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A PROPOSED FREQUENCY DOMAIN SELF ADAPTIVE EQUALIZER

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ABSTRACT

In modern data communication networks , self adaptive equalizers are increasingly required . This is due to the fact that this type of equalizers avoids retransmission of a learning data sequence each time a receiver enters the network. The problem of self adaptive equalization has not yet received sufficient study in the literature where cases of moderate data transmission channels are considered. In this paper we present a self adaptive equalizer capable on equalization of arbitrarily severe distortions in the amplitude-frequency response of the channel . The idea of equalizer relies on the fact that in practical applications, the data signal has a flat spectrum. This enables the identification of the amplitude-frequency response of the channel on the basis of measuring the power spectrum of the received signal . The FFT technique is used to perform that measurement and the corresponding equalization. Computer simulations of the proposed equalizer have shown its ability to deal with drastically distorted channels.

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I- INTRODUCTION

Conventional adaptive equalizers are adapted by starting the transmission with a short training sequence of data known in advance by the receiver. The equalizer then uses the difference between its output and the known data to adjust the tap weights . This arrangement is suitable for point-to-point data communications . However in modern multipoint data communication networks where many receivers may be connected to the same transmitter,it is not desirable to interrupt data transmission and send a training sequence every time a receiver enters the network (e.g. it was not powered on during initial network synchronization,or after a local break) . This fact initiates the need for self adaptive equalizers (SAE's) that do not need a training sequence for the start up period but can rather perform at any time the equalization directly on the data sequence. The subject of self adaptive equalization has not yet received a sufficient study in literature. Indeed we are aware of only five papers [1-5] in that domain.

In this paper we propose an SAE capable of equalization of arbitrarily severe distortions in the amplitude-frequency responce of the channel. Equalization of amplitude-frequency response is sufficient for correct data recovery in a wide class of practical channels. The idea of the proposed SAE relies on the fact that in practical applications,the data signal has a flat spectrum. This enables the identification of the amplitude-frequency response of the channel on the basis of measuring the power spectrum of the received signal . The Fast Fourier Transform (FFT) technique is used to perform that measurement and the corresponding equalization .

The paper is organized as followes . In section II the idea of the proposed SAE is presented . The implimentation of the idea is given in section III. Section IV is devoted to the analysis of the adaptation algorithm of the SAE. In section V the performance of the proposed SAE is studied with the help of computer simulations . In section VI conclusions of the paper are presented .

II- BASIC IDEA OF THE PROPOSED SAE.

To explain the idea of the proposed SAE,consider the discrete baseband equivalent of the data transmission system depicted in figure 1. In that figure a_k is the data transmitted at the k^{th} baud interval,and $\hat{a}_k$ is the estimate of a_k.

Let $S_a(f)$,$S_x(f)$, and $S_y(f)$ be the power spectral density functions (p.s.d.f.) associated with the data sequence $\{a_k\}$, sampled output of channel $\{x_k\}$, and equalizer output $\{y_k\}$. In the presentation of the idea we assume that $\{a_k\}$, $\{x_k\}$,and $\{y_k\}$ are stationary random processes , and that the channel is noiseless.

The latter assumption is nearly fulfilled in telephone data transmission channels where the intersymbol interference (ISI) is the main annoying fenomenon and the noise is a secondary one that may be neglected.

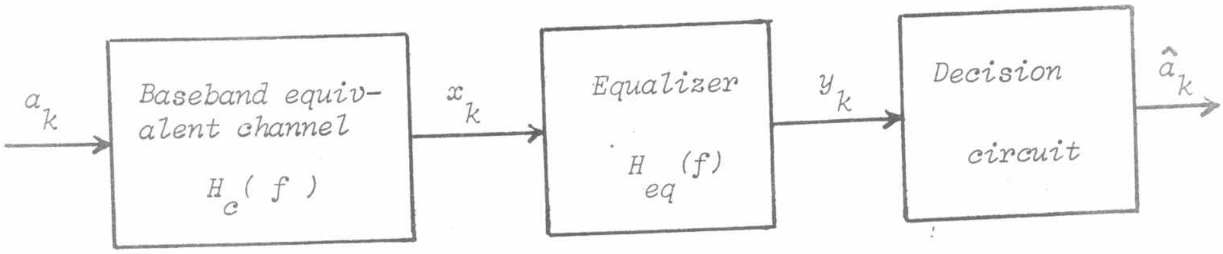

Fig.1. Discrete baseband equivalent of a data transmission system.

