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Optimal Antenna Tracking for LEO-GEO Cross-Link

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ABSTRACT

This paper presents a proposal for an attitude control system for Low Earth Orbit (LEO) satellite antenna that keeps tracking of an existing geo-stationary (GEO) communication satellite as a data relay satellite. The proposed scheme can increase the communication availability with a remote sensing LEO satellite. Moreover, the proposed system improves both the communication coverage area of the LEO satellite and its scanning availability in the real time. This improvement is a real need in the remote sensing application. The use of cross-link in data relay systems requires accurate control system in order to enable tracking the data relay satellite (GEO satellite). The proposed system permits the data relay system to work properly and efficiently. The proposed data relay system provides the real time data accessing, data integrity, and wider coverage area than the obtained one from typical LEO-ground station scheme.

KEY WORDS

Spacecraft, Attitude Control, Satellite

1. INTRODUCTION

The attitude control system for a typical LEO-ground station scheme is simpler and has less complexity than the LEO satellite, which used in data relay systems. The tracking and maneuverability is almost rare and the location of the ground station is predetermined along with the orbit parameters [1, 2]. On the other hand, in data relay systems the maneuverability and tracking of the GEO satellite by the LEO satellite requires complex algorithm in order to keep tracking with the GEO satellite for almost the whole satellite period [3, 4]. This paper contains five sections. Section two presents a proposal scheme that controls and updates the attitude of the LEO satellite antenna system to maintain its directivity towards GEO antenna. This system has two modes, a coarse alignment mode, where a LEO satellite antenna is used to track the GEO satellite using the ground track simulator. Furthermore, a fine alignment mode using monopulse-tracking techniques is also discussed. When the tracking antenna is aligned with the GEO satellite, it sends "COMMUNICATE ON"

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signal to the communication antennas to initiate ordinary data transmission session with the GEO satellite. When the antenna becomes out of alignment, it sends "COMMUNICATE OFF" signal to the communication antennas to shut down communication sessions.
This paper consists of six sections. Section two deals with the proposed control system architecture. Section three deals with studying the analysis and derivation of the equations required for establishing the tracking antenna control system. Moreover, section four contains the simulation results. These results are considered as a measure of performance for the proposed control system. Finally, section five highlights the obtained results.

2. PROPOSED ANTENNA CONTROL SYSTEM

A proposed scheme for antenna control system is depicted in Figure 1. The proposed scheme is based on Kalman filter implementation during the period when LEO satellite in GEO coverage area. The role of the Kalman filter is to estimate and predict the GEO satellite direction. It also extracts and smoothes the measured relative angles between the GEO-LEO satellites from noisy observations [6].

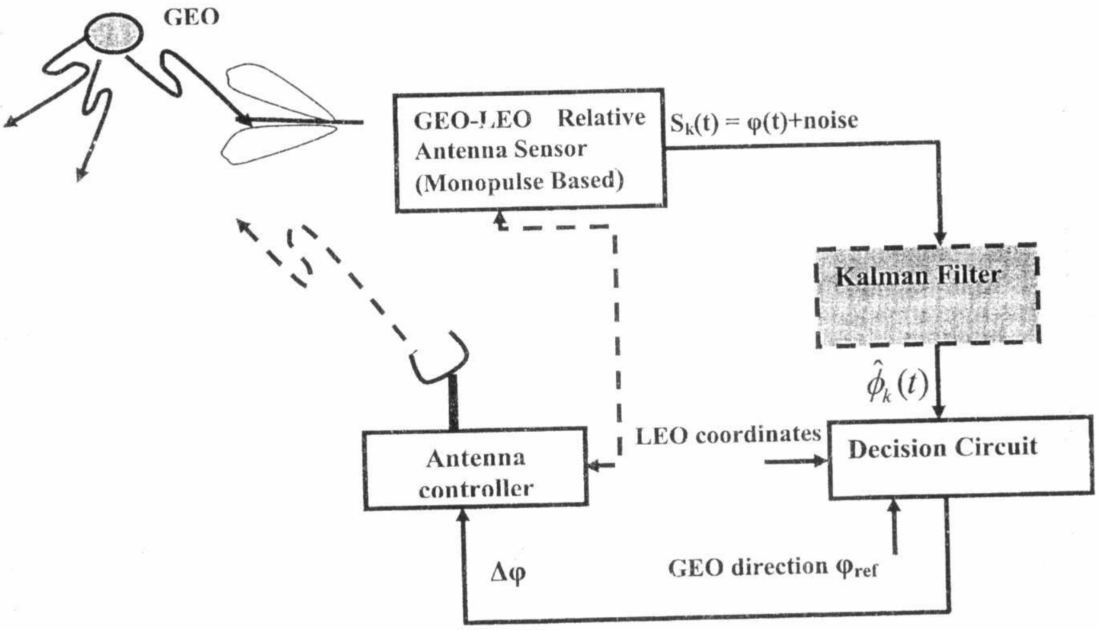


Fig.1. The proposed control antenna scheme

The basic module for the proposed antenna control system is mainly a monopulse sensor [4-7]. It is used to track the GEO direction as the LEO satellite is in the coverage area of the GEO satellite. It delivers the measured value to the kalman filter module in order to smooth and estimate the GEO direction. The Kalman filter delivers the estimated direction to a decision circuit module. This decision circuit provides an alignment correction values (Δφ) to keep tracking of GEO satellite [3].

3 ANALYSIS OF THE PROPOSED SYSTEM

The output of the monopulse transponder can be written as:

$$S_k(t) = \phi_k(t) + n(t) \quad (1)$$

Where

$\varphi_k(t)$....is the relative direction angle of the GEO satellite with respect to the LEO satellite tracking antenna.

n(t).... represents the additive white noise due to the LEO orbit perturbations.

