MILITARY TECHNICAL COLLEGE
CAIRO - EGYPT

OPTICAL FIBRE
DUOBINARY SYSTEM DESION

M.I. BADR

ABSTRACT

A calculation and optimization method for optical fibre transmission systems using the principle of digital duobinary transmission is presented. When the received optical pulses overlap, baseband equalization can be used to separate the pulses with a practical but significant increase in required optical power. This required power penalty is calculated versus the total rms spreading which is related to the ratio between the baseband and optical cut - off frequency via the input and equalized pusle shape, when using PIN diodes or APD's.

AUTHOR : Maj. Eng. Mohamed Ibrahim Badr, General Headquarter's Signal Corps.

INTRODUCTION

It is a well - known fact that signals transmitted in a digital transmission system over optical fibres suffer from prolongation in time caused by fibre dispersion (intermodal, interamodal, material dispersion). Depending on the type of fibre employed (multimode with step - index or graded - index profile, single - mode fibre) and the optical wavelength used for signal transmission. Dispersion effects lead to intersymbol interference (ISI), which results in a reduction of the energy confined per bit. The ISI can be removed by the equalization at the receiving end [I].

SYSTEM MODEL

Fig.1 depicts the principle of a duobinary transmission system. The " data input " binary sequence $\{a_k\}_{k=1}^{\infty}$ to be transmitted is firest precoded into binary sequence $\{b_k\}_{k=0}^{\infty}$ via the coder with the components :

$$b_k = \begin{cases} 0 \quad \text{or } 1 & , \quad k = 0 \\ a_k \oplus b_{k-1} & , \quad k = 1 , 2 , 3 , \ldots \end{cases} \qquad (1)$$

Where $\oplus$ is an operator for modulo - 2 addition.

This precoding is only a transformation of sequence $\{a_k\}$ into sequence $\{b_k\}$, aimed at avoiding error propagation in the decoder [2].

The radiation power signals, generated according to sequence $\{b_k\}$ by an optical source (light emitting diode - LED or semiconductor injection laser diode - ILD) are coupled into the fibre. The light power propagates along the fibre according to the principle of total internal reflection. During propagation, the optical signal pulses suffer attenuation - a (bulk - a_i and distributed - a_j) and dispersion.

The optical signals are detected by the photodetector (S_i PIN photodiodes and S_i or G_e avalanche photodiodes - APD) then, they are amplified in the preamplifier and shaped in the duobinary equalizer, put under decision and decoded using the following rule :

When b_k and b_{k-1} are both 1's the output is 0 .
When b_k and b_{k-1} are different the output is 1 .
When b_k and b_{k-1} are both o's the output is 0 . (2)

OPTICAL RECEIVER

Fig. 2 shows a fairly typical receiver, in schematic form, consisting of an avalanche photodiode, an amplifier and an equalizer.

The amplifier is modeled as an ideal high gain (A), infinite - impedance amplifier with an equivalent shunt capacitance and resistance at the input and with two noise sources referred to the input, the noise sources will be assumed to be white, Gaussian, and uncorrelated.

The power falling upon the detector will be assumed to be of the form of a digital pulse stream.

$$P(t) = \sum_k b_k\, h_p\, (t-kT_r) \qquad (3)$$

Where b_k takes on one of two values for each integer value of k, T_r is the pulse spacing, $h_p\ (t-kT_r)$ = input pulse shape and positive for all time t,

If $h_p(t-kT_r)$ is normalized, and that, $\int_{-\infty}^{\infty} h_p(t-kT_r)\, dt = 1$, therefore b_k represents the energy confined in the pulse k.

In this paper we assume that $h_p(t)$ is Gaussian in shape and given by:

$$h_p(t) = \frac{1}{\sqrt{2\pi}\ T_r \alpha}\ \mathrm{EXP}\left\{ -t^2 / \left(2 \left(\alpha T_r \right)^2 \right) \right\} \qquad (4)$$

The output voltage $V_{out}(t)$ is in the form :

$$V_{out}(t) = i_s(t) * k_{iv} \cdot A\, h_{fe}(t) * k_{vv} \cdot h_{eq}(t) \qquad (5)$$