In practical applications the data sequence is usually a sequence of uncorrelated zero-meam discrete random variables. Thus, the autocorrelation function of the data is

$$R_a(n) = \begin{cases} a^2 & n=0 \\ 0 & n\neq 0 \end{cases} \quad ; \; a^2=E\,[\,a_k^{\,2}\,].$$

where $R_a(n)= E(a_k\ a_{k+n})$. Hence $S_a(f)$, which is the Discrete Fourier Transform (DFT) of $R_a(n)$ is given by

$$S_a(f)=a^2 \quad , \quad f \in \left[-\frac{1}{2T} \, , \, \frac{1}{2T} \right] \tag{1}$$

One has

$$S_x(f)=S_a(f)\ |H_c(f)|^2 = a^2\ |H_c(f)|^2 \tag{2}$$

Thus

$$|H_c(f)| = \sqrt{S_x(f)}/a \tag{3}$$

The equation (3) signifies that the amplitude-frequency response $|H_c(f)|$ of the channel can be completely identified on the basis of $S_x(f)$ which is available . The amplitude-frequency response $|H_{eq}(f)|$ of the ideal equalizer is the reciprocal of $|H_c(f)|$. Hence

$$|H_{eq}(f)| = a/\sqrt{S_x(f)} \tag{4}$$

of (8) may be replaced by $a/\sqrt{(W_l^n + \epsilon)/N}$, where ϵ is an arbitrary small positive number .

The sampled frequency response of the equalizer is multiplied by the DFT of $\{x_k\}$ to yield the DFT of $\{y_k\}$. Time samples y_k are then restored by an inverse FFT processor.

IV - ANALYSIS OF THE ADAPTATION ALGORITHM

The algorithm (7) can be written in the form

$$W_l^n = (1-\mu)\; W_{l-1}^n + \mu \left| x_l^n \right|^2 \tag{9}$$

The equation (9) is equivalent to figure 3 , in which $\{W_l^n\}$ is the response of the discrete filter

$$H(z) = \mu/(1-(1-\mu)z^{-1}) \tag{10}$$

to the sequence $\left|x_l^n\right|^2$; $l = 1,2,\ldots.$

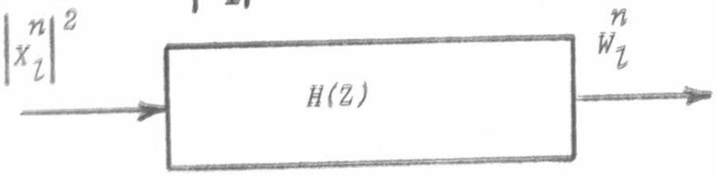

Fig.3. The equivalent discrete filter of (9)

In order that the algorithm (9) may be stable , the poles of the filter (10) should lie inside the unit circle in the Z-plane, i.e. $|1-\mu|$ should be less than unity . Hence μ should satisfy

$$0 < \mu < 2 \tag{11}$$

The amplitude- frequency of the filter (10) is

$$\left| H\,(e^{jwT}) \right| = \mu/\sqrt{1+(1-\mu)^2-2(1-\mu)\cos wT} \tag{12}$$

This response is plotted in figure 4,at different values of μ.