Then, the state vector $x(t)$ of the LEO satellite can be expressed as:

$$\dot{x}(t) = Ax(t) + Bu(t) + w(t)$$
$$y = Cx(t) + Du(t) + v(t) \quad (2)$$

Where

x......... is a state vector of size n_x
u.........control vector of size n_u
y ....output vector of size n_y
w...... disturbance (process noise) vector of size n_w
v....... sensor noise vector of size n_v
A...... state transition matrix of size $n_x x n_x$
B ......penalty vector of size $n_x x n_u$

The Kalman filter output ($\hat{\phi}_k$) can be expressed as [5-6]:

$$\hat{\phi}_k(t) = cx(t) \quad (3)$$

Where

x(t).....is known as the state vector of the state space model of the LEO satellite.
C ......is the output matrix giving the ideal (noiseless) connection between the measurement and the state vector at time (t).

The state vector $x(t)$ can be updated according to following equation [6, 8, 9] and it is defined as [10].

$$\hat{x}_{k+1} = \hat{x}_k + G_k(\phi_{ref} - \hat{\phi}_k) \quad (4)$$

Where

G_k......is the optimal Kalman filter gain, defined as:

$$G_k = PC^T(CPC^T + R_e)^{-1} \quad (5)$$

P..... is known as an error covariance matrix associate with the state $\hat{x}_k$, and expressed as

$$P = E\{(x_k - \hat{x}_k)(x_k - \hat{x}_k)^T\} \quad (6)$$

R_e .... is defined as a covariance matrix of measurement error due to output noise V(n), which is assumed to be white Gaussian sequence with known covariance structure. It is given by [9-10]:

$$R_e = E\left\{ n_k) n_k^T \right\} \qquad (7)$$

The direction misalignment in the LEO satellite antenna platform can be written as:

$$\varepsilon(t) = \phi_{ref} - \hat{\phi}(t) \qquad (8)$$

Where

$\varepsilon(t)$ .... misalignment error at any instant of measurement period
φ_{ref} ....the GEO satellite original direction.
$\hat{\varphi}$ (t)....... the estimated direction at any instant of measurement period

$$MeasErr = S_k(t) - \phi(t) \qquad (9)$$

$$MeasErrCov = \frac{1}{N}\Sigma(MeasErr)^2 \qquad (10)$$

$$EstErr = \hat{\phi} - \phi_{ref} \qquad (11)$$

$$EstErrCov = \frac{1}{N}\Sigma(EstErr)^2 \qquad (12)$$

An improvement factor (IMP) in the output signal to noise ratio is written as:

$$IMP = EstErrCov - MeasErrCov \qquad (13)$$

Where

MeasErr ....the misalignment error along the measurement period
MeasErrCov ....the error covariance before filtering (measurement error)
EstErr............... the instantaneous estimated error
EstErrCov .... the error covariance after filtering (estimation error)
N .... the number of time samples

4. SIMULATION RESULTS

The proposed antenna control system depicted in figure 1 is preformed and evaluated using computer simulation. A Matlab software package is used for system implementation and performance evaluation [9, 10]. The proposed system is assumed to have a second order transfer function [11]. It is defined as:

$$H(s) = \frac{1}{S^2 + 2S + 1} \qquad (14)$$

The state space model of the proposed system can be expressed as:

$$\dot{x} = \begin{bmatrix} -2 & -0.25 \\ 4 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0.5 \\ 0 \end{bmatrix} u \qquad (15)$$

The system output can be written as

$$y = \begin{bmatrix} 0 & 0.5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \qquad (16)$$

The Simulink block diagram of the proposed system in figure 2 shows the actuator antenna system model and the proposed control system using the optimal Linear Quadrature Regulator (LQR) [10]. The quadratic cost function is given by:

$$J(u) = \int_0^\infty x^T Q\, x + u^T R u + 2x^T N u \;\; dt \qquad (17)$$

Solving the quadratic cost function gives the optimal gain matrix K that satisfies the feedback law,

$$u = -kx \qquad (18)$$

In order to obtain the solution of the associated Ricatti equation (S), equation (17) is differentiated and the resultant equation can be expressed as:

$$A^T S + SA - (SB + N)R^{-1}(B^T S + N^T) + Q = 0 \qquad (19)$$

Where

N ....= 0 For the continuous-time state-space model, $\dot{x} = Ax + Bu$
S ....solution of the associated Ricatti equation.
R ....penalty weight matrix
Q............ state weight matrix

