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## CALCULATION OF AERODYNAMIC HEATING OF A CONICAL FOREBODY ALONG FLIGHT TRAJECTORY

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### **ABSTRACT**

The phenomenon of aerodynamic heating is created as a consequence of body-flow interaction. It is originated from viscous flow characteristics inside the boundary layer in the vicinity of body. The adjacent flow temperature increases, leading to an increase in body wall temperature depending on flow regime, surrounding air density, body velocity, wall material, flight time, etc.. The present work provides an illustrative calculation procedure based on closed-form relations. A computer program was implemented to be used as a quick, easy, and accurate calculation tool for the expected wall temperature distribution and its variation with flight time for a conical fore-body during a complete flight at zero incidence. The code was applied on a number of real cases with a variety of flight conditions. Results were compared with published and previous work and showed very good agreement.

KEY WORDS: Aerodynamic heating, boundary layer theory, viscous flow

## **NOMENCLATURES**

MOINTIA	OLATO
а	Speed of sound
Cf	Skin friction coefficient
Ср	Specific heat at constant pressure
h	Heat transfer coefficient
i	Enthalpy
kn	Knudsen number
1	Mean free path length
Ln	Nose generation
M	Mach number
Nu	Nusselt number
p, P	Pressure
Pr	Prandtl number
q	Heat transfer rate, dynamic pressure
r	Recovery factor
Re	Reynolds number
St	Stanton number
t	Time
Т	Temperature
V	Velocity
β	Angle of inclination, effective Mach
ν ν	Specific heat ratio
γ	Boundary layer thickness, thickness
0	5.00 a B S

Kinetic viscosity coefficient	v	
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nc	Incompressible
!	Mean free path, local
_	Laminar
Ν	Nose
m	Mean
0	Stagnation, sea level
r	Recovery, radiation
s S	Shock wave
T	Turbulent

## 1. INTRODUCTION

As the missile flies through the atmosphere, it experiences heat transfer from heated gas, adjacent to the wall, to the wall and vice versa as a consequence of flow stagnation inside the boundary layer in the vicinity of the flying body [1,2,3].

Being able to predict the net amount of heat transferred to the missile body during flight has become a vital step in missile design activities, Fig. 1, since almost all modern missiles carry very delicate guidance and control devices and/or carry very sensitive warheads. Controlling the amount of heat transferred to the missile onboard equipment and the maximum allowable temperature in missile interior is

In the present work, a mathematical model was developed for the estimation of aerodynamic heating over a conical forebody with a zero angle of attack. The mathematical model is based on closed form governing equations that govern the body-flow interaction according to the flow regime. The boundary layer theory governs these relations in the dense atmosphere, while the molecular kinetic theory describes these relations better in cases of high rarefaction or sharp decrease of air

A computer code was implemented having the following characteristics:

- Evaluating the drag forces acting on a complete missile body.
- Predicting the trajectory of a medium- to- long range unguided ballistic missile under the assumptions of: point mass body flying in a one plane with zero angle of attack with a constant mean thrust or a thrust -time table in a stagnant air around a fixed circular Earth
- Estimating the aerodynamic heating parameters with time along the complete trajectory. These parametrs are: the net heat flow rate and the temperature profile over a conical forebody.

The code was applied on a number of real missiles and the results were compared with previous works and showed a very good agreement.

## 2. MATHEMATICAL MODEL

The model being constructed in this study adopts the thin skin (wall) assumption. The conical fore-body, Fig. 2, is divided into a finite number of segments.

The net local value of heat transfer rate is determined according to the appropriate governing theory. This value is used in a heat transfer problem to modify the wall temperature within the time increment previously determined.

