$$A^{(1)} = \left[\frac{NAN_R}{B^{\dagger}AN_R} \vdots \frac{NAB_R}{B^{\dagger}AB_R} \right] \tag{9}$$

$$B^{(1)} = [0 \vdots I]^T \tag{10}$$

$$F^{(1)} = [F_1^{(1)} \vdots F_2^{(1)}] ; \; |F_2^{(1)}| \neq 0 \tag{11}$$

and

$$\left[\frac{N}{B^{\dagger}}\right] [N_R \vdots B_R] = I_n \tag{12}$$

Consequently the ARLB is dictated by the eigenstructure of the set of PMP's amongst which the first-order one is

$$G_1 = F^{(1)} B^{(1)} = F_2^{(1)} \tag{13}$$

and as $|F_2^{(1)}| \neq 0$, the system will possess ARLB of first-order only. Therefore, a judicious choice of $F_2^{(1)}$ as

$$F_2^{(1)} = \alpha_1 I \quad , \alpha \text{ real and positive} \tag{14}$$

will satisfy requirements a) and b) stated before, since all asymptotes will be aligned with the negative real axis. Remarking that eqn (13) is completely insensetive to the variations in the transition matrix A,i.e. to δA, the requirement c) is also satisfied.

So far, the choice of $F_1^{(1)}$ is arbitrary and this remaining degree of freedom should be properly exploited to achieve other objectives. One way of such an exploitation is to assign appropriately the dominating finite-zeros [3-4] .

Despite the simplicity of the formentioned solution, it is clear from practrical experience that feedback design is not trivial . This is true, in our case, because of two main reasons:

(1) loop-gains cannot be made arbitrarily high over arbitrarily large frequency range. Rather they must satisfy certain performance tradeoffs and design limitations. A major performance tradeoff, for example, concerns command and disturbance error reduction versus sensor noise-error reduction.

(2) the choice of $F_1^{(1)}$ is not transparent and may, even, pose a considerable problem when the dimension of matrix A is augmented .

Fortunately this handicap can be overcome when eqn. (14) for $F_2^{(1)}$ determination is fitted into the CAD scheme , Fig.3, described in detail in reference [3] . In rare cases the above stated performance tradeoffs may fail to achieve a satisfactory compromise, particularly when limitation upon feedback gains are very tight and/or some slow modes are required to be insensetively assigned. To confront this situation, an alternative procedure is suggested below. This procedure relies on the concepts of input/output decoupled zeros [15].

Consider the ill-defined model-representation eqn. (1), where the (nxn) dimension matrix A involves μ uncertain parameters. Apply matrix condensation technique [19] , δA can be expressed as

$$\delta A = U \Delta W \tag{15}$$

where U, W are constant full-rank matrices of dimension n μ_1,μ_2 n respectively and $\Delta$is an ($\mu_1 x \mu_2$) dimension matrix including all uncertain parameters of δA :μ_1 μ_2 =μ. Introducing a state feedback F results in the closed-loop polynomial equation

$$|sI-A_o-U\Delta W+BF|=|sI-A_o+BF|\,|I-W(sI-A_o+BF)^{-1}U\Delta|= 0 \qquad (16)$$

Now consider the system $\{A_c, U,W\}$ with

$$A_c = A_o-BF \qquad (17)$$

and associated $t_{f}m$ $G_\delta(s)$:

$$G_\delta (s) = W (sI-A_c)^{-1} U = W (sI-A_o+BF)^{-1}U \qquad (18)$$

Hence, eqn (16) can be rewritten into

$$|sI-A_c| \; |I-G_\delta(s)\Delta| = 0 \qquad (19)$$

This last equation steadily yields the following result:

" A sufficient condition for the set of complex-frequencies $\{s_j\}$to represent a set of insensitive closed-loop modes for system $\{A,B,F\}$ is that s_j should constitute simultaneously a subset of the closed-loop poles of the well-defined system $\{A_o,B,F\}$ and a subset of the finite-zeros of the ill-defined t_fm $G_\delta(s)$ such that

$$|sI-A_c| = |sI-A_o+ BF| = 0 \quad \nabla\; s\; \varepsilon\{s_j\} \qquad (20)$$

and

$$|G_\delta(s)| = |W (sI-A_c)^{-1} U |= 0 \quad \nabla\; s\; \varepsilon\{s_j\} \qquad (21)$$

