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A NOVEL MULTIPARAMETER TOLERANCE MEASURE

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ABSTRACT

The sensitivity of the response of a network to component tolerances is an important practical consideration in the design of active RC networks. Deviations from the nominal element values can distort the response and result in an undesirable or unstable circuit. In order to evaluate the sensitivity of the network a measure should be defined. In order for the measure to be practical all parameters variations must be taken into account. In this paper a new multiparameter tolerance measure is introduced. The measure has been applied to evaluate the sensitivity performance of canonical multiple feedback active RC filter with single operational amplifier. The measure proves to be efficient and a very useful tool for comparing networks from points of view of sensitivity.

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INTRODUCTION

Network synthesis is accomplished via two steps:(1)Approximation step (2)Realization step.The second step involves the choice of a circuit configuration.Available to day a large number of equivalent circuit configurations which are capable of realizing a given transfer function Given this difficult task of selecting a circuit configuration it is valuable to identify criteria that can be used as a figure of merit for determining the goodness of a circuit configuration.One of the most important criteria for comparing equivalent circuit configurations and for establishing their practical utility in meeting desired requirements is sensitivity.In practice real circuit components will deviate from their nominal design values,due to manufacturing tolerances,environmental changes such as temperature and humidity and chemical changes which occur as the circuit ages.Component deviations of these types cause circuit transfer function or response to deviate away from the specified function.

Mathematically,we can relate changes in the response Δr to variations in the elements Δx in the following manner [1]

$$\Delta r/r = S \Delta x/x \qquad (1)$$

In active filter,large S and large $\Delta x/x$ can alter the response of a filter beyond recognition and may even in some cases cause instability. By computing sensitivity,such problems can be identified before the network is constructed.We define S as the sensitivity of r to variations in x .Idealy,we would like $\Delta r/r = 0$ or at least as small as possible.Small $\Delta r/r$ implies that either S,$\Delta x/x$ or both are small.In practice precision elements (i.e.small $\Delta x/x$) implies an expensive network, with cost inversely proportional to $\Delta x/x$.Sensitivity S,which is a function of only the circuit configuration,and the element values,can often be reduced so that $\Delta r/r$ is acceptable at no cost penalty.In fact the smaller we make S the cheaper the circuit becomes to manufacture, thus the cost is reduced.

SENSITIVITY MEASURES

In practice,when we consider the sensitivity of transfer function to component variation we must take into account the variation of a number of parameters.Then,we must find a suitable measure $M(\omega,x)$ In order that the measure to be practical,all parameter variations must be included.This problem has been dealt with by many authors. For example Hakimi,and Gruz [2] defined the measure of multiparameter sensitivity;

$$M = \max_{\Delta x_i = 0}^{\Delta x_i = \delta_i} \frac{\Delta F/F}{\prod_{i=1}^{n} \delta_i} \qquad (2)$$

Where δ_i is the maximum admissible deviation of the i-th element from its nominal value.

Schoeffler [3] proposed a frequently used expression of measure of multiparameter sensitivity by means of the sum of the values:

$$M = \sum |S_x^F|^2 \qquad (3)$$

In the following we shall define our sensitivity measure, also we can use the term "Tolerance Measure" because it can be also applied for greater changes of element values.

TOLERANCE MEASURE

As we have seen before, many authors have tried to find a proper multiparameter sensitivity measure. We have mentioned few of them. We could mention more Goldstein and Kuo[4], Biswas and Kuh[5]. For lower order transfer functions of course we can easily find an explicit expression for the sensitvities to changes of either passive or active elements of the filter. For higher orders these expressions may be also derived, but they are complicated and they do not offer a clear insight to the study of the sensitivity of the given filter. From the great variety of the proposed measures the formula recommended by Schoeffler[3] has found a wide spread use, because of its simplicity.
The relative change of the amplitude response due to a change of an element x_i is defined as:

$$\frac{|\Delta T(j\omega)|}{|T(j\omega)|} = \frac{|T(j\omega, x_i+\Delta x_i)| - |T(j\omega, x_i)|}{|T(j\omega, x_i)|} \qquad (4)$$

