1 ROBUST FILTERS FOR TIME DELAY ESTIMATORS
2 WITH BOUNDED SPECTRAL CLASSES AT THE INPUTS

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4 ABSTRACT

Optimum filters are used to enhance the performance of the generalized cross correlator (GCC) used in time delay estimators (TDE). The performance of these TDE can deteriorate considerably for deviations in the input power spectral densities (PSD's) from their assumed nominal values. In this paper we consider the bounded spectral classes of PSD's which are useful models when the input PSD's are not precisely known. For these classes, robust filters for TDE are derived for two optimum criterions separately. These criterions are maximizing the signal to noise ratio and minimizing the mean square error. The robust filters obtained achieve better performance for any inputs within the assumed PSD's. Numerical examples given illustrate the theoretical results.

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1. INTRODUCTION

Estimating and tracking the time delay between two stochastic signals is encountered in many fields such as radar,sonar, communications and measurment. Localization and tracking of a source may be determined directly from the time delay measurements |1|.
Optimum|2| or heuristicfilters|1|are used to enhance the performance of the GCC used to estimate the time delay. The design of these filters requires the exact knowledge of input signal and noises spectra, which is practically difficult. To encounter this difficulty either adaptive |3| , non-parametric |4| or robust |5| filters may be used.
This paper deals with the design of robust filters for TDE when the input signal and noises power spectra are from the bounded spectral classes. Robust solutions for these cases of spectral classes were given for TDE using Eckart filter |5|. Here, rabust solutions for other two optimum criterions are obtained. Numerical examples are given to illustrate the benefits of using robust filters.

II. PROBLEM FORMULATION

Asignal received from a radiating sourse at two spatially separated sensors has the following mathematical model:

$$r_1(t) = s(t) + n_1(t) \qquad (1a)$$

$$r_2(t) = as(t-\tau) + n_2(t) \qquad (1b)$$

The input signal s(t) and the corrupting noises $n_1(t)$ and $n_2(t)$ are real, jointly stationary, radnom, uncorrelated processes with PSD's $S(\omega)$, $N_1(\omega)$ and $N_2(\omega)$ respectively. The signal and noises PSD's at the sensors input of the GCC used in TDE can be considered as one of the following classes of PSD's :

i) The bounded P-point spectral classes.
ii) The band model. (The bounded spectral classes)
iii) The ϵ model (Contaminated classes).
The bounded P-point spectral classes are considered the general form for the other two classes. The bounded P-point spectral classes are characterised as follows:-

$$C_s : S_L(\omega) \Big|_{a_{j-1}}^{a_j} \leqslant S(\omega) \Big|_{a_{j-1}}^{a_j} \leqslant S_U(\omega) \Big|_{a_{j-1}}^{a_j} \quad , j = 1,2,\ldots,P.,$$

$$\frac{1}{\pi} \int_{a_{j-1}}^{a_j} S(\omega)\, d\omega = b_j \sigma_s^2 \text{ and } \sum_{j=1}^{P} b_j = 1 \qquad (2a)$$

$$C_{N_i} : N_{iL}(\omega) \Big|_{a_{j-1}}^{a_j} \leqslant N_i(\omega) \Big|_{a_{j-1}}^{a_j} \leqslant N_{iU}(\omega) \Big|_{a_{j-1}}^{a_j} \quad , j = 1,2,..,P.,$$

$$i=1,2., \ \frac{1}{\pi} \int_{a_{j-1}}^{a_j} N_i(\omega)\, d\omega = V_{ji} \cdot \sigma_{n_i}^2 \text{ and } \sum_{j=1}^{P} V_{ji} = 1 \qquad (2b)$$

Where:-

C_s and C_{N_i} are the classes of PSD's of signal and noises; Subscripts L,U refer to lower and upper PSD's; σ_s^2 & σ_{ni}^2 the total power of signal and noises n_1, n_2 respect.

III. ROBUST FILTERS

a) Definition:
The most robust filter apply aminimax or game theoretic formulation to find a scheme which optimizes the worst case performance over classes of allowed input signal and noise defining a pair of least favourable PSD's for which optimum filters are designed.
b) Robust filters for bounded P-point spectral classes: Robust filters for these classes are considered for the following two optimum conditions:-
i) Maximizing the expected signal peak relative to the output noise:
The optimum filter characterestics for this condition W_{MSN} is given by |2|:

$$W_{MSN}\Big|_{a_{j-1}}^{a_j} = \frac{S(\omega)}{N_1(\omega)N_2(\omega) + S(\omega)\,(N_2(\omega) + \alpha^2 N_1(\omega)) + \alpha^2 S(\omega)}\Big|_{a_{j-1}}^{a_j}$$

$$= \frac{S_t(\omega)}{N_t(\omega)}\Big|_{a_{j-1}}^{a_j} \qquad (3a)$$

Where:

$$S_t(\omega) = \text{Total signal PSD} = S(\omega) \qquad (3b)$$

$$N_t(\omega) = \text{Total noise PSD} = N_1(\omega)N_2(\omega)+S(\omega)(N_2(\omega) + \alpha^2 N_1(\omega)) + \alpha^2 S^2(\omega) \qquad (3c)$$