The concept of decoupled-zeros |15| , provides a physical interpretation of this result since $\{s_j\}$ emerges as a set of input-decoupled zeros associate with the uncontrollable modes of $\{A_c$, U$\}$or/and a set of output-decoupled zeros associate with the unobservable modes of $\{A_c,W\}$. Therefore, the above approach resembles in its objective classical robust controller design techniques commonly adopted in SISO systems [8]where filters are fitted both in feedback and feedforward pathes to decouple the effect of uncertainty. There remains to determine the number of insensitive-assignable modes. From eqn (13) one can generally assign ,under the condition $|F_2^{(1)}|\neq 0$ all m fast modes by properly assigning the eigenstructure of $F_2^{(1)}$and adjusting a high feedback-gain. Since a bilinear transformation may generally be devised to map a set of fast-modes (infinite-poles)onto a set of slow-modes (finite-poles) and vice-versa, one can, intuitively, state that:at least m insensitive finite poles of the m-inputs controllable dynamic system A,B can be arbitrarily assigned by an appropriate linear state-variable feedback.

Remark:Eqn (20) and (21) are solvable for m_1 values of s where $m_1 \geq m$, however amongst those m_1 insensitive poles only m can be arbitrarily placed. Moreover the above stated condition is sufficient but not necessary. A more convenient necessary and sufficient condition could be derived by investigating the uncontrollable , unobservable subspaces of $\{A_c,U,W\}$. This will be the subject of our next paper.

Outer-Loop Design

Having combined appropriately all accessible states via a linear feedback operator, our design framework is completed by devising a feedforward controller that injects required gains over the operating bandwidth in order to meet the final design objectives.

The procedure that is employed herein is essentially pragmatic and stems from the generalized Nyquist/Bode techniques. Classical well-known compensation methods may be employed to design, generally, a PID approximate commutative controller [2-9-11]. One annoying feature still in the consideration of model uncertainty. Let us consider eqn(2,a) and (2,b) representing ill-defined models. Researches in robust stability, e.g. reference [13], have shown that the perturbed closed-loop system remains stable when $G_o(s)$ is stable and

$$\|\Delta G(s)\| < 1/\,\|[1+G_o(s)^{-1}]^{-1}\| \qquad \nabla\ s \in D \qquad (22)$$

in multiplicative case, and

$$\|\Delta G(s)\| < 1/\,\|[I+G_o(s)]^{-1}\| \qquad \nabla\ s\varepsilon\ D \qquad (23)$$

in additive case, where $\|.\|$ deroles the standard Euclidean vector norm. Those inequalities do not constitute a suitable design tool as they contribute very little to system structural synthesis and they can serve merely for assessing the robustness of the closed-loop . However when combined together with the generalized Nyguist stability criterion they help to extend this criterion to the case of uncertain systems.

If in eqn's (22) and (23) the spectral norm $\|.\|_2$ is taken, then the robustness of each configuration is characterized by the maximum principal gain $\sigma[\Delta\ G(s)]$, or equivalently referred to as singular-value. Based upon the sigular-value decomposition (SV), Macfarlane and his cooperators [12-14] has deduced a sufficient condition for stability, by replacing the charactarastic-loci (loci of the eigenvalues of G(s)) in the generalized Nyquist criterion, by the principal region outlined by their maximum and minimum values. More tight (both necessary and sufficient) conditions have been, further derived [7] . The drawback with this "SV Approach", however, is that it does not lend itself for the purpose of design.

A recent paper [6] has generalized the characteristic-loci (CL) method to the case of both additive and multiplicative unstructured perturbations and thus, establishs connections between the CL and SV methods. The relevalent stability criterion retains the simplicity of the Nyquist criterion, it provides immediate graphical information on tolerance to uncertainty and gain/phase margins, and it enhances procedures for robust design. This approach is known as "Eigenvalue Inclusion Regions Approach"

IV. DESIGN-EXAMPLE

The proposed approach was applied to design a flight-control system of a real high-speed aircraft based upon an 11-th order linearized model, with the aid of the CAD-Package implemented on VAX-VMS computer of the Control Department-ESE.