Where x_i is the nominal value of the element
Δx_i is its variation from the nominal value
If we assume small variation from the nominal we can express the relative change of the amplitude response as:

$$\left|\frac{\Delta T(j\omega)}{T(j\omega)}\right| = S_{x_i}^{|T(j\omega)|}\frac{\Delta x_i}{x_i} \qquad (5)$$

Consider now a second order Sallen and Key low pass filter shown in Fig.1. The open circuit voltage transfer function can be found to be:

$$T(s) = \frac{U_2}{U_1} = \frac{K}{s^2C_1C_2R_1R_2 + s[C_1R_1 + C_1R_2 + C_2R_2(1-K)] + 1} \qquad (6)$$

Where K is the gain of the voltage controlled voltage source.
If we consider a Chebyshev response with 1db ripple in the passband and if we choose a value of amplifier gain K=2.0, and equal valued capacitors normalized to the value 1.0. Then the normalized values of resistors will be:

$R_1 = 0.995668$ $\qquad$ $R_2 = 0.910967$

Then the relative change of the transfer function is calculated in the frequency range of interest using (5) assuming $\Delta x/x=1\%$. The dependance of this relative change on frequency is shown in Fig.2

Our tolerance measure will be defined (see Fig.2) as[6] :

$$M_T = \sum_{i=1}^{N} m_i \qquad (7)$$

Where
$$m_i = \max\left|\frac{T(x_i+\Delta x_i) - T(x_i)}{T(x_i)}\right|$$

N is the number of elements

APPLICATION OF THE MEASURE

The recommended measure has been applied to two cases. First case is a 3rd order Butterworth low pass filter with equal resistors normalized to the value 1.0. The network is shown in Fig.3 The relative deviation of the amplitude response has been calculated using a computer program in the frequency range of interest for different values of VCVS gain K These deviations were calculated by the program assuming 1% and 10% tolerances of the elements. The tolerance measure was calculated and the dependance $M_T = f(K)$ is shown in Fig.5 and Fig.6. A second network is chosen, a 4-th order B.W. low pass filter with equal resistors shown in Fig.4. The result $M_T = f(K)$ is shown in Fig.7.

CONCLUSION

A new tolerance measure was introduced. This measure proves to be the simplest, efficient and graphically interpretable. By means of this measure the performance of the studied structures was evaluated, compared from the point of view of the sensitivity of the amplitude response to element variations. From the obtained dependances we can conclude that:

1. We can find a certain range of gain K for which sensitivity is very small.
2. The sensitivity is extreme at the limits of the gain range.
3. There is a value of gain K can be found for which the sensitivity is minimum and hence the parameters of the network with minimum sensitivity could be found.

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[1] M.S. Ghausi, K.R. Laker "Modern Filter Design Active and Switched Capacitors" Prentice-Hall 1981.

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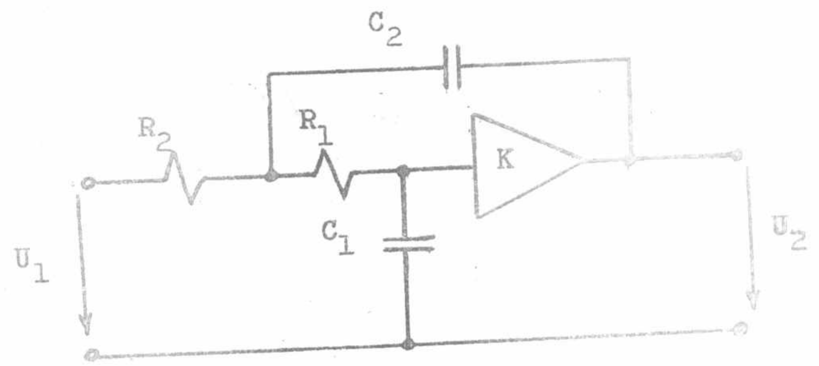