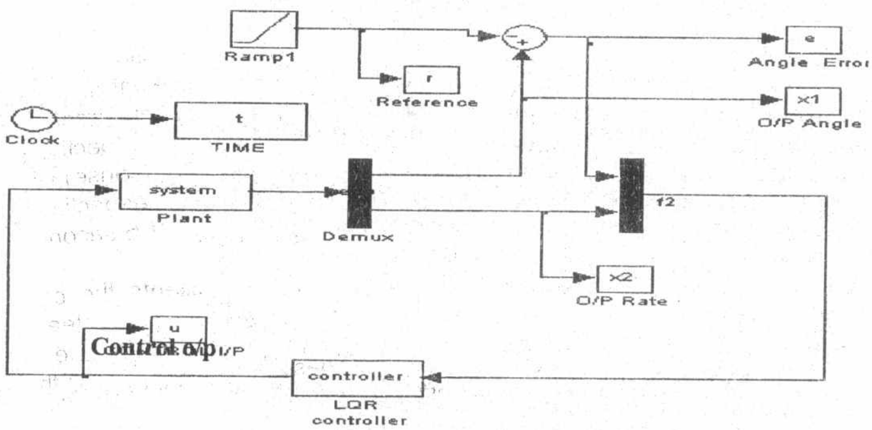
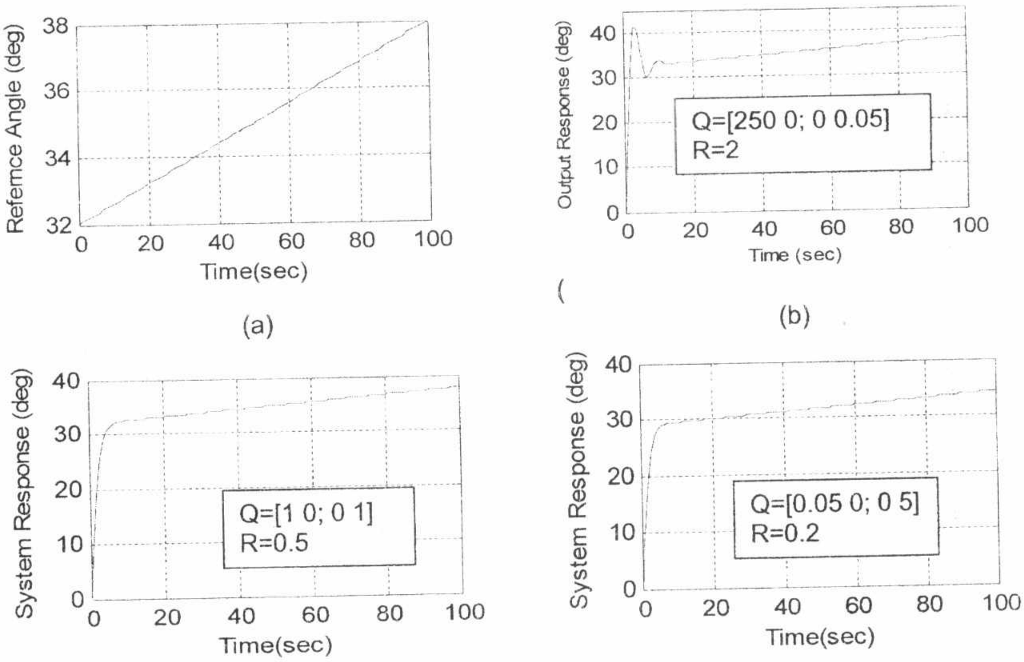


Fig.2. Simulink Representation of the Proposed System Attached with LQR controller

The performance of the proposed system depicted in figure 2 is measured and evaluated using different values for the state weight (Q) and penalty weight (R). These matrices are controlling the solution of the cost function (Ricatti) equation as shown in equation (17). The proper choice of these matrices ensures good performance of the proposed system such that the critical response is achieved. In figure 3, different values are assigned to (Q, R) and the system noise-free response is evaluated for these values, while the reference angle remained constant.



(c) (d)

Fig.3. Simulation Results for Closed Loop Noise–Free System with different values of Q, R

Figure (3.a) represents the reference input (antenna direction) as function of time. This input will remain constant while varying the state and penalty weight matrices respectively and monitor the output response. Figure (3.b) represents transient overshoot response when Q is set to [250 0; 0 0.05] and R is set to 2 respectively. Figure (3.c) depicts the transient response without overshoot (critical response) with settling time of 3 seconds, while Q is set to [1 0; 0 1], R is set to 0.5 respectively. Figure (3.d) depicts transient over damped response with settling time of 5 seconds, while Q is set to [0.05 0; 0 5] and R is set to 0.2.

The best choice for R and Q is the second case, where it represents the best convergence to the desired pseudo angle tracking with fast settling to the steady state in 3 seconds with the compromise to keep the control torque in the range of 1Nm [11]. The margins where R and Q are tested were based on analogies for the environment, where this control system will be used [9].

The sources of simulated noise that might encounter the proposed system can be either from orbit perturbations or due to thermal noise generated due to antenna operation [8].
In figure 4, the sources of noise are depicted. It is compulsory to study the effect of both the process and the sensor noises effects on system response.

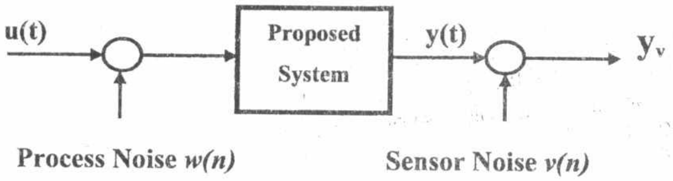


Fig.4. The proposed System Sources of Noise

Both process and sensor noises are represented by white Gaussian noise in a form of vectors *w(n)* and *v(n)* respectively. These vectors are represented as follows:

$$w(n) = \sqrt{Q_1} * randn(n,1) \qquad (20)$$

$$v(n) = \sqrt{R_1} * randn(n,1) \qquad (21)$$

Where
n ....The length of the time vector (t)
Q_1, R_1 ...=1, the noise covariances ,which represent the noise spectrum over the sampling period [10]