The least favourable pair for the total signal and total noise PSD's (S_{t_r} and N_{t_r}) are defined according to eq.(3) and theorem (1) below. The robust filter W_{MSN_r} is the optimum filter for the pair S_{t_r} and N_{t_r} and is given by eq.(3a)

ii) Minmizing the mean square error:
The optimum filter characterestic (W_{LMS}) for this condition is given by:

$$W_{LMS}\Big|_{a_{j-1}}^{a_j} = \frac{2\alpha^2 S^2(\omega)}{N_1(\omega)\,N_2(\omega) + S(\omega)(N_2(\omega) + \alpha^2 N_1(\omega)) + 2\,\alpha^2 S^2(\omega)}\Big|_{a_{j-1}}^{a_j}$$

$$= \frac{S_t(\omega)}{S_t(\omega) + N_t(\omega)}\Big|_{a_{j-1}}^{a_j} \qquad (4a)$$

Where:

$$S_t(\omega) = \text{total signal PSD} = 2\,\alpha^2\,S^2(\omega) \qquad (4b)$$

$$N_t(\omega) = \text{total noise PSD} = N_1(\omega)\,N_2(\omega) + S(\omega)(N_2(\omega) + 2\alpha^2 N_1(\omega)) \qquad (4c)$$

The least favorable pair for the total signal and total noise PSD's ($S_{t_r}(\omega)$ and $N_{t_r}(\omega)$) are defined according to eq.(4) and theorem (1) below. The robust filter for this condition is defined as the optimum filter for the pair $S_{t_r}(\omega)$ and $N_{t_r}(\omega)$ and is given by eq.(4a).

c) Theorem (1):
Robust filters for the bounded P-point spectral classes are the optimum filters for a pair of PSD's $S_{t_r}(\omega)$ and $N_{t_r}(\omega)$ and $N_{t_r}(\omega)$ defined according to one of the following:-

i) If $K_{s_j} \leqslant K_{n_j}$ exist satisfying:

$$P_{S_{t_j}}(x)\big|_{K_{s_j}} = d_j\pi\,\sigma^2_{S_t} = X\int_{\alpha_{j1}(x)} N_{tL_j}(\omega)\,d\omega + \int_{\alpha_{j2}(x)} S_{tU_j}(\omega)\,d\omega + \int_{\bar{\alpha}_j(x)} S_{tL_j}(\omega)\,d\omega \qquad (5a)$$

$$P_{N_{tj}}(x)\big|_{K_{n_j}} = h_j\pi\sigma^2_{n_t} = \frac{1}{X}\int_{B_{j1}(x)} S_{tL_j}(\omega)\,d\omega + \int_{B_{j2}(x)} N_{tU_j}(\omega)\,d\omega + \int_{\bar{B}_j(x)} N_{tL_j}(\omega)\,d\omega\,. \qquad (5b)$$

then,

$$S_{tr_j}(\omega) = \begin{cases} K_{s_j} N_{tL_j}(\omega) & ,\ \omega\varepsilon\,\alpha_{j1}(K_{s_j}) \\ S_{tU_j}(\omega) & ,\ \omega\varepsilon\,\alpha_{j2}(K_{s_j}) \\ S_{tL_j}(\omega) & ,\ \omega\varepsilon\,\bar{\alpha}_j(K_{s_j}) \end{cases} \qquad (6a)$$

$$N_{tr_j}(\omega) = \begin{cases} \frac{1}{K_{n_j}} S_{tL_j}(\omega) & ,\omega\varepsilon\ B_{j1}(K_{n_j}) \\ N_{tU_j}(\omega) & ,\omega\varepsilon\ B_{j2}(K_{n_j}) \\ N_{tL_j}(\omega) & ,\omega\ \varepsilon\bar{B}_j\ (K_{n_j}) \end{cases} \qquad (6b)$$

ii) Otherwise with,

$$K_j = \frac{d_j\pi\sigma^2_{st} - \int_{B_{j2}} S_{tL_j}(\omega)\,d\omega - \int_{\alpha_{j2}} S_{tU_j}(\omega)\,d\omega}{h_j\pi\sigma^2_{n_t} - \int_{B_{j2}} N_{tU_j}(\omega)\,d\omega - \int_{\alpha_{j2}} N_{tL_j}(\omega)\,d\omega} \qquad (7a)$$

then,

$$S_{tr_j}(\omega) = \begin{cases} K_j N_{tL_j}(\omega) + S_e(\omega) & ,\ \omega\varepsilon\alpha_{j1}(k_j) \\ S_{tU_j}(\omega) & ,\ \omega\varepsilon\alpha_{j2}(k_j) \\ S_{tL_j}(\omega) + S_e(\omega) & ,\ \omega\varepsilon\ B_{j1}(K_j) \\ S_{tL_j}(\omega) & ,\ \omega\varepsilon\ B_{j2}(K_j) \end{cases} \qquad (7b)$$

