Military Technical College
Kobry El-Kobba,
Cairo, Egypt

11-th International Conference
on Aerospace Sciences &
Aviation Technology

FLUTTER CONSTRAINT FOR AIRCRAFT CONCEPTUAL DESIGN USING RESPONSE SURFACE METHODOLOGY

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ABSTRACT

Flutter constraint, applicable to aircraft conceptual design, is constructed using response surface methodology. It is presented by the critical flutter speed, as a function of wing torsion stiffness, root chord, sweep, mass ratio, taper ratio, aspect ratio, center of gravity location and radius of gyration. The constraint function is a quadratic response surface polynomial. The D-optimal design is used to find the best combinations of design points required to determine the function coefficients. The Regier number criterion is used to calculate the critical flutter speed at these design points. Analysis of variance is used to remove the unreliable terms from the function. To match the Regier number criterion, two constraint functions suitable for subsonic aircraft with traditional wing are constructed. The first one is applicable to aircraft with low sweepback wing while the second one is applicable to aircraft with moderate sweepback wing. As a case study, the constraint function is applied within the conceptual design of a subsonic aircraft leading to a considerable weight saving.

KEY WORDS

Aircraft Conceptual Design, Flutter and Response Surface Methodology

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INTRODUCTION

Increasing size, speed and performance requirements and economical pressure to reduce aircraft operational costs can no longer be met by traditional design processes. In particular, the impact of aeroelastic effects on aircraft design demands the use of multidisciplinary design concepts and optimization strategies to develop flutter free structures while maintaining good multipoint performance characteristics [1]. Flutter identified at a late stage of the product development causes severe economic and technological problems. To avoid these problems, a flutter constraint should be applied in the conceptual design stage. To decrease the computations burden, designers have always been looking at the use of approximations. Response surface methodology has recently gained popularity as a powerful way to replace the objective or the constraint functions in the design process. The response surface model is a simple and smooth function where a limited number of computational analyses are used to determine the function coefficients. Although the computational expense of creating a response surface model may be significant, this cost is incurred prior to the use of the model in numerical optimization. Thus, this model may be evaluated hundreds or thousands of times during an optimization process without significant computational expense.

Solving the flutter equations is not suitable during the conceptual design stage because it needs detailed information about aircraft structures, which are not available in the early design stages. Mukhopadhyay [2, 3], Dunn et al [4] and Frueh [5] introduced a flutter criterion which is particularly applicable during conceptual design. This criterion is based on Regier number, which depends on the stiffness of the wing and characteristics of the fluid in which the wing is operating, in particular, the density and the speed of sound. In this work, the criterion based on the Regier number is used to calculate the critical flutter speed at the design points which is required to construct the response surface function.

CONSTRUCTION OF THE FLUTTER CONSTRAINT USING RESPONSE SURFACE METHODOLOGY

The flutter constraint can be modeled as a function of design variables and design parameters. The design parameters are those parameters that affect flutter constraint, and can be calculated from other design variables not included directly in the flutter constraint. One important example of design parameters is the center of gravity location which has a significant effect on the flutter critical speed. It is not a direct design variable but its value can be calculated using other design variables such as the location of wing spars or engine location for wing mounted engines. In this work, the used design variables are wing sweep (Λ), aspect ratio (A), taper ratio (λ) and wing root chord (C_r). The used design parameters are the uncoupled torsional frequency (ω_α), mass ratio (μ), center of gravity location and radius of gyration. Wing root chord, uncoupled torsional frequency and the speed of sound (a) is combined in one non-dimensional variable known as modified Regier number ($\frac{C_r\omega_\alpha}{a}$).

Several steps are done to suite the Regier number criteria. First, two-flutter constraint functions are modeled; one for a wing with low sweepback angle (less than 20°), while the other for a wing with moderate sweepback angle (from 20° to 40°). Second, a non-dimensional center of gravity ratio and radius of gyration ratio is used instead of the center of gravity location and radius of gyration respectively. The center of gravity ratio is obtained by dividing the center of gravity position by the wing chord at 75% semispan while the radius of gyration ratio is obtained by dividing the radius of gyration by the wing chord at 60% semispan. Third, the lower and upper bounds of the design variables are selected within the application range of the criterion. The lower and upper bounds of the design variables and parameters are listed in Table1.