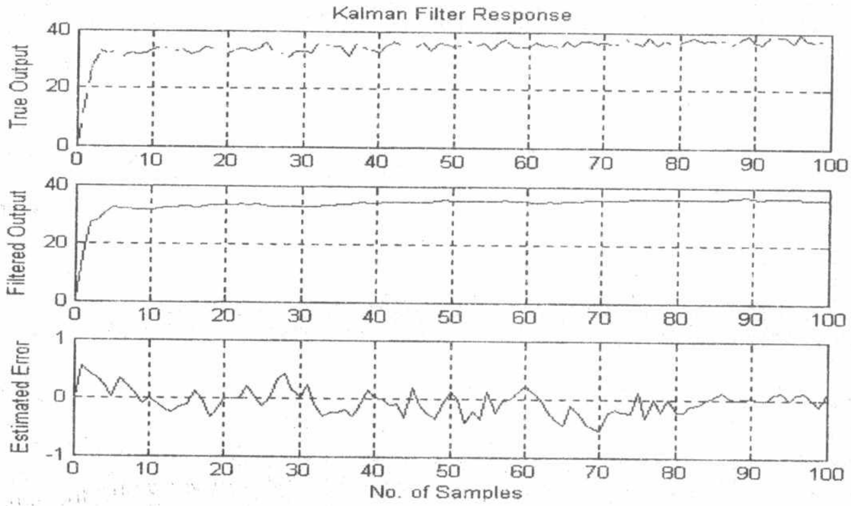


Fig.5. Proposed System Enhanced Response

In figure 5, the noise-free system response is plotted without using Kalman filter, where the ripples are obvious. The response is smoother in the following curve, where Kalman filter is being used. The third curve shows the amount of error which had been eliminated by the use of Kalman filter.

It is obvious that the proposed optimal LQR controller still have a great effect in reducing the effect of the process noise as well as sensor noise as depicted in Figure 5. it is shown that the noise level is low and its range is between (-0.5007 , 0.5825) along the time frame. This is confirmed by calculating the error covariance (*MeasErrCov*). This parameter indicates the amount of difference in evaluating the system response in the presence of noise and the system response without taking the noise in consideration [10].
Equation (10) yields that;

$$MeasErr\ Cov = 1.1138$$

This is considerably low and can be tolerated, since it represents 0.0111 error probability per sample along the time frame. The Kalman filter outputs are the state estimate ($\hat{x}(n)$) and the filtered output (y_e). In order to evaluate the filtered response (y_e), another subroutine is added to the control program [11]. This subroutine contains simulated functions, which return a state-space model of the desired Kalman filter (kalmf). Finally, close the sensor loop by connecting the plant output (y_v) to the filter input (y_v) with positive feedback.
So to evaluate the response of the proposed enhanced system, the following Matlab code is executed in the main control algorithm in order to evaluate the true response and the filtered response:

$$output = lsim\ (SimModel,[w,v,u]) \qquad (22)$$

Computing the error covariance after filtering yields the amount of remaining error after filtering. This is expressed as:

$$EstErrCov = 0.8075$$

Comparing (*EstErrCov*) with (*MeasErrCov*) evaluates the amount of improvement of the system response after filtering, as stated in equation (13).

$$IMP = 1.1138 - 0.8075 = 0.3063$$

This means that there will be 0.003063 error probability per sample along the time frame after using Kalman Filter.

In order to measure the effectiveness of the Kalman filter insertion to the system, different levels of the noise are used. As the noise level increases at the input, the system response is affected. This effect is propagated at the system output. As shown in Figure 6, three different cases are illustrated for system output response, where the overall noise level (process noise and measurement noise) are -5db, -10db,-15db respectively. It is obvious that the error in output response is directly proportional to the noise level. At 5 db noise level, the ripples level in the output response is the lowest for the three cases shown. Its range is between (-0.5007, 0.5825) along the time sampling period.

Figure 7 shows the system output response using Kalman filter at different noise levels the aforementioned noise levels.

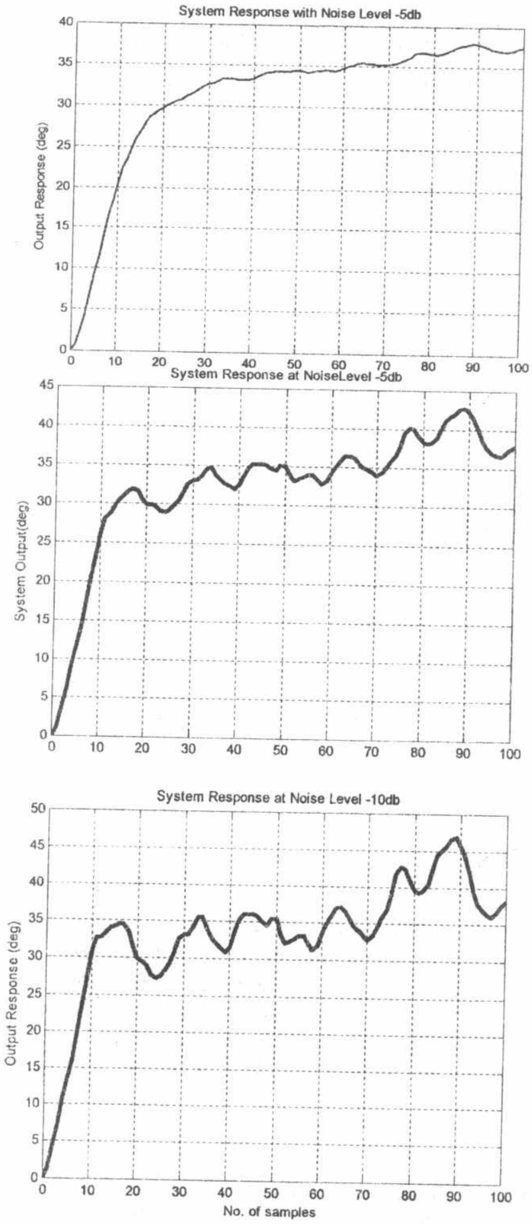


Fig.6. The noise effect on the system response noise Level of (5db, 10db,15db)