$$N_{tr_j}(\omega) = \begin{cases} \frac{1}{K_j} S_{tL_j}(\omega) + N_e(\omega) & ,\omega \in B_{j1}(k_j) \\ N_{tU_j}(\omega^j) & ,\omega \in B_{j2}(k_j) \\ N_{tL_j}(\omega) + N_e(\omega) & ,\omega \in \alpha_{j1}(k_j) \\ N_{tL_j}(\omega) & ,\omega \in \alpha_{j2}(k_j) \end{cases} \qquad (7.c)$$

iii) If neither (i) or (ii) above yields the robust solution, then if 1_{s_j}, 1_{n_j} are solutions of:

$$Q_{st_j}(x)\big|_{1_{s_j}} = d_j \pi \sigma^2_{st} = x \int_{b_{j1}(x)} N_{tU_j}(\omega)\, d\omega + \int_{b_{j2}(x)} S_{tL_j}(\omega)\, d\omega + \int_{\bar{b}_j(x)} S_{tU_j}(\omega)\, d\omega \qquad (8a)$$

$$Q_{nt_j}(x)\big|_{1_{n_j}} = h_j \pi \sigma^2_{nt} = \frac{1}{x} \int_{a_{j1}(x)} S_{tU_j}(\omega)\, d\omega + \int_{a_{j2}(x)} N_{tL_j}(\omega)\, d\omega + \int_{\bar{a}_j(x)} N_{tU_j}(\omega)\, dw \qquad (8b)$$

with $1_{sj} > 1_{nj}$ necessarily, then we have:

$$S_{tr_j}(\omega) = \begin{cases} 1_{s_j} N_{tU_j}(\omega) & ,\omega \in b_{j1}(1_{s_j}) \\ S_{tL_j}(\omega) & ,\omega \in b_{j2}(1_{s_j}) \\ S_{tU_j}(\omega) & ,\omega \in \bar{b}_j(1_{s_j}) \end{cases} \qquad (9a)$$

$$N_{tr_j}(\omega) = \begin{cases} \frac{1}{1_{nj}} S_{tU_j}(\omega) & ,\omega \in a_{j1}(1_{n_j}) \\ N_{tL_j}(\omega) & ,\omega \in a_{j2}(1_{n_j}) \\ N_{tU_j}(\omega) & ,\omega \in \bar{a}_j(1_{n_j}) \end{cases} \qquad (9b)$$

Finally if no solution is obtained for 1_{s_j}; $S_{tr_j}(\omega)$ can be picked to be $S_{tU_j}(\omega)$ when $N_{tU_j}(\omega) > 0$ and arbitrary otherwise, and if no solution is obtained for 1_{n_j} then $N_{tr_j}(\omega)$ can be picked to be $N_{tU_j}(\omega) > 0$ and aribtrary otherwise.

d) Notes:

i) Subscribt j defines the frequency band a_{j-1} to a_j.

ii) S_{tj} , N_{tj}, $\sigma^2_{st_j}$ and σ^2_{ntj} are defined from the given signal and noise PSD's $S_j(\omega)$, $N_{1j}(\omega)$ and $N_{2j}(\omega)$ according to the optimum condition used i.e. eq.(3) or eq.(4).

iii) N_{tL_j} & N_{tU_j} are defined bycalculating S_{tj} and N_{tj} resulting from different combinations of signal PSD's (S_{Lj} or S_{Uj}) with noises PSD's (N_{1Lj} or N_{1Uj} and N_{2Lj} or N_{2Uj}). The spectral combination which gives the lower total noise power on the subband is N_{tLj}, while that which gives the maximum total noise power is N_{tUj}. The same procedure is used to define S_{tLj} and S_{tUj}.

iv) The band model and the ϵ model spectral classes are special cases of the bounded P-point spectral classes[5].

IV. NUMERICAL EXAMPLES

Different numerical examples are solved to show the difference between using the optimum filters and the robust filters in TDE. The signal and noises input to the GCC sensors are supposed to be of the ε model spectral classes. The results obtained show that when the optimum filter W_{MSN} is used the degradation in SNR from that when the optimum inputs are used reaches 80 % through all the band of the PSD's inputs, while when the robust filter W_{MSN_r} is used the degradation in SNR is not more than 10% all over the band of PSD's inputs from that due to optimum inputs.
When W_{LMS_r} is used the increase in the mean square error reaches 50% from that due to optimum inputs, while when W_{LMS} is used the MSE increases 110% all over the whole band of PSD's inputs w.r.t optimum inputs.

V. CONCLUSION

Robust filters designed for TDE with bounded spectral classes at the inputs are the saddle point solutions for these classes.

VI. REFERENCES

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