Where $i_s(t) = R \cdot E(g) \cdot P(t)$ = detector output carrent, k_{iv} is a factor describing the current to voltage conversion in the amplifier, k_{vv} = factor describing the voltage to voltage conversion in the equalizer, $h_{eq}(t)$ = equalizer impulse response, $*$ indicates convolution and $h_{fe}(t)$ = amplifier input circuit current impulse response given by :

$$h_{fe}(t) = F^{-1} \left\{ \frac{1}{1/R_t + jwC_t} \right\} \qquad (6)$$

Where $R_t = R_a // R_b$ = total detector parallel load resistance, R_b = detector biasing resistance, R_a = amplifier input resistance, $C_t = C_a // C_d$ = total detector parallel load capacitance, C_d = junction capacitance of the diode, C_a = amplifier input capacitance, $R = \eta e / h\nu$ = responsivity of detector and $E(g) = G$ = mean detector gain.

If we assume that A is very large $= 1 / k_{iv} \cdot k_{vv}$, then $V_{out}(t)$ is given by :

$$V_{out}(t) = i_s(t) * h_{fe}(t) * h_{eq}(t)$$

$$= \sum_k b_k\, h_{out}(t-kT_r) + n(t) \qquad (7)$$

Where n(t) represents deviations (or noises) of $V_{out}(t)$ from its average and $h_{out}(t-kT_r)$ = equalizer output pulse waveform. For duobinary transmission $h_{out}(t)$ is given by :

$$h_{out}(t) = \left[4\cos(\pi t / T_r) \right] / \left[\pi (1-4t^2/T_r^2) \right] \qquad (8)$$

$$\text{and } H_{out}(f) = \begin{cases} 2\,T_r \cos(\pi f T_r) & , |f| \leq 1/(2T_r) \\ 0 & , |f| > 1/(2T_r) \end{cases} \qquad (9)$$

NOISE CALCULATIONS

We can now calculate the variance of n(t), the noise portion of the output voltage $V_{out}(t)$ in Fig. (2) as follows :

$$N = E\left[n^2(t) \right] = E\left[V_{out}^2(t) \right] - E^2\left[V_{out}(t) \right] \qquad (10)$$

The noise, N, depends upon the coefficients $\{b_k\}$ and upon the time t. This is a property which distinguishes fibre optical systems from many other systems where the noise is signal and time independent. The set of times $t = \{ kT_r - T_r/2 \}$ represents the sampling times for $V_{out}(t)$ and the decision depends upon the vaules of $\{b_k, b_{k-1}\}$ and N. We shall assume that the equalized pulse $h_{out}(t)$ is normalized, then the mean value of $V_{out}(t)$, at $t = -T_r/2$ is given by :

$$b_k h_{out}(-T_r/2) = 2b_{max}, \; b_o \text{ and } b_{-1} \text{ are both 1's .}$$

$$= b_{max} + b_{min}, \; b_o \text{ and } b_{-1} \text{ are different.}$$

$$= 2b_{min}, \; b_o \text{ and } b_{-1} \text{ are both 0's .}$$

$$\text{and, } h_{out}(kT_r - T_r/2) = 0 \;, \; k \neq 0, -1 \qquad (11)$$

That is, the equalized pulse stream $V_{out}(t)$ has no ISI at the sampling times $(kT_r - T_r/2)$, $k \neq 0,1$, $b_k = b_{max}$ when receiving 1's and $b_k = b_{min}$ when receiving 0's, this due to the non ideal modulation process.

We shal define the two - sided spectral density of the amplifier - current noise source $i_a(t)$ as S_i and the two-sided spectral height of the amplifier - voltage noise source $e_a(t)$ as S_e. The two - sided spectral density of the Johnson - current noise source $i_b(t)$ associated with R_b is $2KT/R_b$, where K is Boltzmann's constant and T is the absolute temperature. The output noise is given by [3]:

$$n(t) = V_{out}(t) - E\left[V_{out}(t) \right] = n_s(t) + n_r(t) + n_i(t) + n_e(t) \qquad (12)$$

Where $n_s(t)$ = output noise due to the random poisson process nature of the current $i_s(t)$ produced by the detector, $n_r(t)$ = output noise due to the Johnson noise current source $i_b(t)$ of the resistor R_b, $n_i(t)$ = output noise due to the amplifier input current noise source $i_a(t)$, and $n_e(t)$ = output noise due to the amplifier input voltage noise source $e_a(t)$.
The variance of n(t) is given by [3]:

$$N = E\left[V_{out}^2(t)\right] - E^2\left[V_{out}(t)\right] = E\left[(V_{out}(t) - E(V_{out}(t)))^2\right]$$
$$= E\,(n_s^2(t)) + E\,(n_r^2(t)) + E(n_i^2(t)) + E\,(n_e^2(t)) \qquad (13)$$