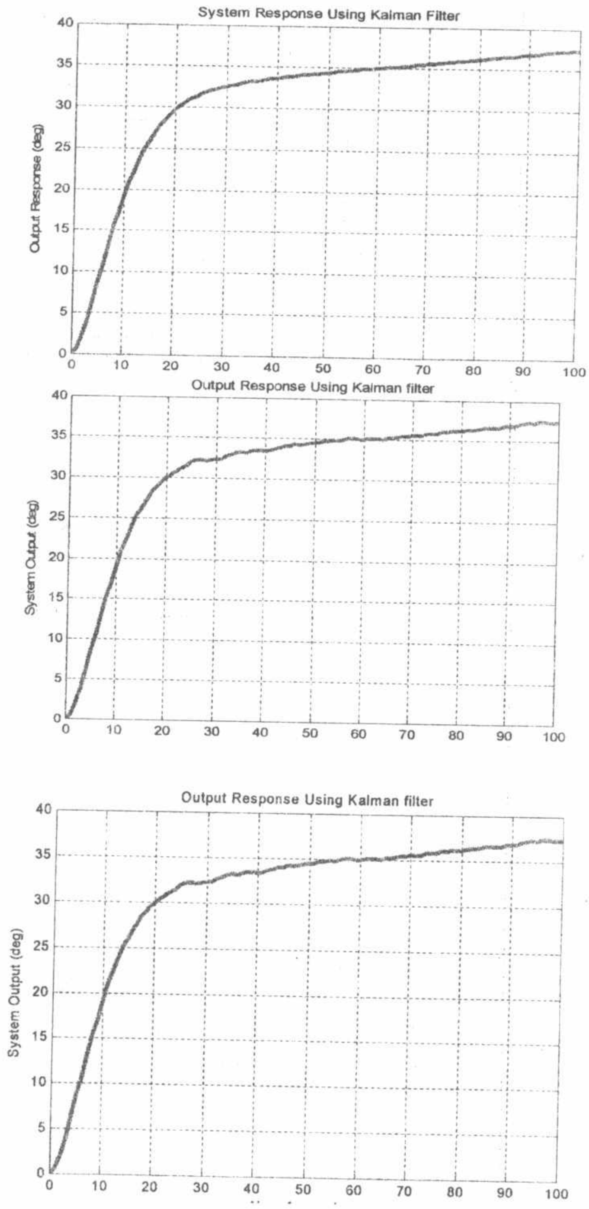


Fig.7.The System output using Kalman Filter

The covariance of the estimation error is illustrated in Figure 8 in order to check when the system reaches steady state. It is clear that the convergence time when the noise level was 5 db (first curve) is minimum (2 seconds). The convergence time is directly proportional to the noise level as shown in figure 8.

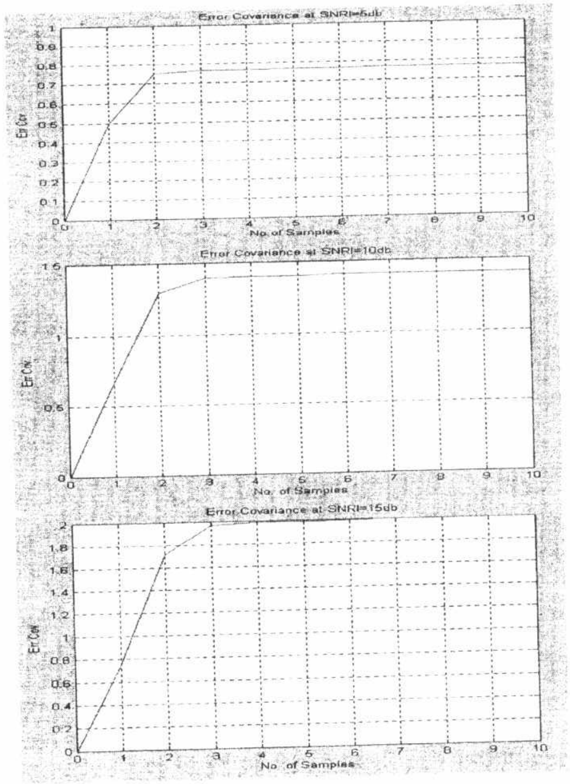


Fig.8. Error Covariance of the Estimation Error

From these figures, it can be deduced that as the noise level gets higher, the system takes more time in reaching steady state. So for noise level of 5db, the system will reach steady after about two seconds, while for noise level of 10, 15db, it takes about tree seconds to reach the steady state. So, this means that approximately at the third second of the specified time period, the fine alignment begins, and the tracking antenna assembly starts locking on the GEO satellite according to the field of view of the LEO tracking antenna.

5. CONCLUSION

It is apparent that that the proposed system is stable and robust. Moreover, the system response is performed and evaluated for different levels of Signal to Noise Ratio (SNR). The system output with Kalman filter is significantly improved and exhibits large improvement in the output response. It is proved that the proposed optimal Linear Quadrature Regulator (LQR) controller has a great effect in reducing the effect of the process and sensor noises. The LQR controller is implemented for various values of the state weight matrix (Q) and the penalty weight matrix (R). Hence the optimal proposed system can be applied as the backbone of any data relay system utilizing LEO-GEO cross-link scheme. It can be also implemented for real time reconnaissance system, so as to ensure data transfer terminal in the real time. It will be also of greater effect on early warning networks and fast data accessing systems.

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A New Method for the Control and Synchronization of Simple Chaotic systems With Only One Nonlinear Term of one Variable

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ABDEL-MONEM ABDEL-BARY** AND ABDEL-AZEEM MOHAMED**

Abstract

In this paper, we present a new algorithm for controlling a class of chaotic system that contains only one nonlinear term of one variable. Coupling depends on the choice of the drive signal or its configuration .The coupling term depends upon the nonlinear term which makes the largest conditional Lyapunov exponents of the response system negative. The comparison between the implementation of the present method and the Pecora and Carroll method is given. This comparison shows the advantages of the present method

Keywords: Chaotic dynamical system, synchronization, control of Chaos, Lyapunov exponents.