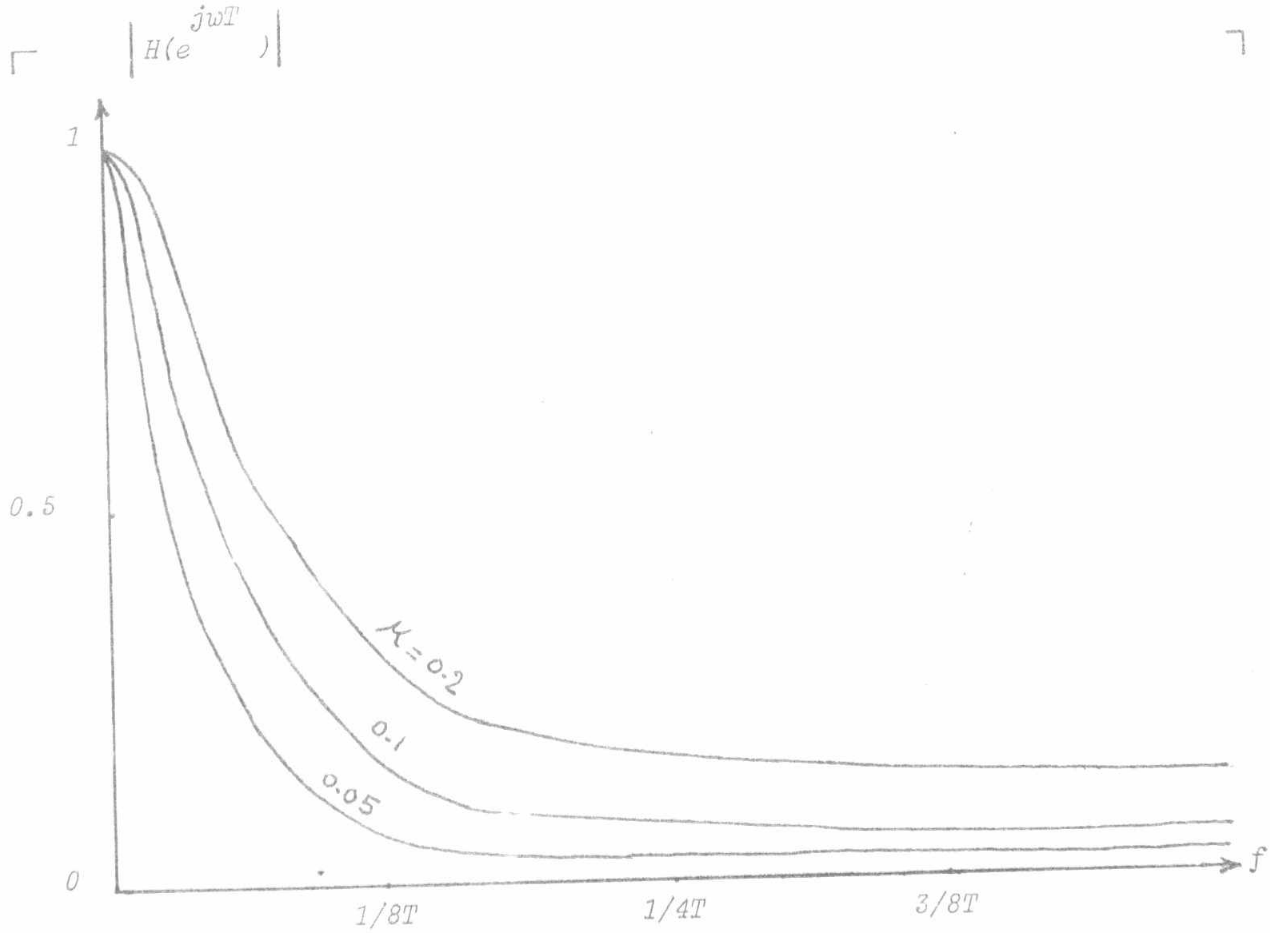

Fig .4. Amplitude- frequency response of the filter (10)