In a sufficiently small volume of the design space, any function may be approximated by a quadratic polynomial with good accuracy. Although this is certainly not true for all cases, RSM becomes prohibitively expensive when cubic and higher-order polynomials are chosen for problems involving several variables. In contrast, Quadratic polynomials are easy to implement and provide the capability of modeling curvature of the actual function. The flutter constraint function can be modeled using a quadratic response surface polynomial as follow:

$$M_F = c_0 + \sum_{i=1:p} c_i x_i + \sum_{\substack{i=1:p-1 \\ j=i+1:p}} c_{ip-\frac{i(i+1)}{2}+j}\, x_i x_j + \sum_{i=1:P} c_{3(p+1)+i}\, x_i^{\,2} \qquad (1)$$

Where x_i are the design variables, c_i are the polynomial coefficients, and M_F is the critical flutter Mach number. Quadratic response surface function with six design variables has 28 terms. Estimating the unknown coefficients require k analyses, where $k \geq 28$. These analyses should be done at k combinations of design points. The designer should determine the value of the constraint function at these points to determine the values of the function coefficients. For the six design variables listed in Table 1, the allowable range is discretized at equally four spaced levels. Therefore, the full factorial design provides 4096 possible combinations. The D-optimality criterion is used to select a small set of design points (Design space). The D-optimal design points are a collection of sample sites for which the determinant of the moment matrix $|X^T X|$ is maximized over all possible site locations where X is the vector of length 28 corresponding to the form of the x_i's terms in the polynomial model. This implies a good estimate of the regression coefficients in the model [6]. An iterative numerical optimization method is employed to find the k locations which maximize $|X^T X|$. The least squares regression technique is then used to determine the values of the function coefficients which achieve the minimum root mean square error.

The flutter constraint function is constructed using different numbers of design points to investigate the effect of the number of design points. Flutter constraint functions are calculated using 28, 35, 75 and 150 D-optimal combinations. The value of the critical flutter speed is calculated using the constructed constraint function and compared to the basic values calculated using the Regier number criterion. The reference aircraft parameters are aspect ratio of 6, taper ratio of 0.6, modified Regier number of 0.7, mass ratio of 60, center of gravity ratio of 0.45, and radius of gyration ratio of 0.5. These reference values are the average of the lower and upper bounds of the design variables except the value of the modified Regier number which was chosen to keep the value of the critical flutter speed within the required range. The root mean square error is calculated when only one-design variable or parameter is changed from its reference value. These errors are listed in Table 2 and Table 3 for wing with low sweep angle and moderate sweep angle respectively. Case 1 presents the values of the root mean square error when aspect ratio varies from 4 to 8, case 2 presents these errors when taper ratio varies from 0.2 to 1 while case 3 presents theses errors when the center of gravity ratio varies from 0.35 to 0.55.

The previous results show that the model accuracy is poor for 28 experiments. The constraint function accuracy improved when the number of design points is increased to 75. When the number of design points is increased from 75 to 150, the model accuracy is increased slightly in some cases and decreased slightly in the other cases. It can be concluded that 75 design points is suitable for constructing a response surface model for a flutter constraint with 28 terms.

To improve the model accuracy, the Mach number is decreased in two steps. In the first step the range of Mach number is divided into two equal zones from 0.4 to 0.7 and from 0.7 to 1, while in the next step, this range is divided into three equal zones from 0.4 to 0.6, from 0.6 to 0.8 and from 0.8 to 1. Therefore, each flutter constraint consists of two functions in the first step and three functions in the second step, where each function is suitable for a certain Mach number zone. To investigate how the accuracy of the flutter constraint improves due to the decrease in speed interval, the variation of critical flutter speed with aspect ratio, taper ratio, and center of gravity location are calculated using constraint functions constructed at different speed intervals, and compared to the critical flutter speeds calculated using the Regier number criterion. The root mean square error for these variations is listed in Table 4 and Table 5 for wing with low sweep angle and moderate sweep angle respectively. The results show that a significant improvement in the model accuracy is achieved by decreasing the speed interval.