Fig.1 Sallen and Key Low Pass Filter

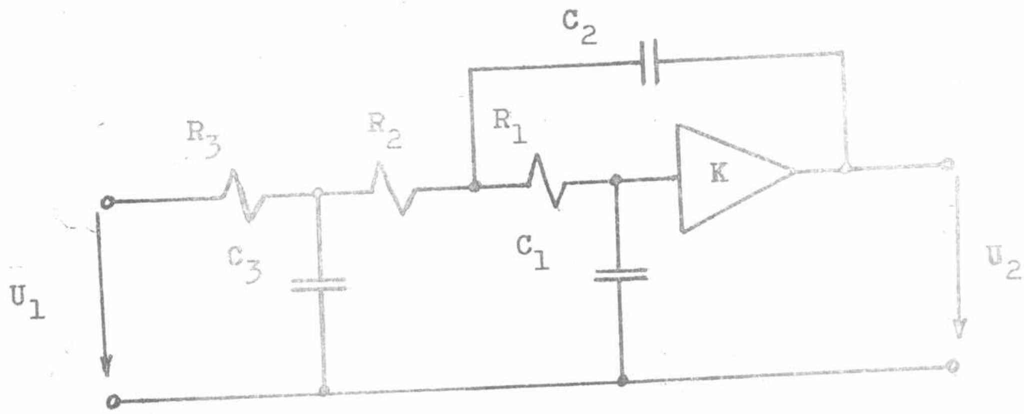

Fig.3 3rd Order B.W. Low Pass Filter

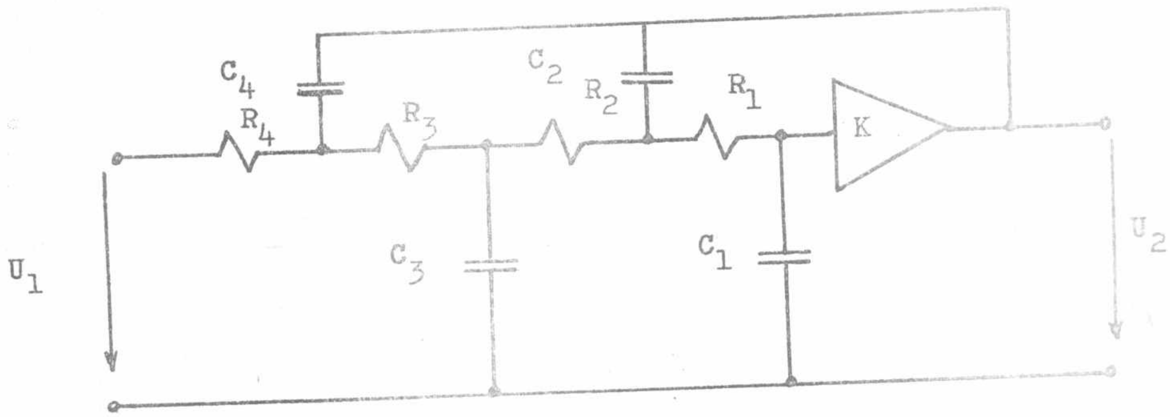

Fig.4 4th Order B.W. Low Pass Filter