Now , it is desired that the algorithm (7) converges to the mean of $|X_1^n|^2$. Hence the filter of figure 3 , should pass only the D.C component of the sequence $|X_1^n|^2$ .From this point of view, μ should be chosen close to zero due to figure 4 . On the other hand it is desired that the algorithm (7) has a small convergence time . This means that transient response of the filter (10) should be rapid . The speed of the transient response is related to the -3 db frequency w_3,

$$w_3 = \frac{1}{T} \cos^{-1}\left[1-(\mu^2/(2-2\mu))\right]; \quad 0 < \mu < 0.8 \qquad (13)$$

Equation (13) is plotted in figure 5. On the basis of that figure, a large value of w_3, and hence a small convergence time , implies the use of relatively large values of μ . Notice that as w_3 increases the filter passes frequency components of $|X_1^n|^2$ other than the desired D.C one , and thus W_1^n will fluctuate around the mean of $|x_1^n|^2$. The power of these fluctuations increases as W_3 increases . Hence , as μ increases the convergence accuracy of the algorithm (7) deteriorates. Thus the choice of μ is a comprpromize between the convergence speed and the convergence accuracy. One may ensure both rapid and accurate convergence through the use of a large value of μ at the start of adaptation, and a small one after the elapse of few tens of baud intervals . Simulation results in the next section verifies this idea.

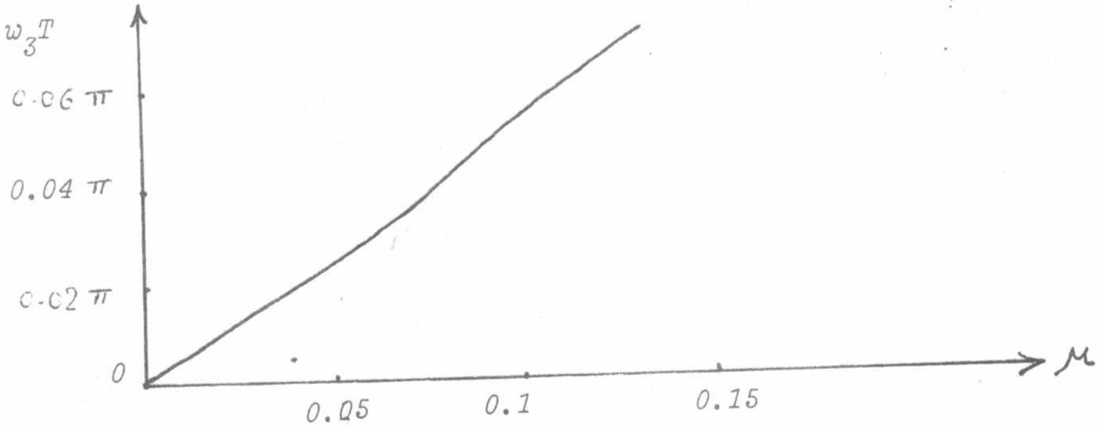

Fig.5.Dependence of w_3 on .

V- COMPUTER SIMULATIONS

V- 1 Modeling

The data sequence $\{a_k\}$ is modeled by a set of independent discrete random variables, each assumes one of the four values $\{-3,-1,1,3\}$ equally likely.

The additive noise n_k is modeled by a stationary sequence of independent, zero- mean Gaussian randon variables with variance σ^2. The signal to noise ratio (SNR) is defined by

$$\text{SNR} = 10 \log E(|a_k|^2)/\sigma^2$$

The channel is modeled by the finite impulse response filter

$$x_k = \sum_{i=-2}^{i=2} h_i a_{k-i} + n_k$$

where

$h_{-2} = 0.1$, $h_{-1} = 0.2$, $h_0 = 0.9$, $h_1 = 0.2$, $h_2 = 0.1$

This model has a peak distortion , δ = 67% where δ is defined by [6]

$$\delta = \sum_{i \neq 0} |h_i| / |h_0| .$$

The channel corresponding to this model is free from phase distortion. Other channel models are considered in [7] .