The variance of n(t) = N, is *generally* a function of t and b_k, for our receiver model is given by [1] :

$$N = (h\cdot\nu/\eta)^2 \Big[(\eta\cdot E(g^2)/\, h\cdot\nu\cdot E^2(g))\cdot\int_{-\infty}^{\infty} H_p(f)\left[(H_{out}(f)/H_p(f)) * (H_{out}(f)/H_p(f))\right]\cdot e^{j2\pi ft}\cdot \underbrace{\sum b_k\cdot e^{-j2\pi fkT_r}}_{\text{shot noises}}\, df +$$
$$(1/e^2E^2(g))\cdot(\underbrace{2KT/R_b + S_i}_{\text{thermal noises}} + \underbrace{e^2\cdot E(g^2)\cdot\lambda_d}_{\text{shot noises}})\cdot\int_{-\infty}^{\infty}\left|H_{out}(f)/H_p(f)\right|^2 df + (S_e/e^2\cdot E^2(g))\int_{-\infty}^{\infty}\left|H_{out}(f)\cdot(1/R_t + j2\pi fC_t)/H_p(f)\right|^2 df\Big] \qquad (14)$$

Where λ_d = dark current, rate of counting (electrons per second), $E(g^2) = F_G\, E^2(g) = G^{2+x}$, F_G is the noise figure of APD and x is the excess noise factor [3].
The worst case of noise NW (b_o, b_{-1}) for duobinary system is given by :

$$NW\,(b_o, b_{-1}) = \max_{\{b_k\},\ k\neq 0,\,-1} (N\,(b_k)),\quad t = -T_r/2 \qquad (15)$$

REQUIRED OPTICAL POWER

We shall assume that the output voltage of the equalizer at

the decision instants is approximately a Gaussian random variable. We define $\sigma_0^2 = NW(b_{min}, b_{min})$; $\sigma_1^2 = NW(b_{min}, b_{max}) = NW(b_{max}, b_{min})$ and $\sigma_2^2 = NW(b_{max}, b_{max})$.
For threshold levels F_1 ($2b_{min} < F_1 < b_{max} + b_{min}$) and F_2 ($b_{max} + b_{min} < F_2 < 2b_{max}$), the probability of error, P_e can be put in the following form :

$$P_e = (1/4)\, Q\left((F_1 - 2b_{min})/\sigma_0\right) + (1/2)\left[Q\left((F_2 - (b_{max} + b_{min}))/\sigma_1\right) + Q\left(((b_{max} + b_{min}) - F_1)/\sigma_1\right)\right] + (1/4)\, Q\left((2b_{max} - F_2)/\sigma_2\right) \quad (16)$$

$$\text{Where } Q(u) = (1/(2\pi)^{1/2}) \int_u^{\infty} e^{-x^2/2}\, dx \quad (17)$$

The optimum F_1, F_2 corresponding to P_e minimum is given by:

$$(F_1 - 2b_{min})/\sigma_0 = ((b_{max} + b_{min}) - F_1)/\sigma_1 = C_1 ,$$

Thus

$$F_1 = (\sigma_0 b_{max} + (\sigma_0 + 2\sigma_1) b_{min})/(\sigma_0 + \sigma_1) \quad (18)$$

and

$$(F_2 - (b_{max} + b_{min}))/\sigma_1 = (2b_{max} - F_2)/\sigma_2 = C_2 ,$$

Thus

$$F_2 = (\sigma_2 b_{min} + b_{max}(2\sigma_1 + \sigma_2))/(\sigma_1 + \sigma_2) \quad (19)$$

From (18) and (19) we get the following relation :

$$C_2 = \alpha_c C_1 \quad , \alpha_c = (\sigma_0 + \sigma_1)/(\sigma_1 + \sigma_2) \quad (20)$$