Analysis of variance (ANOVA) is used to remove unreliable terms from the model function. The unreliable terms are the terms which when removed from the function the model accuracy is improved. The process is repeated until the stopping criterion is reached. This is an essential step in the response surface model generation process, as it not only reduces the size of the model function, but it may improve its performance. ANOVA provides a measure of the uncertainty in the coefficients of the constraint function [7, 8]. This uncertainty estimation is provided by several hypothesis tests. The t-statistic [9] is used in the present work. There are several ways of selecting the appropriate subset of terms in a response surface model. The most widely used methods are: forward selection, backward elimination, and stepwise regression. In this work, one-step backward regression is performed for the flutter constraint functions to determine the optimum structure of the polynomial that provides the lowest errors. Each time the term with the lowest value of the t-statistic is removed from the model, and the mean, R.M.S and maximum errors of the response surface model are checked using either the same design points (fitting errors) or a randomly selected set of points. Figure 1 and Fig. 2 are a sample from the result. The complete set of results can be reviewed in reference [1].

The analysis indicates that the analysis of variance does not improve the model accuracy. Although the model accuracy is not impaired by using a fewer number of terms, but there is no need to use these models because analysis of variance must done after the function construction. Therefore, decreasing the number of terms will not decrease the computations required to build the model function. The model with a reduced number of terms is used only if removing some terms improves the model accuracy, which does not occur in the case of flutter constraint. Therefore, the flutter constraint functions contain all its original terms, i.e. 28 terms. The values of the function coefficients are listed in Table 6 and Table 7 for wing with low sweep angle and wing with moderate sweep angle respectively

CASE STUDY

To investigate the applicability and effect of using the flutter constraint, it is implemented in the conceptual design of a business jet aircraft. The conceptual design of this aircraft was submitted to the Department of Aerospace Engineering, Faculty of Engineering at Cairo University as a graduation project [10]. The wing structure is a box beam with two spars. In this study, the effect of flutter constraint on the selection of the wing taper ratio and position of wing front spar is discussed. The selected values for the taper ratio are 0.2, 0.33, 0.4, and 0.5, while the front spar location as a percent from the chord is varied from 0.05 to 0.3. The wing root chord is changed according to the taper ratio to maintain the value of the wing area and wing aspect ratio without change.

The wing structural arrangement is shown in Fig. 3. The wing is divided into two regions. The first one extends from RIB A to RIB B at 50% semispan. The second one extends from RIB B to the wing tip. The stingers are arranged along the constant percentage lines. The number of stringers is abruptly changed at 50% semispan. Upper and lower skin has equal thickness. The skin thickness is constant along each region. The flutter analysis is carried out in the start of the cruise condition only. The considered fuel weight is the total fuel weight minus the fuel weight required for engine warm-up, taxing, takeoff, and climb. The spanwise and chordwise distribution of the lift forces on the wing surface is calculated using the Vortex Lattice Method.

The analysis is carried out in two steps and repeated for different taper ratios and front spar locations. In the first step, the sizes of the wing structural elements are determined based on the strength criteria only. In the second step, the flutter constraint is applied. If the flutter constraint is not satisfied, the skin thickness is increased until a flutter free wing is achieved. The weight of the main structural elements, corresponding to each step, is shown in Fig. 4. The figure shows that the choice of the best combination of the taper ratio and front spar position is affected by the flutter constraint. If the flutter constraint is not considered, the selected values will not be optimal. For example, if a taper ratio of 0.33 is selected, the optimum point will be point A with a structural weight of 374 kg. Point A is not a flutter free design point so this point must be changed to get a flutter free wing. In the late design stages, the designer does not have the freedom to change the spar location significantly. Therefore, the wing rigidity should be increased to reach point B with a structural weight of 434 kg. On the other hand if flutter constraint is considered, point C will be selected. It satisfies the flutter constraint with a structural weight of 399 kg. Using the flutter constraint in the conceptual design process leads to 8% reduction in the wing weight.