V-2. Simulation results

Performance dependence on μ and N

The channel is assumed noiseless ; effect of noise is considered at the end of this section . Both transient and steady state performance are studied. The first is specified by T_c while the latter is specified by E. T_c is defined as the first time after which the section error rate P_l does not exceed 6% ; P_l is defined by

P_l = Number of errors in the data recovered during the l^{th} section/ N.

The steady state error rate E is defined as

E = Number of errors on a long window (15NT) after convergence / 15NT

The dependence of T_c on μ and N is shown in figure 6, while the steady state error rate performance is shown in figure7 .

From these figures , it is clear that as μ increases, T_c decreases , while E increases and vice versa . thus the choice of μ is a compromize between E and T_c performance. As mentioned in section IV , it is possible to ensure both rapid and accurate convergence through the use of more than one value for μ . To verify this idea, we have simulated the case where

$$\mu = \begin{cases} 0.2 & \text{for } l \leq 10 \\ 0.025 & \text{for } l > 10 \end{cases}$$

and the obtained performance is

$$T_c = 512\ T \qquad \text{and} \qquad E = 2\text{x}\ 10^{-3}$$

The proposed SAE working with a fixed $\mu = 0.025$ ensures the same value of E but after a ten times longer convergence time.

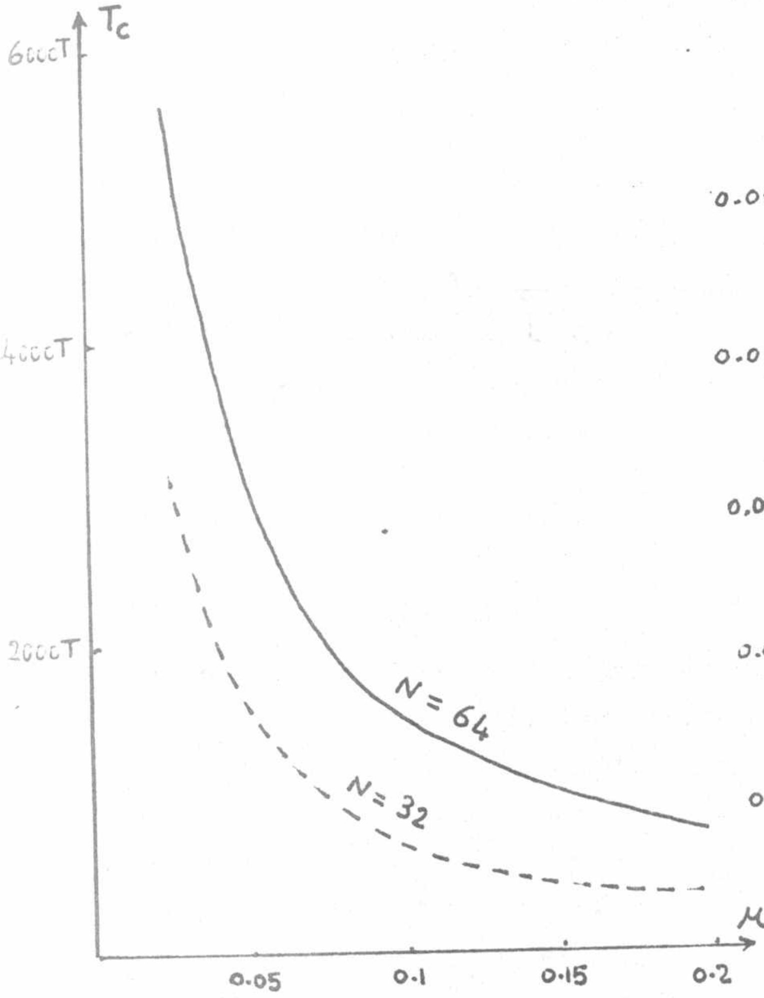

Fig.6. Dependence of T_c on μ and N

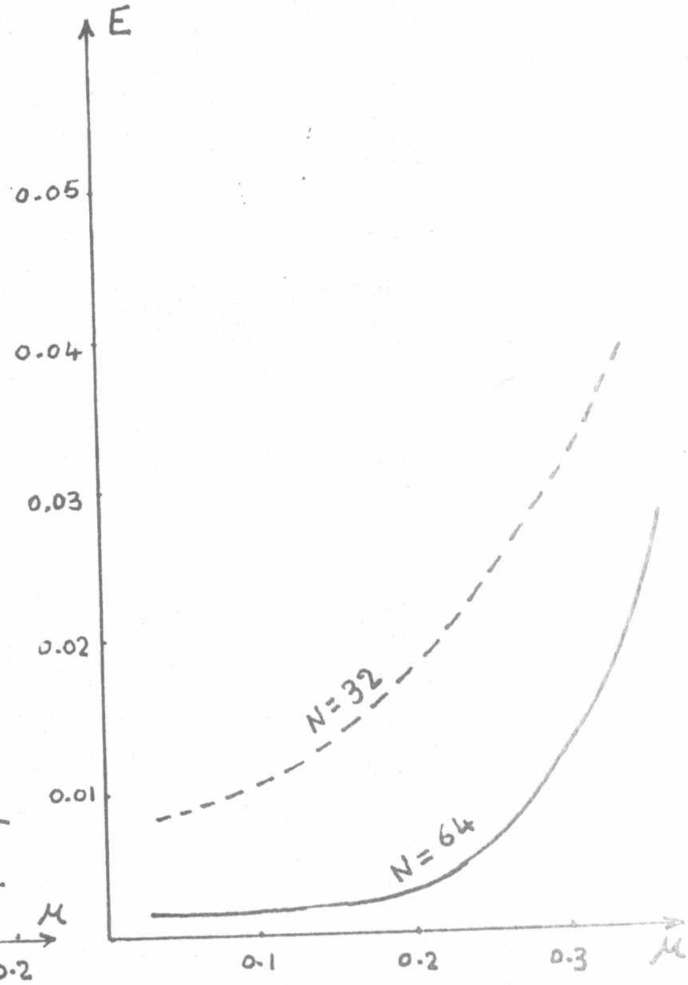

Fig.7. Dependence of E on μ and N