The MRL ofthe five longitudinal-motion modes of the uncompesated aircraft are shown Fig.4 . Having employed the proposed technique all fast-modes are attracted asymptotically to the negative real axis as it is illustrated in Fig. 5.

Fig.6 and Fig.7 illustrate the time response of the nominal model under step demand in ground speed V_s and pitch angle Θ .The robustness of the resulted feedback configuration is confirmed by simulation results Fig.8 and Fig.11.

It is noteworthy that the theoretically insensitive modes V_s and θ becomes rather slightly-sensitive. This is naturally resulted from the practical constraint upon loop-gains.

V. CONCLUSION

A feedback control problem can be attacked by many different methods amongst which an experienced designer should select the best solution from some practical viewpoint. Uncertainty should play a critical decisive role in such a selection. Consequently, a hybrid approach that embraces the generalized classical techniques together with the state-space geometric methods can constitute a powerpul CAD approach that is compatible with a large class of robust-controller design.

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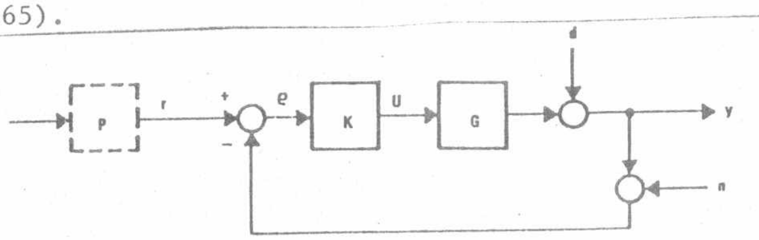


Fig. 1.

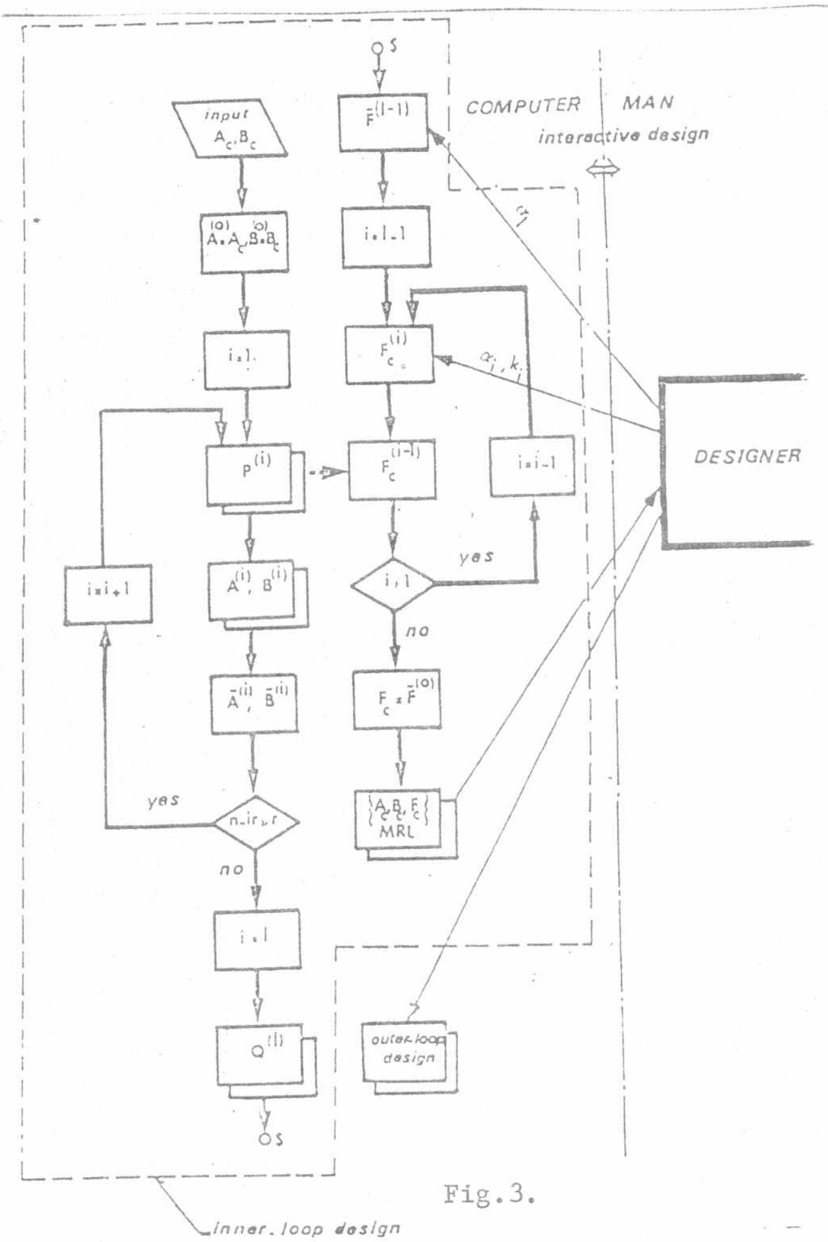
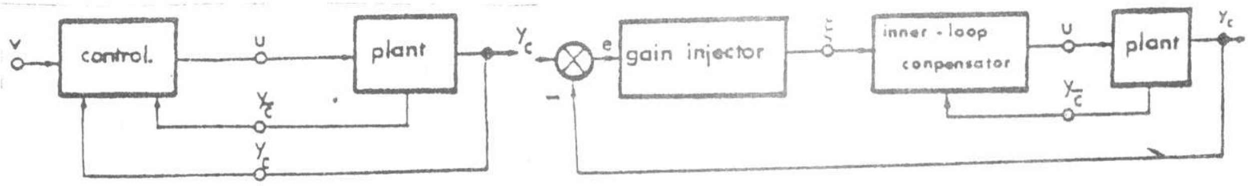