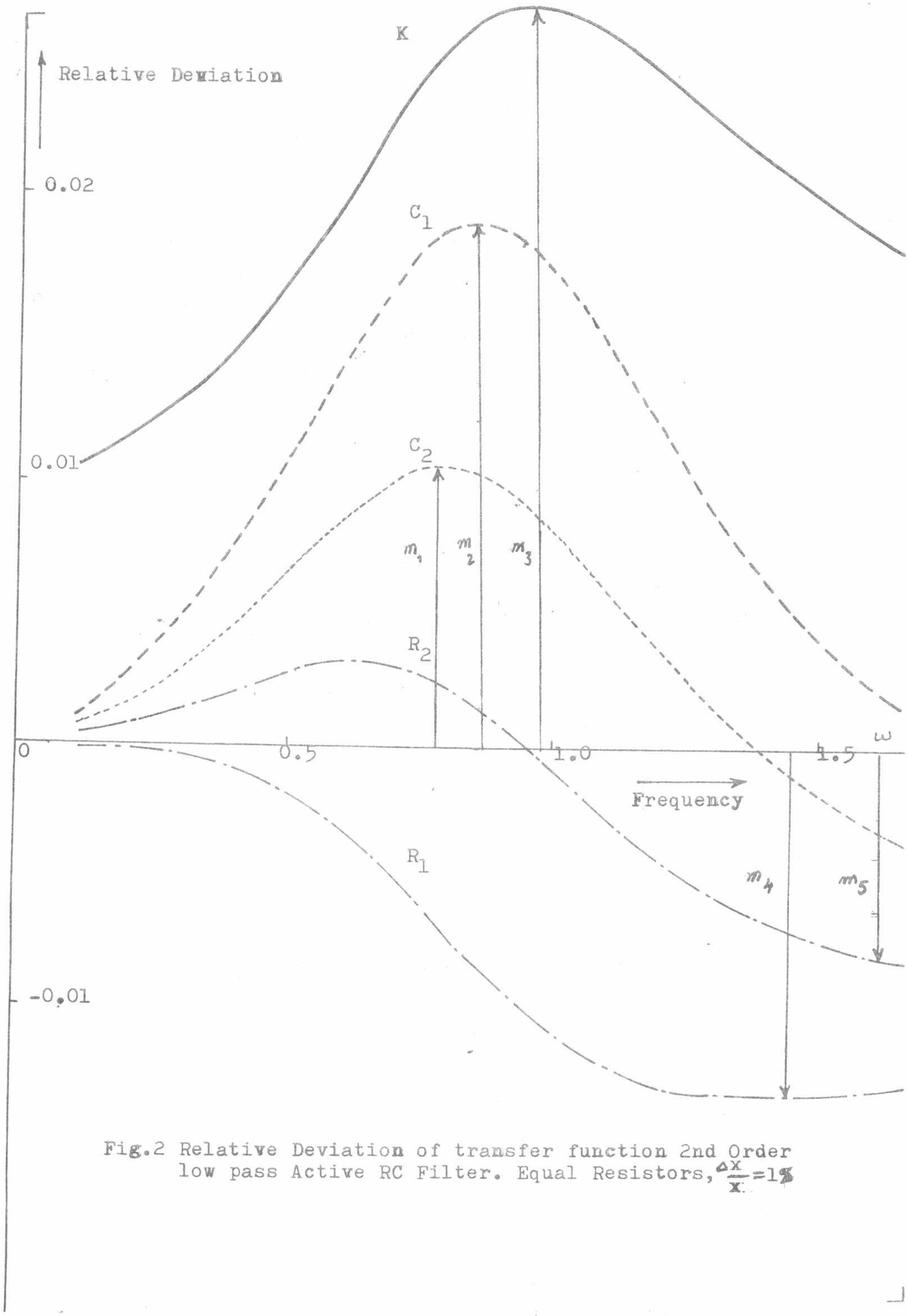

Fig.2 Relative Deviation of transfer function 2nd Order low pass Active RC Filter. Equal Resistors, $\frac{\Delta x}{x}=1\%$