The figures 6 and7 show also that as N increases , T_c increases, while E decreases. Thus the choice of N is a compromize between the convergence speed and technical simplicity on one side and the error rate performance on the other one.To underline the benefit of the proposed SAE we mention that such SAE with N=64 and $0.025 \leq \mu \leq 0.15$ reduces the error rate from 0.17 (that corresponds to the case of no equalization) to the value $2\text{x}\ 10^{-3}$

(i.e. 100 times smaller).

Effect of channel noise

Two values for SNR are considered ; 20 and 25 db. The results of the proposed SAE with N = 64 and μ = 0.1 are

T_c = 1728T , E= 0.01 when SNR = 20 db.

T_c = 1664T , E= 0.004 when SNR = 25 db.

From these results , the following conclusions are drawn:

1 - The proposed SAE works in the presence of noise; e.g. when SNR is 20 db, the proposed SAE with N=64 and μ =0.1 reduces the error rate from 0.18 (without equalization) to 0.01.

2 - Both T_c and E increase as the channel noise increases , and vice versa.

VI - CONCLUSIONS

1 - The proposed SAE can equalize severe distortions in the amplitude-frequency response of the channel .

2 - As μ increases , T_c decreases while E increases. Thus the choice of μ is a compromize between E and T_c.

3 - As N increases , T_c increases while E decreases. Thus the choice of N is a compromize between T_c and technical simplicity on one side and E on the other one.

4 - Fast convergence and small steady state error rate can be attained by using more than one value for μ .

5 - The proposed SAE works in the presence of noise. Both T_c and E increase as the channel noise increase and vice versa.

6 - The proposed SAE can work in the presence of mild distortions in the phase-frequency response of channel [7] .

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