1-Introduction

There are many methods for controlling chaos such as feedback and synchronization techniques [1],[2] [3],[4]. The synchronization of two chaotic dynamical systems occurs when the trajectories of one of the systems will converge to those of the other system at the same time or the two systems show the same behavior at the same time. Different methods for synchronization such as complete synchronization, phase synchronization, lag synchronization and generalized synchronizations have been presented in [3],[5],[6],[7].

To achieve synchronization, there are many methods for linking chaotic systems. Two of these methods are the linear diffusive coupling, as initially suggested by Fujisaka and Yamada [8],[9], and driving coupling, introduced by Pecora and Carroll[5], [6],[7]. Another additive coupling is offered by the open-plus-closed-loop method to control and synchronize chaotic systems developed by Jackson and Grosu[10],[11] [12] .

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Concerning stability, there are two criteria which are most commonly used for stability of synchronized chaotic motions, these two criteria are the Lyapunov function criterion [13] ,[17]and the conditional Lyapunov exponents . In many practical cases, Lyapunov functions cannot be found, even for systems that possess a stable manifold of synchronized motions for a broad range of parameters of coupled systems, and/or the coupling itself. In contrast with Lyapunov functions, the analysis of conditional Lyapunov exponents [13],[14] is quite straightforward and can be easily employed, even for rather complicated systems. The presence of synchronization can occur if the conditional Lyapunov exponents of the response system are negative.

The control of chaotic system by synchronization is the aim of this work. This method is suggested to force the largest Lyapunov exponent of the response system to be negative. This is achieved by combining the open -loop-closed –loop method and Routh –Hurwitz criteria [15], [16].

2.Synchronization

This section reviews the Pecora and Carroll method and introduces the proposed method for synchronization of chaotic system .

2.1 Pecora and Carroll method.

This method is a widely used approach synchronization problem [5],[6], [7].
Consider an n-dimensional autonomous system governed by equation

$$\frac{dx}{dt} = f(x), \qquad x = (x_1, x_2, \ldots, x_n)^T \tag{1}$$

Divide the system into two parts in an arbitrary way, thus dividing the state vector into $X=[x_D,x_R]^T$. Where, D refers to as the driving subsystem, and R refers to as the response subsystem respectively, then

$$x^{\bullet}{}_D = g(x_D, x_R), \tag{2}$$

$$x^{\bullet}{}_R = h(x_D, x_R), \tag{3}$$

Where,

$$x_D = (x_1, x_2, \ldots, x_m)^T,$$

$$x_R = (x_{m+1}, \ldots x_n)^T,$$

$$g = [f_1(x), \ldots, f_m(x)]^T,$$

$$h = [f_{m+1}(x), \ldots, f_n(x)]^T.$$

Pecora and Carroll suggested building an identical copy of the response subsystem and driving it with the X_D variable coming from the original system.
In such a method, we have the following compound system of equation

$x^{\bullet}{}_D = g(x_D, x_R)$, (m-dimensional)- drive (4)

$x^{\bullet}{}_R = h(x_D, x_R)$ (k-dimensional)- drive (5)

$x^{\bullet\prime}{}_D = h(x_D, x'_R)$, (k-dimensional)- response (6)

2.1.1Theorem 1[5]

The subsystems X_R and X'_R will synchronize only if the conditional Lyapunov exponents are all negative.

Where, X_R is the response of the master system and X'_R is the response of the slave system. Under the right conditions, the X'_R variable will converge asymptotically to the X_R variable and continue to remain in step with instantaneous value of $X_R(t)$. Here the drive or master system controls the response or slave system through X_D component. If all conditional Lyapunov exponents of the response system are negative then the synchronization occurs otherwise the synchronization does not occur if at least one of the conditional Lyapunov exponents is positive.

2.1.2 Lyapunov exponents [13],[14],[15]

The Lyapunov exponent measures the growth of small perturbation of the difference between two systems. We shall introduce briefly how to calculate the Largest Lyapunov exponents of the whole system after synchronization.

The largest Lyapunov exponents of the whole system can calculate the distance between the aided system and the original system which is $\sqrt{\sum_{i=1}^{2N} \partial^2 x_0(i)}$. With the evaluation of time, the distance will be expanded along the largest eigenvalues directions, so the largest Lyapunov exponents λ can be obtained as follows

$$\lambda = \lim_{t \to \infty} \frac{1}{t} \ln \sqrt{\sum_{i=1}^{2N} \partial^2 x_t(i)} \Big/ \sqrt{\sum_{i=1}^{2N} \partial^2 x_0(i)} \qquad (7)$$

where, 2N is the dimension of the whole system , $\sqrt{\sum_{i=1}^{2N} \partial^2 x_0(i)}$ is the initial distance between the aided system and the original system and $\sqrt{\sum_{i=1}^{2N} \partial^2 x_t(i)}$ is the distance between the aided system and the original system after time t.

2.2 The proposed method of Synchronization

Some undesirable characteristics of the Pecora and Carroll method are the existence of the positive conditional Lyapunov exponent and the dependence on the configuration of the drive signal. The proposed method avoids this problem by choosing the values of system parameters that make the largest Lyapunov exponents negative using the concept of Routh- Hurwitz criteria and the dependence on the choice of drive signal is discarded.

The method depends upon the method of open- plus-closed-loop for control of dynamical systems (OPCL) [10],[11],[12]which is summarized as follows

Consider the dynamical system is given by:

$$\frac{dx}{dt} = F(x,t), \quad \left(x \in R^n\right). \tag{8}$$

If one wants to entrain the solution of a dynamical systems to some goal behavior, g(t), using linear feedback , in order to obtain $\lim_{t\to\infty}(x(t)-g(t))=0$, then the dynamics are in the form

$$\frac{dx}{dt} = F(x,t) + D(t,x,g) \tag{9}$$

where, $D(t,x,g)$ is the drive term and is given by:

$$D(t,x,g) = \frac{dg}{dt} - F(g,t) + \left(A - \left.\frac{\partial F}{\partial x}\right|_{x=g} \right)(x-g), \tag{10}$$

Where A is a constant matrix with eigenvalues having negative real part also D is some suitable matrix in this case g(t) has to be restricted to some solution of

$$\frac{dg}{dt} = F(g,t) \tag{11}$$

for more details see [10],[11],[12].