The flow regime; continuum, slip, or free molecular flow dictates the governing theory. Obviously, the regime of the flow must be specified first according to the local value of Knudsen number

The local value of Knudsen number is evaluated for both laminar and turbulent flows, respectively as follows [3]:

$$Kn = 0.264\sqrt{\gamma} \text{ M}/\sqrt{Re} \qquad \qquad \text{If } Re \leq 5e5 \qquad \qquad \text{(1)}$$

$$Kn = 3.39\sqrt{\gamma} \text{ M/Re}^{4/5}$$
 If  $Re > 5e5$  (2)

5

where Re the local value of Reynolds number (Re =  $\frac{Vx}{v}$ ), and x is the position of point of calculations from the nose tip, Fig. 2 . If Knudsen value is less than or equal to 0.01 the flow is considered to be in viscous continuum regime, and is said to be in the free molecular flow regime if Knudsen exceeds unity and to fall in the "intermediate" slip flow regime if Knudsen has the values ranging from 0.01 to unity.

# 2.1. Aerodynamic Heating in Continuum Flow Regime

## Non-viscous (edge) properties

The non-viscous flow properties at the edge of the boundary layer are evaluated relative to the free stream values as follows [2]:

$$\frac{T_{\delta}}{T_{\infty}} = \left(1 - \widetilde{v}_{\epsilon}^{2}\right) \left(1 + \frac{\gamma - 1}{2} M_{\infty}^{2}\right) \tag{3}$$

 $\tilde{V}_c = \frac{V_\delta}{V_{\rm cons}}$  is the ratio between the velocity downstream of the shock wave and a

certain hypothetical velocity  $V_{max} = \sqrt{2\gamma(p_o/\rho_o)/(\gamma-1)}$  with  $p_o$  and  $\rho_o$  are the stagnation pressure and density of the flow behind the shock wave. The value  $\widetilde{v}_c$  is given in charts as a function of free stream Mach number and the cone semi apex angle [2].

$$\frac{p_{\delta}}{p_{\infty}} = V_{o} \left( \frac{T_{\delta}}{T_{\infty}} \right)^{\gamma/(\gamma-1)} \tag{4}$$

with 
$$v_o = \frac{1}{\left[\left(1+\epsilon\right)\!K_o^2 - \epsilon\right]^{\left(1-\epsilon\right)/2\epsilon} \left(\epsilon + \frac{1-\epsilon}{K_o^2}\right)^{\left(1+\epsilon\right)/2\epsilon}}$$

where  $\epsilon=(\gamma-1)/(\gamma+1)$  and  $K_o$  =  $M_\infty$  sin  $\theta_s$  , with  $\theta_s$  is the angle that the shock wave makes with the free stream flow direction. The value of  $\,$   $\,$   $\,$   $\,$  depends on the value of  $K_1 = M_{\infty} \, \delta_c \,$  according to the following rule,

$$\begin{aligned} & \text{K}_{o} = \left(1 + \frac{\gamma_{\infty} + 1}{2} \text{K}_{1}^{2}\right)^{1/2} & \text{if } \text{K}_{1} \leq 6 \\ & \text{K}_{o} = \left[0.39 - 0.01 \text{log}\left(\frac{p_{\infty}}{p_{\text{SL}}}\right)\right] + \left[1.03 + 0.005 \text{log}\left(\frac{p_{\infty}}{p_{\text{SL}}}\right)\right] \text{K}_{1} & \text{if } \text{K}_{1} \geq 6 \end{aligned}$$

where  $p_{\text{\tiny SL}}$  is the value of atmospheric pressure at sea level conditions.

the value of atmospheric pressure at each 
$$\frac{c_{p_{\delta}}}{c_{p_{\infty}}} = \left(\frac{T_{\delta}}{T_{\infty}}\right)^{0.1}$$
 (6)

$$\frac{\mu_{\delta}}{\mu_{\infty}} = \left(\frac{\mathsf{T}_{\delta}}{\mathsf{T}_{\infty}}\right)^{0.7} \tag{8}$$

$$= \sqrt{\frac{(\gamma - 1)}{1}} \left(\frac{1}{1}\right) \tag{9}$$

$$M_{\delta} = \sqrt{\frac{(\gamma - 1)}{2} \left(\frac{1}{1 - \widetilde{V}_{c}^{2}} - 1\right)}$$
 (9) 
$$\frac{V_{\delta}}{V_{\infty}} = \frac{M_{\delta}}{M_{\infty}} \frac{a_{\delta}}{a_{\infty}}$$
 (10)

## Boundary layer properties

The wall enthalpy  $i_w$ ; at each point is then evaluated as a function of temperature and pressure [2].