From equations (18 - 20) relation (16) becoms :

$$P_e = (3/4)\left[Q(C_1) + Q(\alpha_c C_1)\right] \quad (21)$$

If we assume that the conditional error probabilities associated with both symbols (0) and (1) are equal and $b_{min}=0$, the upper bound of $\sigma_i = \sigma_2\ \forall i$, $\alpha_c = 1$ and the probability of error is then given by :

$$P_e = (3/2)\left[Q(C_1) \right], \quad C_1 = b_{max} /(2\sigma_2)$$

and $F_1 = b_{max}/2$, $F_2 = 3b_{max}/2$ (22)

For specific P_e required, C_1 can be found from Q function tables, the optimum gain for minimum b_{max} can be calculated. We assume that for long string of bits, the probability of transmitting 0's = probability of tronsmitting 1's = 0.5, then the average required optical power at the receiver input is given by :

$$P_r = b_{max} /2T_r \tag{23}$$

REPEATER SPAN

The maximum repeater span in general case is given by :

$$L = (P_t - P_r) /a \ , \quad a = a_i + a_j \tag{24}$$

Where P_t = average transmitted optical power, a = total fibre attenuation in dB/Km, this consists of bulk attenuation a_i plus distributed attenuation a_j [4].
In [3] we see that the required optical power at the receiver is minimum when α is very small (≤ 0.1), when α increases due to optical cut - off, the required optical power increases by value called power penalty, Fig.3 - shows the power penalty in dB calculated for Gaussian $h_p(t)$ and duobinary $h_{out}(t)$ when using PIN diodes or APD's versus f_r/f_t, which is related to the total normalized rms dispersion α of $h_p(t)$ by the following relation [4] :

$$\alpha = \sigma/T_r = (\tau_e /2\sqrt{2})/T_r \ , \quad f_t = 0.53 / \tau_e$$

then $\alpha = 0.188 \ (f_r/f_t)$ (25)

Where P_{ro} is the average required optical power when $\alpha \leq 0.1$, C_d is Constant for numerical approximation ($\cong 0.4$ When using Apd's or $\cong 0.3$ When using PIN Photodiodes).

CONCLUSIONS

We have presented a brief eview for the duobinary system approach based on the Calculation of minimum energy confined per bit"I" for Specific P_e. As a result, the power penalty for duobinary System is less than that for raised cosine system[3], the difference in power penalty increases when rms dispersion increases, i.e. the ratio f_r/f_t increases for the same P_e and *assuming* other system parameters *to be constant.*

ACKNOWLEDGMENT

The outhor Would like to thank Col. Dr. Eng. Ahmed Elosany and Lt. Col. Dr. Eng . Ali Elmoghazy Ali for their Valuable Comments and encouragements.

REFERENCES

[I] M.I. Badr, "Digital optical fibre systems, duobinary & raised conie trade-off", M.Sc degree in communication engineering, Military Technical College, Cairo, 1984 .

[2] K.S. Shanmugam, "Digital and analog Communication Systems", John Wiley and sons, 1979.

[3] S.D. Personick "Reseiver design for digital fibre optic communication systems", Part I&II" Bell syst. Tech. J., vol. 52, pp.834-887, July- Aug. 1973.

[4] J.E. Midwinter, "Optical fibres for transmission", John wiley and sons , 1979.

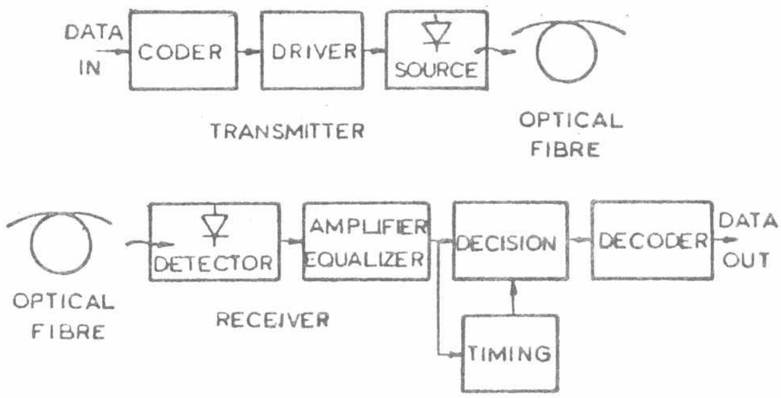

FIG. 1 – Block diagram of an optical fibre digital transmission system.