Fig.3.



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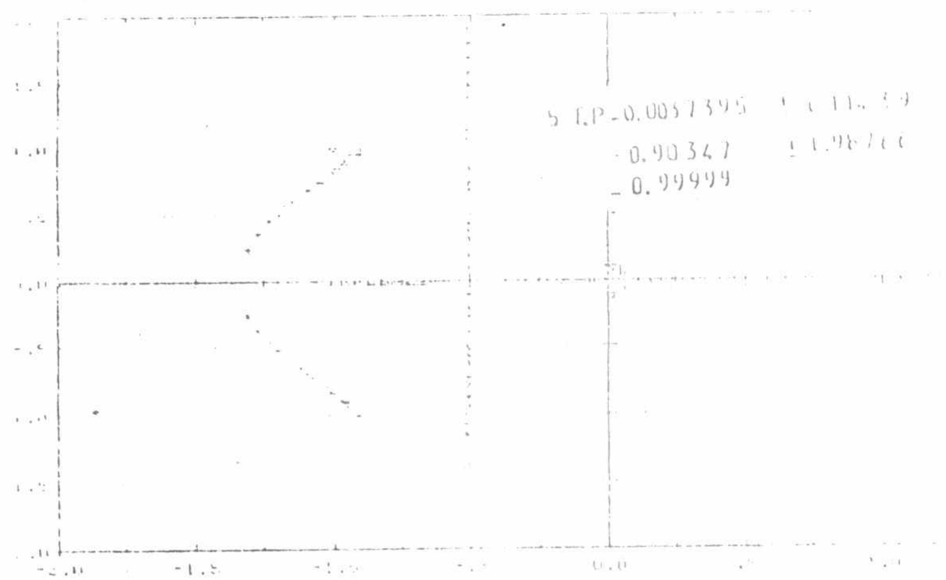