CONCLUSION

Two flutter constraints are constructed, the first one for a wing with low sweptback angle (less than 20°), while the second for a wing with moderate sweptback angle (from 20° to 40°). Each flutter constraint is expressed using three different quadratic polynomials, the first one is suitable for Mach number from 0.4 to 0.6, the second one is suitable for Mach number from 0.6 to 0.8 and the third one is suitable for aircraft with Mach number from 0.8 to 1. The number of analysis points required to find the constraint function coefficients is between 2-3 times the number of polynomial coefficients. The accuracy of the constraint function significantly increases as the interval of speed decrease. Using analysis of variance to omit the unreliable terms from the constraint function does not improve the constraint function accuracy. The possibility and effect of using the suggested flutter constraint in the early design stages are investigated through a case study. The flutter constraint is implemented in the conceptual design of a jet aircraft. The effect of the flutter constraint on the selection of wing taper ratio and front spar location is discussed. A considerable weight saving is achieved.

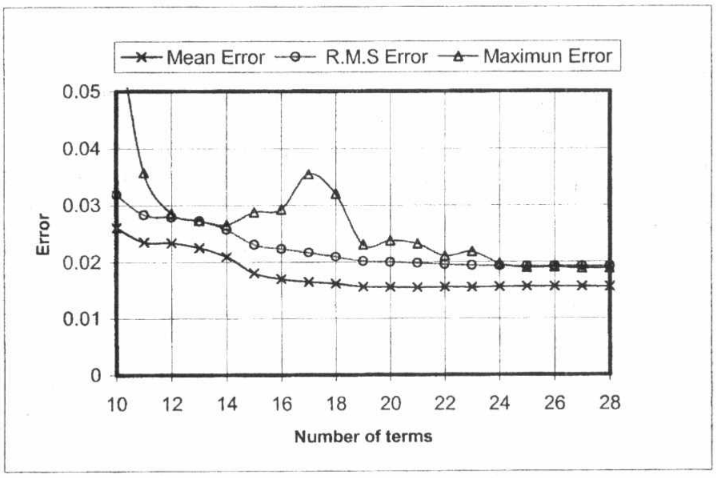

Fig. 1. Fitting errors (computed at the same design points) for wing with moderate sweep (0.8 <M< 1.0)

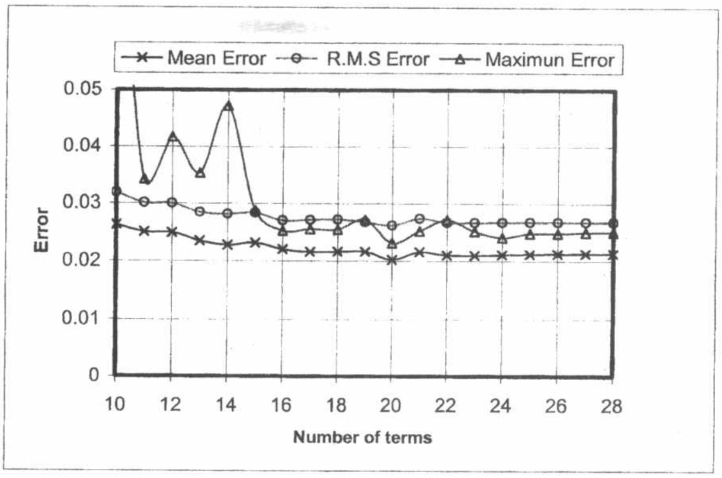

Fig. 2. Fitting errors (computed at random points) for wing with low sweep (0.6 <M< 0.8)