The idea of our proposed method is replacing the nonlinear term in the constant matrix of the jacobian of the master system with the parameter P. Then choosing the value of the parameter P that makes the eigenvalues having negative real part and also satisfying the Routh – Hurwitz criteria for stability then driving the slave system by the drive term as follows

$$\left(A - \left.\frac{\partial F}{\partial x}\right|_{x=y} \right)(x-y). \tag{12}$$

Consider the dynamical system given by ;

$$\frac{dx}{dt} = F(x,t) \tag{13}$$

then the slave system

$$\frac{dy}{dt} = F(y,t) + \left(A - \left.\frac{\partial F}{\partial x}\right|_{x=y}\right)(x-y) \tag{14}$$

where A is a constant matrix with eigenvalues with negative real parts. Then x(t) converges to y(t) for any $\| x(0) - y(0)\|$ small enough.

<u>The algorithm of the proposed method is given by:-</u>

1-Find $J = \frac{\partial F}{\partial x}$ the Jacobian matrix of the master system

2-Find the matrix A by replacing the element containing the variable in the matrix J by the parameter P (coupling term)

3-Find the characteristic equation of the matrix A

4-Apply the Routh- Hurwitz criteria to find the range values of P and substitute it in the matrix A

5-Find the drive term ($A-\left.\frac{\partial F}{\partial x}\right|_{x=y}$)(x-y)

6-Create the synchronization algorithm according to the range of parameter P as follows

The master system $\frac{dx}{dt}$=F(x,t)

The slave system

$$\frac{dy}{dt} = F(y,t) + \left(A-\left.\frac{\partial F}{\partial x}\right|_{x=y}\right)(x-y) \quad (15)$$

7-Test for the synchronization (calculate the largest Lyapunov exponents), the algorithm is coded in C^{++}

2-2-1 Parametric Analysis

The proposed method is based on the analysis of the characteristic polynomial coefficients of the matrix A. Let the characteristic polynomial of A be denoted by:

$$A(s)= a_0\, s_n + a_1\, s_{n-1} + \ldots + a_n = 0, \quad a_0 > 0 \quad (16)$$

As presented in [16], it is possible to derive necessary and sufficient conditions for the existence of eigenvalues with negative real part by applying Routh- Hurwitz conditions. By applying these conditions, it is possible to determine the numerical interval of parameter that insure the eigenvalues in the left hand plane

3 Example of Application

The following system will be analyzed using the suggested method and the P C method

3-1 Chua's circuit with cubic nonlinearity (autonomous system)

The Chua's circuit with cubic nonlinearity is described by:

$$\frac{dx}{dt} = \alpha\,(y - cx - x^3)$$

$$\frac{dy}{dt} = x - y + z \quad (17)$$

$$\frac{dz}{dt} = -\beta y$$

where, α , β and c are the parameters of Chua' circuit, and the system has three equilibrium points $(-\sqrt{-c}\ ,0,\ \sqrt{-c})$,(0,0,0), $(\sqrt{-c}\ ,0,-\sqrt{-c})$. Letting α =10, β=16 and c=-0.143 this values gives the chaotic behavior namely double scroll attractor with largest Lyapunov exponent (0.1317) and shown in Figure (1)

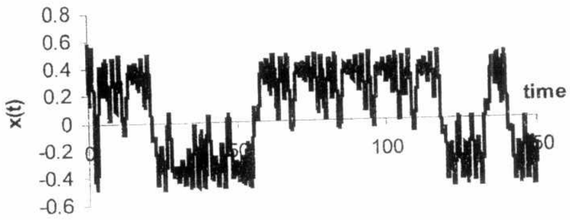


Fig (1-a) The time response of x(t) of Chua system

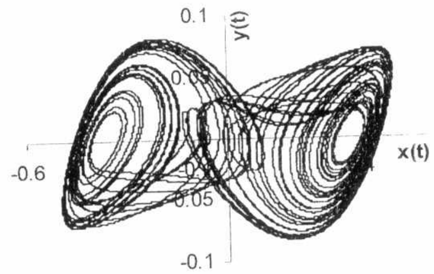


Fig (1-b) The phase plan plot of x(t) versus y(t)(double scrolls)

by applying the concept of The Pecora and Carroll to the system using x- drive configuration we get

$$\dot{x} = \alpha\ (y-cx-x3)$$
$$\dot{y} = x-y+z$$
$$\dot{z} = -\beta y \qquad (18)$$
$$\dot{y}_1 = x-y_1+z_1$$
$$\dot{z}_1 = -\beta y_1$$

and the conditional Lyapunov exponents of the subsystem are (-0.5, -0.5) i.e. the synchronization occurs. Also the Largest Lyapunov exponents of the whole system is zero and the system after synchronization is periodic.