The flow parameters inside the boundary layer may be calculated considering compressibility of gas and ignoring influence of temperature rise or considering the temperature rise and ignoring compressibility [2]. The decision to choose either way depends mainly upon the value of Mach number of the free stream. The "switching" Mach number was found to have the value of 5.2 [5]. The viscous flow properties in each case are evaluated as follows.

## Case1: temperature rise is more significant

The effective flow parameters are evaluated as follows:

$$i^* = 0.5(i_w + i_\delta) + 0.22(i_r - i_\delta)$$
 (11)

here  $i_{\scriptscriptstyle W}, i_{\scriptscriptstyle \mathcal{S}}$  , and  $i_{\scriptscriptstyle r}$  are the wall, edge, and recovery enthalpies, respectively. where

$$i_r - i_\delta = r \frac{V_\delta^2}{2} \tag{12}$$

r is the recovery factor

$$r = (Pr)^a \tag{13}$$

where a=0.5 for laminar flow and a=1/3 for turbulent flow. For calculation purposes, Prandtl number equals 0.71 for air at normal temperature [3].

$$i_{\delta} = c_{p_{\delta}} T_{\delta} \tag{14}$$

T\*; is evaluated from effective enthalpy, figure 4.

$$\frac{\rho'}{\rho_{\delta}} = \frac{T_{\delta}}{T'}$$

$$\frac{c_{\rho}}{c_{\rho_{\delta}}} = \left(\frac{T'}{T_{\delta}}\right)^{0.1}$$
(16)

$$\frac{\mu}{\mu_{\delta}} = \left(\frac{T}{T_{\delta}}\right)^{0.7} \qquad (17) \qquad \frac{\lambda}{\lambda_{\delta}} = \left(\frac{T}{T_{\delta}}\right)^{0.85} \qquad (18)$$

$$\frac{\lambda^{\cdot}}{\lambda_{\delta}} = \left(\frac{T^{\cdot}}{T_{\delta}}\right)^{0.85}$$
(19) 
$$\operatorname{Re}^{\cdot} = \operatorname{Re}_{\delta} \left(\frac{\rho^{\cdot}}{\rho_{\delta}}\right) / \left(\frac{\mu^{\cdot}}{\mu_{\delta}}\right)$$
(20)

7

## The local skin friction drag coefficient

$$c_{f_x} = C \times D \times (Re_x)^n$$
 (21)

The values of C, D, and n have different values according to table 1.

Table 1. Coefficients used to calculate Reynold's number

Parameter	Laminar flow	Turbulent flow
С	$\sqrt{3}$	1.17
D	0.664	0.0578
N	-1/2	-1/5

Local Stanton number;

$$St_{x}^{*} = \frac{1}{2}c_{f_{x}}^{*}(Pr^{*})^{-\frac{2}{3}}$$
 (22)

The heat transfer coefficient; at each point is evaluated from the relation

$$h_{x}^{\star} = St_{x}^{\star}c_{p}^{\star}\rho^{\star}V_{\delta} \tag{23}$$

Finally the heat transfer rate at any point on the cone generator can be calculated from the relation

$$q_o = h_x \frac{\left(i_r - i_w\right)}{c_{o_m}} \tag{24}$$

where  $c_{\text{p}_{\text{m}}}$  is the mean value of specific heats calculated at the recovery and wall temperatures.

## Case2: compressibility is more significant

The local skin friction drag coefficient; for laminar and turbulent flow cases, respectively, is

$$\frac{c_{fL}}{c_{fL}^0} = \left(1 + 0.03M_{\infty}^2\right)^{-\frac{1}{3}} \tag{25}$$

$$\frac{c_{ff}}{c_{ff}^{0}} = \left(1 + 0.12M_{\infty}^{2}\right)^{-\frac{1}{2}} \tag{26}$$

where  $c_{rr}^0$  is the incompressible skin friction drag coefficient which can be calculated through equation 21 and table 1.