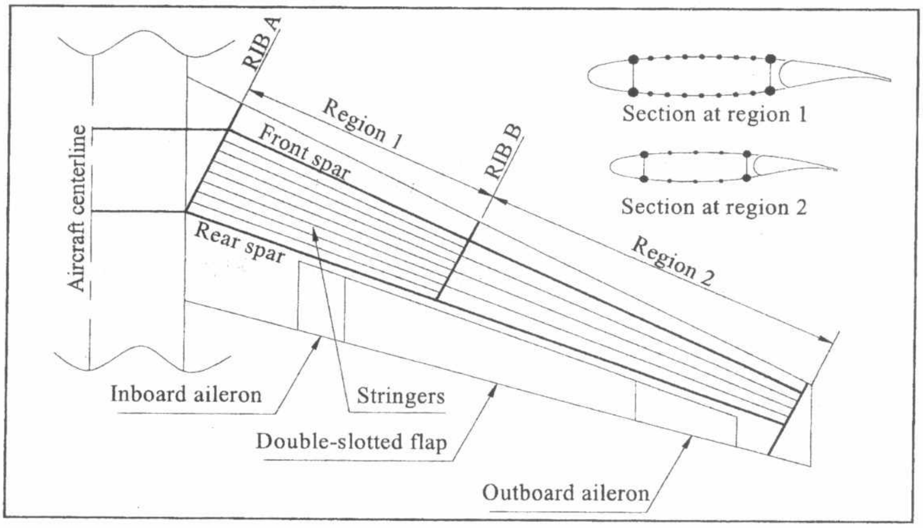

Fig. 3. Wing structural arrangement

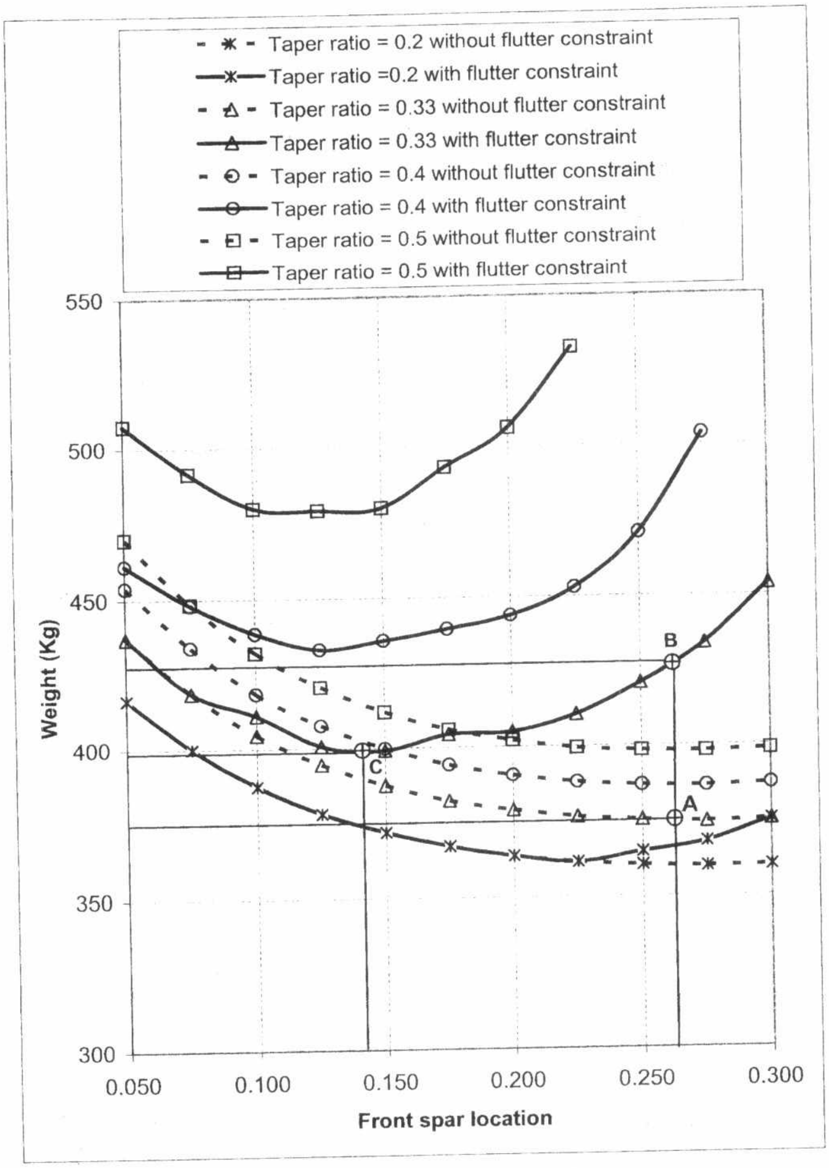

Fig. 4. Weight of the wing structures

Table 1. Design variables and parameters

Design variable / parameters		Lower limit	Upper Limit
x_1	Wing aspect ratio (A)	4	8
x_2	Wing taper ratio (λ)	0.2	1
x_3	Modified Regier number $\left(\frac{C_r\omega_\alpha}{a}\right)$	0.5	2
x_4	Mass ratio (μ)	10	100
x_5	Center of gravity ratio (cg_{ratio})	0.35	0.55
x_6	Radius of gyration ratio (rg_{ratio})	0.3	0.7

Table 2. RMS error for various number of design points (low sweep wing)

Number of design points	28	35	75	150
Case 1	0.136	0.057	0.046	0.032
Case 2	0.147	0.066	0.048	0.031
Case 3	0.142	0.077	0.056	0.035

Table 3. RMS for various number of design points (moderate sweep wing)

Number of design points	28	35	75	150
Case 1	0.148	0.102	0.053	0.068
Case 2	0.172	0.068	0.044	0.059
Case 3	0.171	0.064	0.042	0.057

Table 4: RMS error for various Speed intervals (low sweep wing)

Number of Mach number zones	1	2	3
Case 1	0.046	0.016	0.009
Case 2	0.071	0.060	0.029
Case 3	0.053	0.051	0.021

Table 5. RMS error for various Speed intervals (moderate sweep wing)

Number of Mach number zones	1	2	3
Case 1	0.053	0.024	0.017
Case 2	0.044	0.028	0.016
Case 3	0.042	0.032	0.011

Table 6. The constraint function coefficients for low sweep wing

i	X_i	c_{i-1}		
		0.4<M<0.6	0.6<M<0.8	0.8<M<1
1	1	1.59723977	1.72597424	1.96575368
2	A	0.01807283	-0.01707635	-0.05284119