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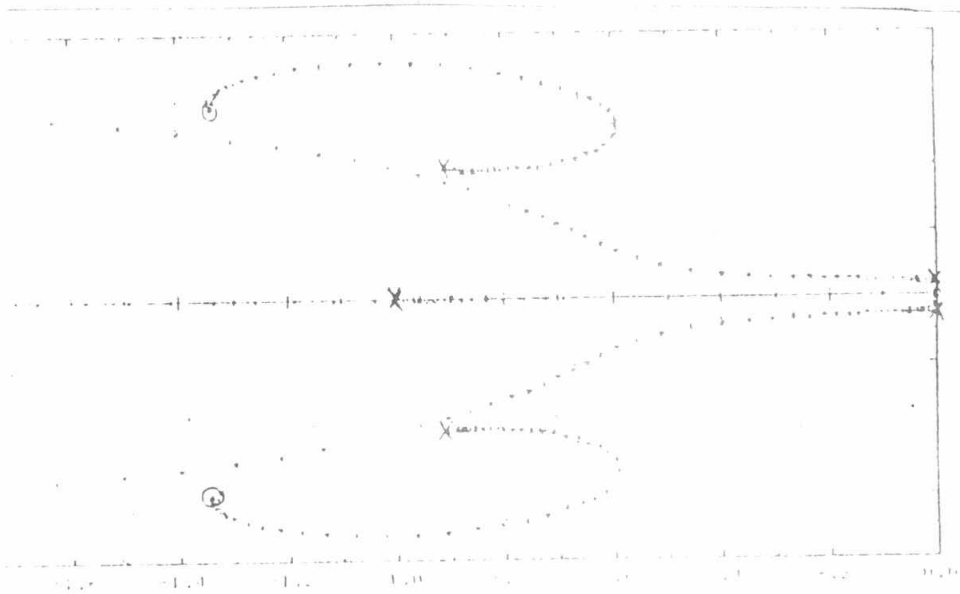

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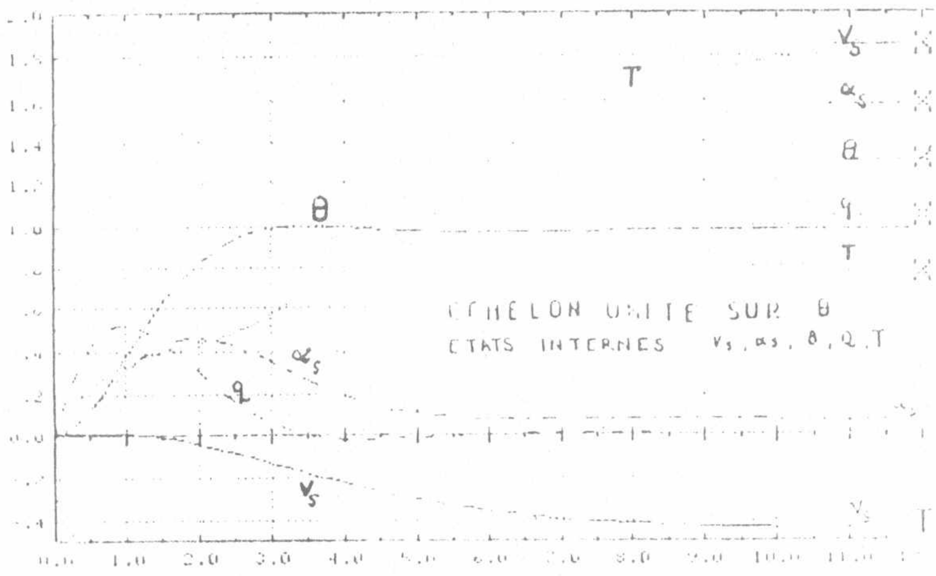

Fig.6.