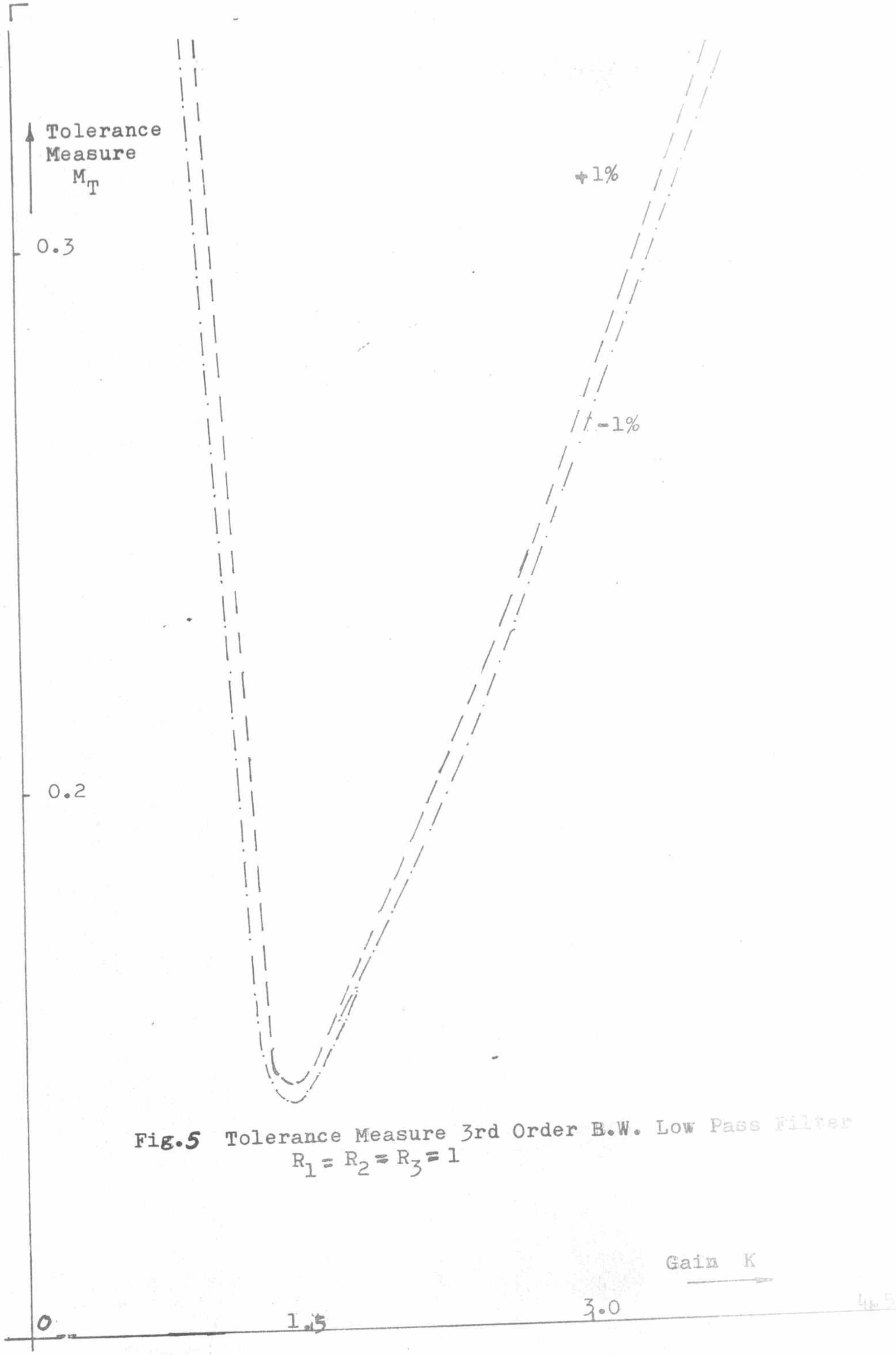

Fig.5 Tolerance Measure 3rd Order B.W. Low Pass Filter
$R_1 = R_2 = R_3 = 1$