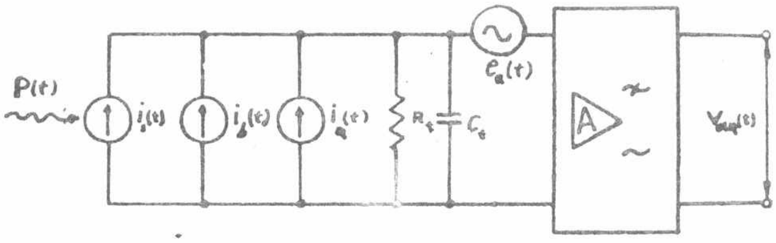

FIG. 2 – Equivalent circuit of an optical receiver.

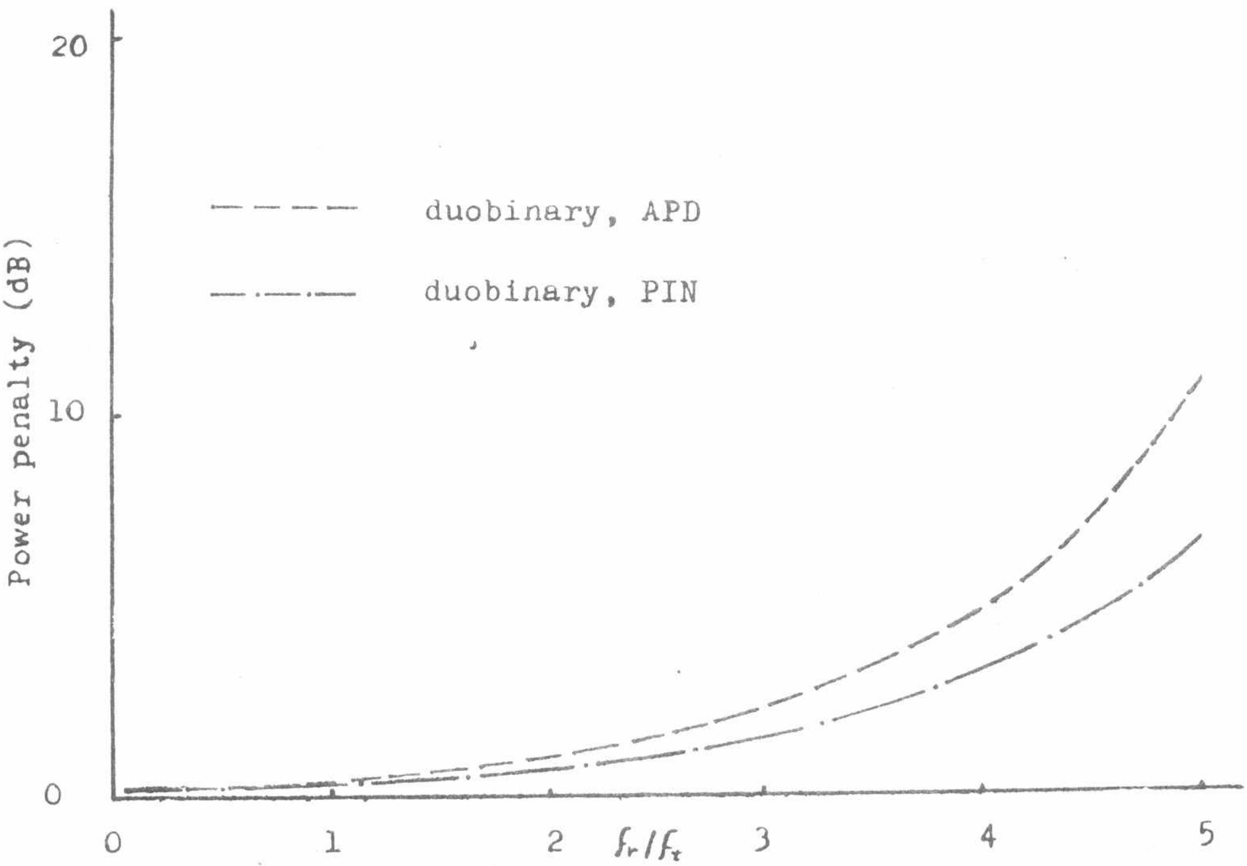

Fig. 3- power penalty versus (f_r/f_t)