Local Stanton number;

$$St_x = \frac{1}{2}c_{f_x}(Pr)^{-\frac{2}{3}}$$
 (27)

The heat transfer coefficient;

$$h_{x} = St_{x}c_{p}\rho V_{\delta}$$
 (28)

Finally the heat transfer rate at any point on the cone generator can be calculated from the relation

$$q_o = h_x \frac{(i_r - i_w)}{c_{p_m}}$$
 (29)

where the recovery enthalpy  $i_r$  is evaluated as  $i_r = (i_r - i_\delta) + i_\delta$ . Again, the enthalpy difference  $(i_r - i_\delta)$  and  $i_\delta$  are given in Eqs. (12) and (14), respectively, and  $c_{p_m}$  is the mean value of specific heats calculated at the recovery and wall temperatures.

## 2.2. Aerodynamic Heating in Slip Flow Regime

Being intermediate regime, the calculations of the heat transfer during the slip flow regime are constructed based upon those of the continuum flow regime. The local heat transfer rate in this regime at any point is evaluated in the following manner.

The length of the mean free path I; at any point is evaluated by

$$l = 1.255v \frac{\sqrt{\gamma}}{a}$$
 (30)

where  $\gamma$  is the specific heat ratio, v is kinetic viscosity and "a" is the speed of sound of the flow.

Heat transfer rate q; is evaluated by the relation [2]

$$q_o = q^c \left( 1 - \frac{1.89}{\overline{\chi}} + \frac{10.7}{\overline{\chi}^2} \right)$$
 (31)

with  $q^{\text{c}}$  is the local heat transfer rate evaluated as if the flow were in the continuum regime, and

$$\overline{\chi} = \frac{2.89\overline{x}}{Re_1} \tag{32}$$

with  $\bar{x} = \frac{x}{l}$ , where x is the local distance from nose apex and Re<sub>l</sub> is the local value of Reynolds number based on the free length l.

### 2.3. Aerodynamic Heating in Free Molecular Flow Regime

According to the molecular kinetic theory, the local heat transfer rate is given by [3]

$$q_{o} = \eta \rho_{i} \sqrt{\frac{\left(RT_{i}\right)^{3}}{2\pi}} \left\{ \left[ e^{-\overline{x}^{2}} + \overline{x} \sqrt{\pi} \left(1 + erf(\overline{x})\right) \right] \left[ \overline{x}_{\infty}^{2} - \frac{\gamma + 1}{2(\gamma - 1)} \cdot \frac{T_{w}}{T_{i}} + \frac{\gamma + 1}{2(\gamma - 1)} \right] + \frac{1}{2} \overline{x} \sqrt{\pi} \left(1 + erf(\overline{x})\right) \right\}$$

$$(33)$$

where  $\rho_i$  and  $T_i$  are the density and temperature of the incident molecules (atmospheric values),  $\eta$  is the accommodation coefficient and it ranges from 0.7 to 0.97 [1] (an average value of 0.85 is assumed),  $T_w$  is the local wall temperature and the error function

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_{0}^{x} e^{-z^{2}} dz \tag{34}$$

and 
$$\overline{x} = M_{\infty} \sqrt{\gamma/2} \sin(\beta) = \overline{x}_{\infty} \sin(\beta)$$
 (35)

with  $\beta$  the inclination angle of surface to flow direction (here it is taken to be equal to the nose semi-apex angle).