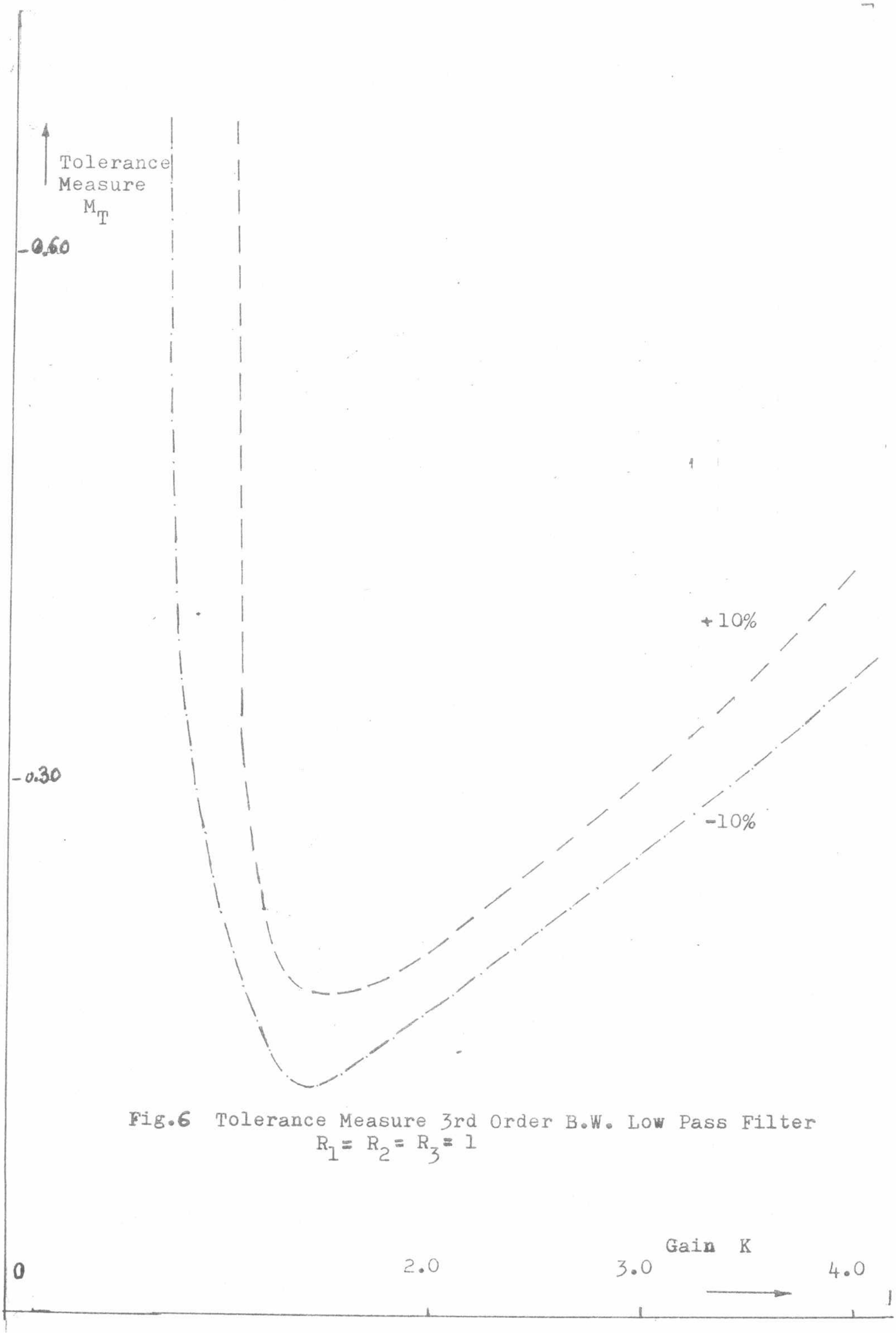

Fig.6 Tolerance Measure 3rd Order B.W. Low Pass Filter
$R_1 = R_2 = R_3 = 1$

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