3	λ	-0.17918044	-0.06648632	-0.11682632
4	$\frac{C_r\omega_\alpha}{a}$	0.80158042	0.78664666	0.51180852
5	μ	0.00882046	0.00970179	0.00589988
6	cg_{ratio}	-8.16750487	-8.12279198	-7.22903756
7	rg_{ratio}	0.85937913	0.75274508	0.67768451
8	$A\ \lambda$	-0.00434031	-0.00099142	-0.00651406
9	$A\ \frac{C_r\omega_\alpha}{a}$	-0.00573479	-0.00549098	-0.01437336
10	$A\ \mu$	-0.00002670	-0.00004801	-0.00017512
11	$A\ cg_{ratio}$	0.01164257	0.02864852	0.06141355
12	$A\ rg_{ratio}$	-0.00516771	-0.00759052	-0.01358376
13	$\lambda\ \frac{C_r\omega_\alpha}{a}$	0.12694896	0.07710863	0.13400200
14	$\lambda\ \mu$	0.00134203	0.00128782	0.00206373
15	$\lambda\ cg_{ratio}$	-0.15988369	-0.34231851	-0.46312834
16	$\lambda\ rg_{ratio}$	0.17015281	0.05391993	0.19084022
17	$\frac{C_r\omega_\alpha}{a}\ \mu$	0.00719669	0.00566999	0.00688793
18	$\frac{C_r\omega_\alpha}{a}\ cg_{ratio}$	-0.96255331	-1.01369573	-1.18560656
19	$\frac{C_r\omega_\alpha}{a}\ rg_{ratio}$	0.41599379	0.46583966	0.44886447
20	$\mu\ cg_{ratio}$	-0.00943683	-0.01328344	-0.01830803
21	$\mu\ rg_{ratio}$	0.00367634	0.00563996	0.00662470
22	$cg_{ratio}\ rg_{ratio}$	-0.66404602	-1.10275601	-1.47820118
23	A^2	-0.00159626	0.00076778	0.00445212
24	λ^2	0.23029173	0.26733282	0.20667963
25	$\left(\frac{C_r\omega_\alpha}{a}\right)^2$	-0.04495169	-0.05454945	0.01921405
26	μ^2	-0.00004627	-0.00004419	-0.00001781
27	$cg_{ratio}^{\ 2}$	8.43710091	8.91718014	8.94842455
28	$rg_{ratio}^{\ 2}$	-0.33943861	-0.12279069	-0.14072465