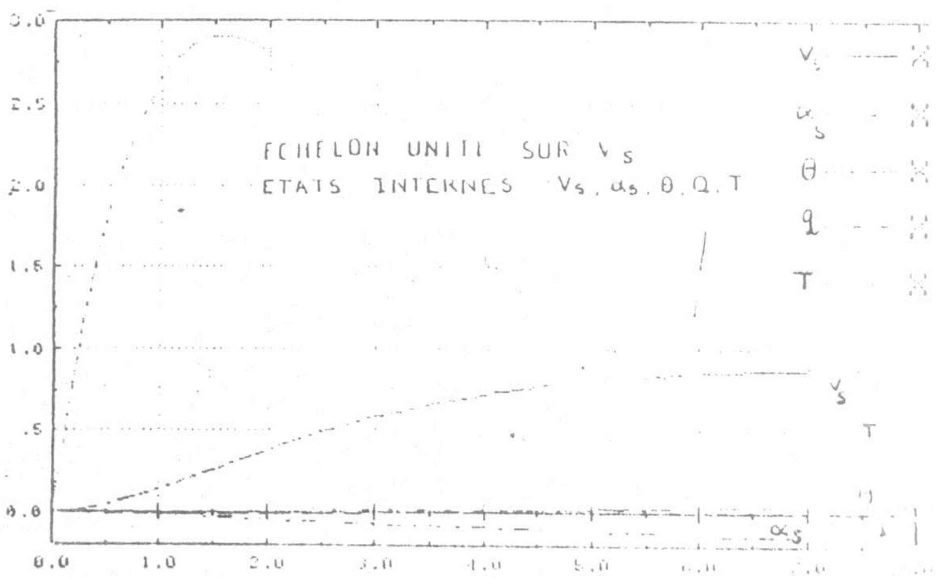

Fig.7.

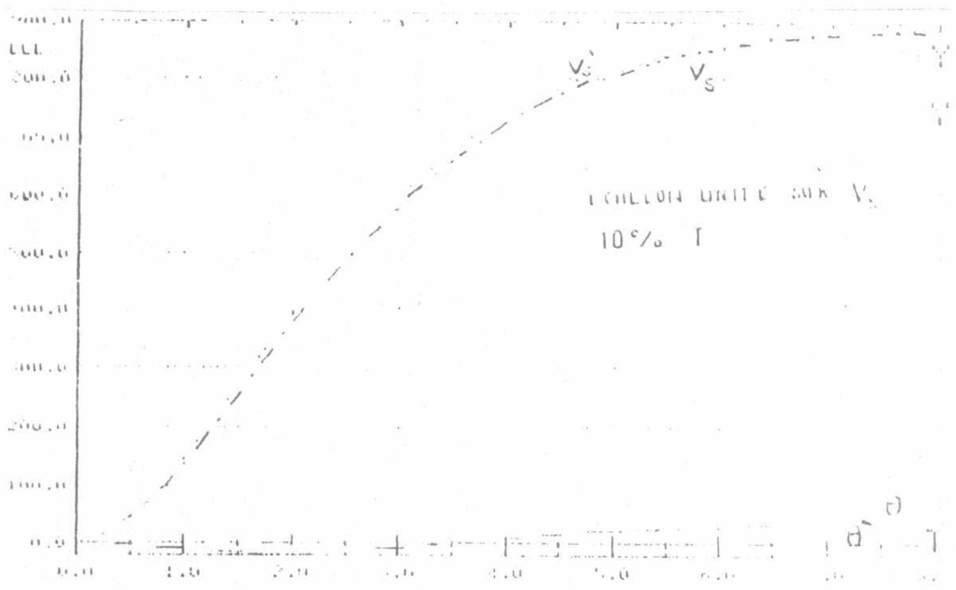

Fig.8.

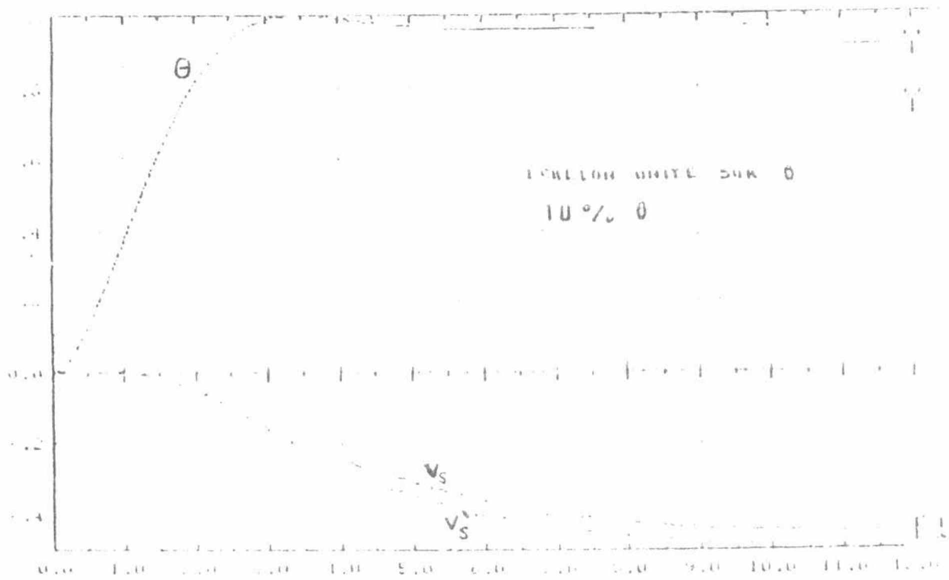

Fig.9.