## 2.4. Aerodynamic Heating at the Stagnation Point

Recovery enthalpy of the flow 
$$i_r$$
;  $i_r = i_{\infty} + \frac{V_{\infty}^2}{2}$  (36)

Free stream stagnation temperature 
$$T_{o1}$$
 [6];  $T_{o1} = T_{\infty} \left( 1 + \frac{\gamma - 1}{2} M_{\infty}^2 \right)$  (37)

Free stream stagnation pressure 
$$P_{o1}$$
 [6];  $P_{o1} = P_{\infty} \left( 1 + \frac{\gamma - 1}{2} M_{\infty}^2 \right)^{\frac{\gamma}{\gamma - 1}}$  (38)

Free stream stagnation density 
$$\rho_{o1}$$
 [6]; 
$$\rho_{o1} = \rho_{\infty} \left( 1 + \frac{\gamma - 1}{2} M_{\infty}^2 \right)^{\frac{1}{\gamma - 1}}$$
 (39)

Stagnation temperature behind the normal shock wave  $T_{o2}$ ; under the assumption of adiabatic flow through a normal shock wave [6]; hence,  $T_{o2} = T_{o1}$ 

Stagnation pressure behind the normal shock wave Po2; [1]

$$P_{o2} = \frac{P_{o1}}{\left[ (1 + \epsilon) M_{\infty}^2 - \epsilon \right]^{\frac{1 - \epsilon}{2\epsilon}} \left( \epsilon + \frac{1 - \epsilon}{M_{\infty}^2} \right)^{\frac{1 + \epsilon}{2\epsilon}}}$$
(40)

where  $\epsilon = \frac{\gamma - 1}{\gamma + 1}$ 

Stagnation density behind the normal shock wave 
$$\rho_{o2}$$
;  $\frac{\rho_{o2}}{\rho_{o1}} = \frac{P_{o2}}{P_{o1}}$  (41)

Velocity gradient at the stagnation point; 
$$\widetilde{\lambda} = \overline{\lambda} \frac{V_{\infty}}{D_{N}}$$
 (42)

where  $D_N$  is the diameter of the nose tip,  $V_\infty$  is the free stream velocity. The dimensionless velocity gradient  $\overline{\lambda}$  is given as a function of flight Mach number [2].

$$\textit{Density ratio;} \quad \overline{\rho}_w = \frac{\rho_w}{\rho_{o2}} \quad \text{for } M_{_{\infty}} \geq 1 \text{ , and } \quad \overline{\rho}_w = \frac{\rho_w}{\rho_{o1}} \quad \text{for } M_{_{\infty}} < 1$$

$$\textit{Viscosity ratio; } \overline{\mu}_w = \frac{\mu_w}{\mu_{o2}} \ \text{for } M_{\scriptscriptstyle \infty} \geq 1 \text{, and } \quad \overline{\mu}_w = \frac{\mu_w}{\mu_{o1}} \ \text{for } M_{\scriptscriptstyle \infty} < 1$$

Heat transfer rate to the wall [2];

$$q_o = \frac{0.763}{Pr^{2/3}} \sqrt{\widetilde{\lambda} \rho_{o2} \mu_{o2}} \left( i_r - i_w \right) \left( \overline{\rho}_w \overline{\mu}_w \right)^{0.1} \qquad \text{for } M_{\infty} \ge 1$$
 (43)

$$q_{o} = \frac{0.763}{Pr^{2/3}} \sqrt{\tilde{\lambda} \rho_{o1} \mu_{o1}} (i_{r} - i_{w}) (\bar{\rho}_{w} \bar{\mu}_{w})^{0.1}$$
 for  $M_{\infty} < 1$  (44)

The radiation heat transfer can be calculated at each point simply adopting the well-known Stefan-Boltzmann law

$$q_r = \varepsilon \sigma T_w^4 \tag{45}$$

where  $\epsilon$ : the wall emissivity and

 $\sigma$ : Stefan-Boltzmann coefficient [ 5.6697 × 10<sup>-8</sup> W/(m<sup>2</sup>.K<sup>4</sup>)].