Table 7. The constraint function coefficients for moderate sweep wing

i	X_i	c_{i-1}		
		0.4<M<0.6	0.6<M<0.8	0.8<M<1
1	1	1.69903936	2.27333340	2.91317491
2	A	-0.02900017	-0.00483005	-0.05524974
3	λ	-0.02894517	-0.24489075	-0.23173968
4	$\frac{C_r\omega_\alpha}{a}$	0.87046862	1.08549302	0.89536692
5	μ	0.00846311	0.01141504	0.00818824
6	cg_{ratio}	-8.59950425	-12.45354861	-13.64469181
7	rg_{ratio}	0.83452171	1.41161088	1.28108746
8	$A\,\lambda$	-0.00107776	-0.00161372	0.00345914
9	$A\,\frac{C_r\omega_\alpha}{a}$	-0.00547202	-0.00145619	-0.00616667
10	$A\,\mu$	-0.00004340	-0.00001530	-0.00004146
11	$A\,cg_{ratio}$	0.02905408	0.00148709	0.04983404
12	$A\,rg_{ratio}$	-0.00730409	-0.00924901	-0.00967206
13	$\lambda\,\frac{C_r\omega_\alpha}{a}$	0.08238385	0.17644058	0.09267842
14	$\lambda\,\mu$	0.00076086	0.00216611	0.00193667
15	$\lambda\,cg_{ratio}$	-0.22041515	-0.43185124	0.36376886
16	$\lambda\,rg_{ratio}$	0.10120842	0.14852183	-0.05578713
17	$\frac{C_r\omega_\alpha}{a}\,\mu$	0.00378644	0.00641220	0.00989809
18	$\frac{C_r\omega_\alpha}{a}\,cg_{ratio}$	-1.02288067	-1.23302358	-1.31354544
19	$\frac{C_r\omega_\alpha}{a}\,rg_{ratio}$	0.51973567	0.44046845	0.43144363
20	$\mu\,cg_{ratio}$	-0.00809572	-0.01555002	-0.02102628
21	$\mu\,rg_{ratio}$	0.00443606	0.00527316	0.00743742
22	$cg_{ratio}\,rg_{ratio}$	-1.18304537	-0.86950404	0.22062648
23	A^2	0.00162489	0.00003243	0.00272272
24	λ^2	0.18538061	0.35941963	0.21168632
25	$\left(\frac{C_r\omega_\alpha}{a}\right)^2$	-0.06215699	-0.10237307	-0.01871781
26	μ^2	-0.00004037	-0.00005342	-0.00003886
27	$cg_{ratio}{}^2$	9.11146008	13.37673064	13.46257094
28	$rg_{ratio}{}^2$	-0.16567594	-0.64999291	-0.85843412

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