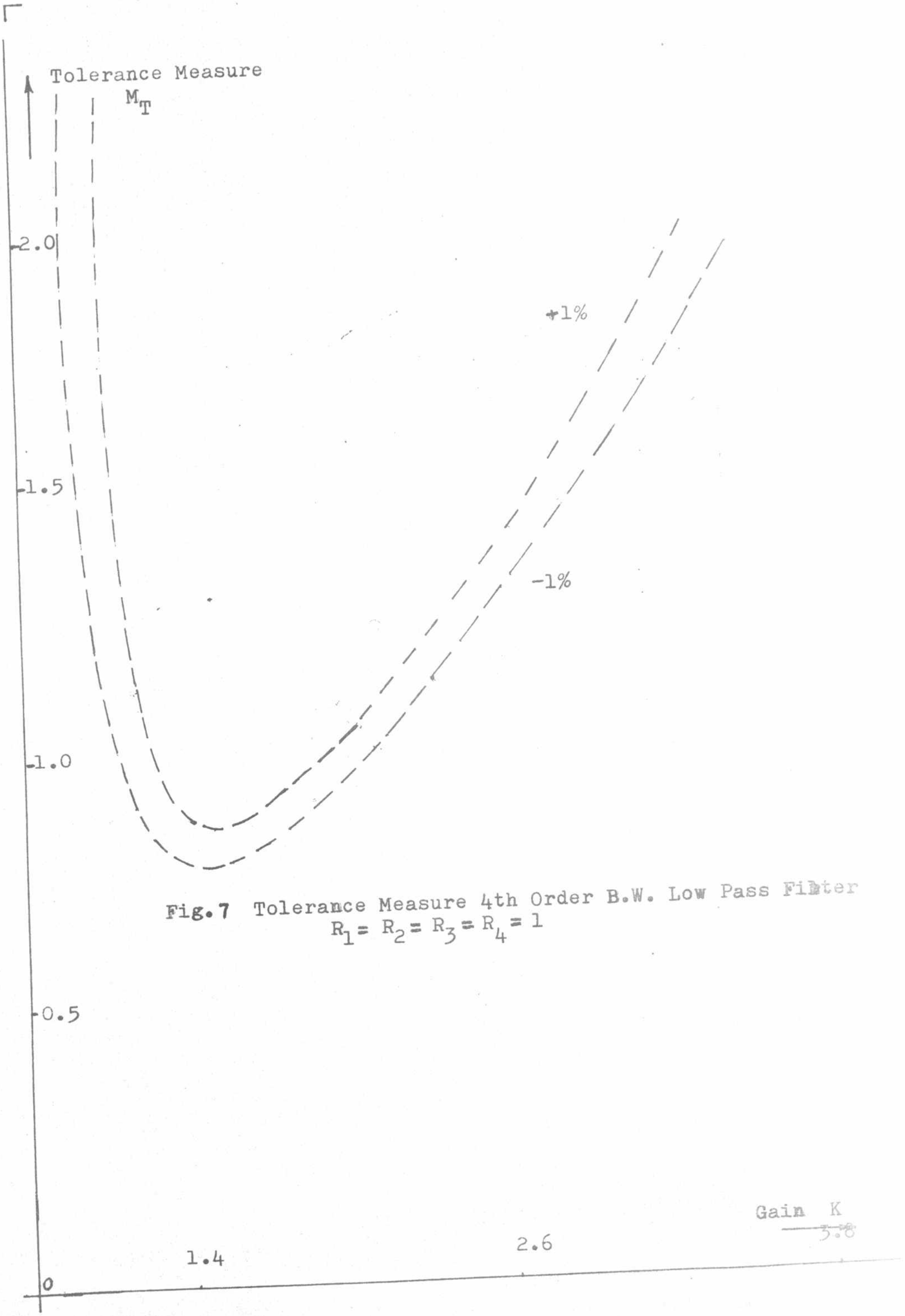

Fig.7 Tolerance Measure 4th Order B.W. Low Pass Filter
$R_1 = R_2 = R_3 = R_4 = 1$