The neat heat flux to the wall is readily evaluated as

$$q_w = q_o - q_r \tag{46}$$

### 2.5. Wall Temperature

The change 0in a thin wall temperature due to heat flux equals

$$\Delta T_{w} = \frac{q_{w}.\Delta t}{c.\rho.\delta}$$
 (47)

where  $q_w$  is the net heat flux to the wall in [W/m²],  $\Delta t$  is the time increment of exposure, c ,  $\rho$  , and  $\delta$  are the specific heat, density, and thickness of wall material. The wall temperature at any instant is

$$T_{w} = (t + \Delta t) = T_{w}(t) + \Delta T_{w}$$
(48)

#### 3. CASE STUDIES

The implemented program was applied to a number of real rockets with a variety of flight conditions such as, flight time from 80 to 600 seconds, ranges from 20 to 1800 km, apogee from 8 to 300 km, maximum Mach number from 2 to 16. Sample results are illustrated in Figs. 3-11.

### • Missile1:

An unguided, single solid stage, fin-stabilized ground-to-ground rocket

- Mean thrust	21225	Ν
<ul> <li>Launching mass</li> </ul>	64.525	Kg
- Propellant mass	19.0	Kg
<ul> <li>Burning time</li> </ul>	1.74	S
<ul> <li>Maximum range</li> </ul>	20.4	Km
<ul> <li>Corresponding altitude</li> </ul>	7.8	Km
<ul> <li>Maximum Mach number</li> </ul>	2.4	

The output, of running these data on the implemented program, is presented in Fig. 3 which displays the wall temperature at three different locations on the missile nose during the flight time. Figure 4 displays the variation with time of flight parameters and wall temperature at 0.1 nose generator length.

AF-01

### Missile 2:

A tactical unguided rocket fired from an oblique launcher.

<ul> <li>Mean thrust</li> </ul>	34656	5 N
- Launching mass	2450	Kg
- Propellant mass	1143	Kg
– Burning time	9.53	sec
- Maximum range	75	Km
<ul> <li>Corresponding altitude</li> </ul>	20	Km
- Maximum Mach number	4.8	

#### • Missile 3:

A long range, single stage, liquid propelled ballistic missile. A typical flight of this missile consists mainly of a short vertical rise path, a gravity turn path, a constant attitude path, and a passive part.

- Mean thrust	133000	N
- Launching mass	5686.5	Kg
- Propellant mass	3612.5	Kg
- Burning time	59.0	S
- Maximum range	300	Km
<ul> <li>Corresponding altitude</li> </ul>	89	Km
- Maximum Mach number	5.3	

### Missile 4:

This missile is a single stage, unguided ballistic missile. It is required to deliver a payload of at least 200 Kg to an apogee of about 300 Km.

- Mean thrust	238674	N
- Launching mass	4043.6 2900	Kg Kg
- Propellant mass	36	S
<ul><li>Burning time</li><li>Maximum range</li></ul>	147	Km
Corresponding altitude	312	Km
Maximum Mach number	8	

### • Missile 5:

An intermediate-range, two-stage, solid propelled, hypersonic ballistic missile.

<ul> <li>Mean thrust</li> </ul>	109500	N
<ul><li>Launching mass</li></ul>	7200	Kg
- Propellant mass	5838	Kg
- Burning time	105	S
<ul><li>Maximum range</li></ul>	1800	Km
<ul> <li>Corresponding altitude</li> </ul>	300	Km
Maximum Mach number	16	

Figures 5-11 show the obtained skin temperatures of the missiles 2-5 and variation of flight parameters respectively during the flight times.

#### 4. VERIFICATION OF RESULTS

A comparison between results of the implemented program and previous work [4] for sample missile 4 is shown in Fig. 12. The results show a sufficient agreement with previous work, the difference gives an error of about 4%.