Figure (2) shows the synchronization of the chaotic system in case of x- drive using PC method

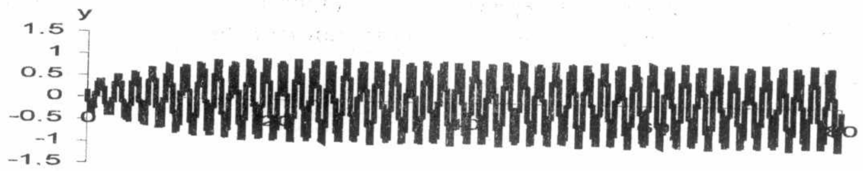


Fig (2-a) The time response of y(t) of synchronized system

Fig (2-b) The time response of y $_1$(t) of synchronized system

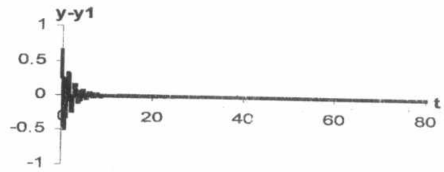
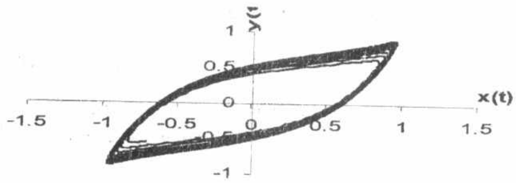


Fig (2-c) The error of synchronization between y(t), y_1(t)



Fig(2-d)The phase plan(x(t) versus y(t)) of whole system after synchronization (periodic)

It is noticed from the above that, using Pecora and Carroll (PC) method, the synchronization does not occur if the conditional Lyapunov exponent of the response system is positive. It is also dependent on the configuration of the drive system. The following explains the synchronization using the PC method for different drive signal of the Chua system that is shown in table 1.

system	Drive signal	Response system	Conditional Lyapunov exponents	Synchronization
Chua system	x	(y, z)	-ve (-0.5)	yes
	y	(x, z)	-ve (- 0.001)	yes
	z	(x, y)	+ve (0.021)	No

TABLE5. I. Conditional Lyapunov exponents for Chua system for different drive signal

To avoid the previous mentioned problems, we now apply the suggested algorithm for the synchronization of the chaotic system. The algorithm selects the drive signal directly from nonlinear part and uses it.
Applying the method outlined above, the master system is given by equation (17).
The Jacobian matrix is given by

$$J := \begin{bmatrix} -30\,x^2 + 1.430 & 10 & 0 \\ 1 & -1 & 1 \\ -16 & 0 & 0 \end{bmatrix} \qquad (19)$$

The coupling terms are deduced from the matrix

$$A := \begin{bmatrix} -30\,P + 1.430 & 10 & 0 \\ 1 & -1 & 1 \\ -16 & 0 & 0 \end{bmatrix}, \qquad (20)$$

where P is a parameter that has to chosen to cause the eigenvalues of A to have a negative real part. The characteristics equation is

$$\lambda^3 + (30\,p-0.43)\lambda^2 + (30\,p-11.43)\lambda+160=0 \qquad (21)$$

with $a_1 = 30P-0.43$, $a_2 = 30\,P-11.43$ and $a_3 = 160$

The Routh – Hurwitz condition a_1, $a_3 > 0$ and $a_1\, a_2 - a_3 > 0$
Given the condition P > 0 (numerical interval for parameter P) , then the slave system is given by

$$\frac{dx_1}{dt} = \alpha\ (y_1-cx_1-x_1^3) + (-1)*(P-30*x^2)(x-x_1)$$
$$\frac{dy_1}{dt} = x_1-y_1+z_1 \qquad (22)$$
$$\frac{dz_1}{dt} = -\beta y_1$$

Figure (3) shows the synchronization behavior of the Chua system using the proposed method

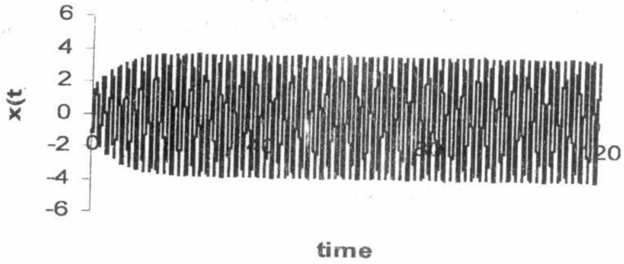


Fig (3-a)The time response of x(t)

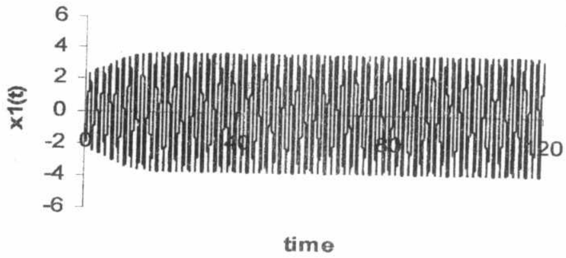


Fig (3-b)The time response of $x_1(t)$

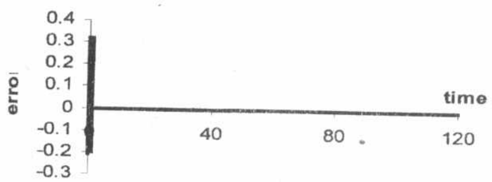
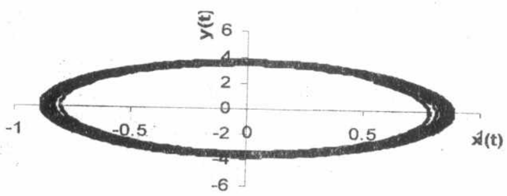


Fig (3-c) The error of synchronization between x(t), $x_1(t)$



Fig(3-d) The phase plan plot of x(t) vs y(t) whole system after synchronization (Periodic behavior, the Largest Lyapunov exponents of the whole system equal to zero)

From the above figures we note that the synchronization process is achieved without the need to select the type of drive signal. The error of synchronization in the proposed method is smaller than that of the method of Pecora and Carroll.

4- CONCLUSION

The proposed method for control and synchronization chaotic system with only one nonlinear term of one variable can be achieved without depending on the configuration of the drive signal besides minimizing the error of synchronization

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