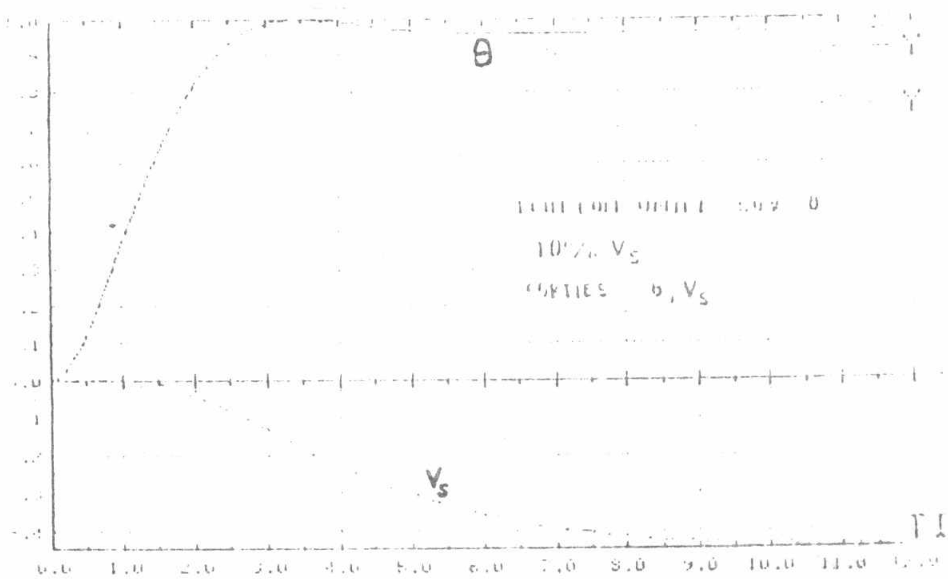

Fig.10.

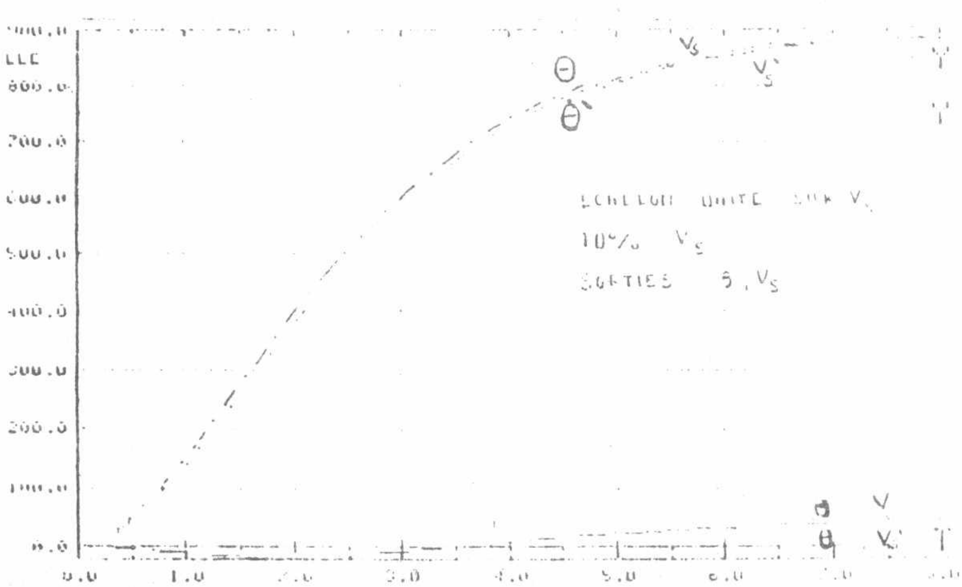

Fig.11.