#### 5. CONCLUSIONS

A computer code was developed in this work so as to facilitate the estimation of aerodynamic heating phenomena. The code can also be used in estimating the complete trajectory and missile drag components as functions of Reynold and Mach numbers. The code results showed good agreement with previous works. Some important conclusions could be deduced through analyzing the case study results:

- The heat transfer rate has a maximum value exactly at the nose tip.
- As the nose bluntness increases, the nose tip temperature decreases continuously for the sane flight parameters.
- As the nose vertex angle increases, the wall temperature increases for the same flight parameters.
- Isolating the effect of altitude variation, the wall temperature follows approximately the behavior of flight velocity. The rate of change of wall temperature approximately behaves like acceleration. If the missile is flying with a positive acceleration, the wall temperature increases with a certain rate, if this positive acceleration decreases in magnitude, the wall temperature increases with a lower rate. The opposite is also true.
- At the moment of reentry, altitudes of about 75- 80 km, the missile experiences a severe heat transfer reaching very high rates.
- The event of maximum wall temperature is most likely to happen during atmospheric flight.

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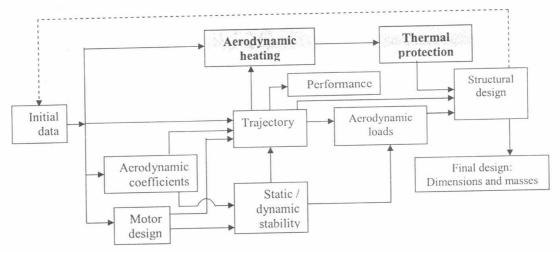


Fig. 1 Scheme illustrating the Missile System Analysis

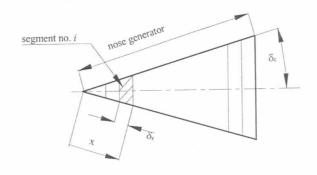


Fig. 2 Segmentation of the nose

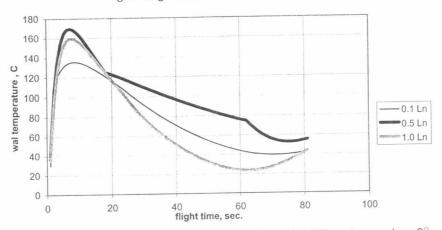


Fig.3 Variation of wall temperature with flight time of missile "1", vertex angle =  $9^{\circ}$ 

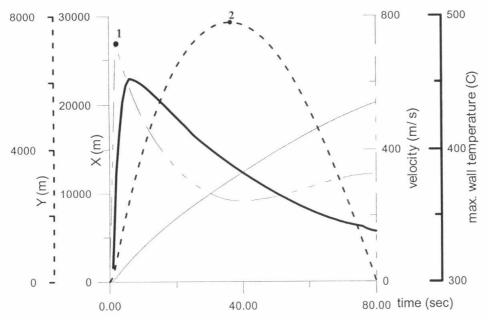


Fig. 4 Variation of flight parameters with flight time of missile "1"

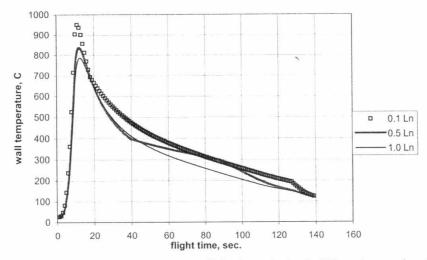


Fig.5 Variation of wall temperature with flight time of missile "2", vertex angle =13  $^{\circ}$ 

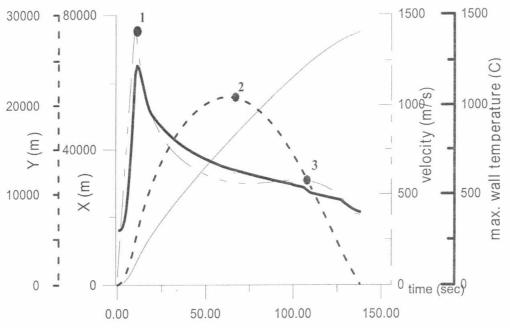


Fig. 5 Variation of flight parameters with flight time of missile "2"

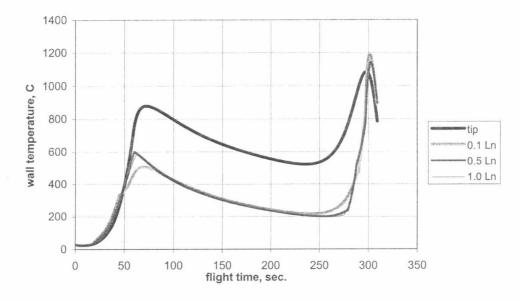


Fig.6 Variation of wall temperature with flight time of missile "3", vertex angle = 11  $^{\circ}$ 

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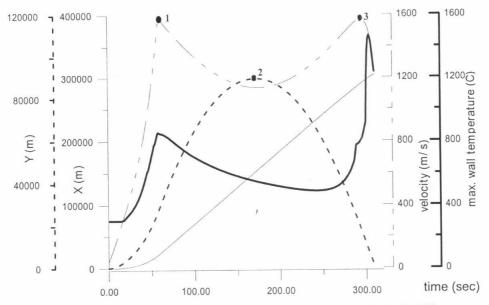


Fig. 7 Variation of flight parameters with flight time of missile "3"

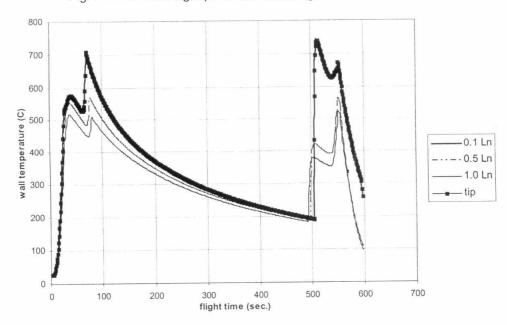


Fig.8 Variation of wall temperature with flight time of missile "4", vertex angle = 11.3  $^{\circ}$ 

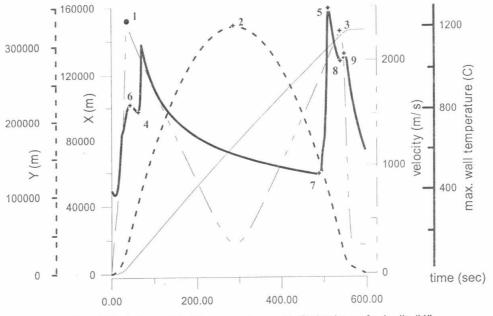


Fig. 9 Variation of flight parameters with flight time of missile "4"

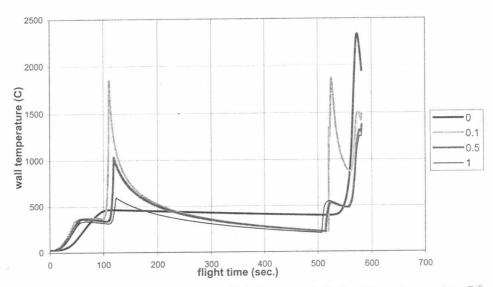


Fig.10 Variation of wall temperature with flight time of missile "5", vertex angle = 7  $^{\circ}$ 

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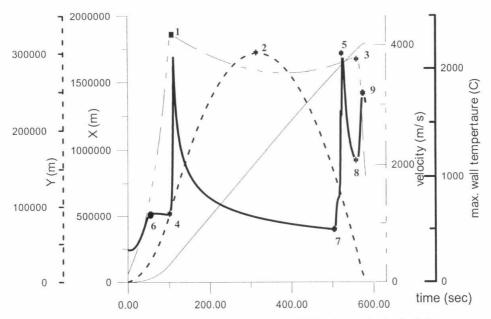


Fig. 11 Variation of flight parameters with flight time of missile "5"

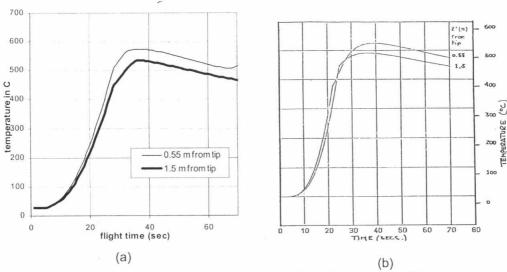


Fig.12 Comparison between (a) results obtained from the code and (b) data presented in previous work [4]