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REAL TIME ADAPTIVE VARIABLE DIMENSION KALMAN FILTER IN MANOEUVRING TARGET TRACKING

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ABSTRACT

Tracking process for manoeuvring targets is still the main problem in Kalman filtering [1-6]. The conventional variable dimension filter used for manoeuvring target tracking is not a practical filter because of the previous state vector estimates for several scans must be recomputed to avoid the filter divergence when a new target model is detected [1,3-5].

In this paper a real time adaptive variable dimension filter is proposed. This proposed filter is without the previous estimates to be recomputed. This proposed filter is also with a less filter divergence level and therefore more practical filter. Such a filter will be more appropriate to use when the target rather rarely experiences a new models. When the target more frequently experiences a new models it will be better to use the constant acceleration filter. This constant acceleration filter is actually a part of the proposed filter in this paper. This constant acceleration filter is also capable of operating in real time with no filter divergence at the expense of slight increasing of the estimate error variance level.

KEY WORDS

Manoeuvring target tracking. Kalman filtering. Variable dimension filter. Adaptive dynamic modeling.

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NOMENCLATURE

x_k – target position at scan k
$\dot{x}_k$ – highly correlated target velocity at scan k
$\ddot{x}_k$ – highly correlated target acceleration at scan k
a_k – plant noise that perturbs the acceleration of the target and accounts for both manoeuvres and other modeling errors.
T – sampling time.
s- second.
m- meter.
z_k - the measured position at scan k.
v_k - the random noise corrupting the measurement at scan k.
$\hat{\boldsymbol{X}}_k$ - state vector estimate
$\hat{\boldsymbol{X}}1_k$ - state vector estimate of the output of real time conventional variable dimension filter.
$\hat{\boldsymbol{X}}2_k$ - state vector estimate of the output of constant acceleration filter.
$\Delta\hat{\boldsymbol{X}}1_k$ - state vector estimate error at the output of real time conventional variable dimension filter.
$\Delta\hat{\boldsymbol{X}}2_k$ - state vector estimate error at the output of constant acceleration filter.
R- Correlation covariance matrix for measurement noise.

INTRODUCTION

It is well known, that the Kalman filter is the optimal state estimator for linear observations of a target with known dynamics and white Gaussian process and measurement noise. However, as soon as a target changes its dynamics, for example, by executing a maneuver, the Kalman filter estimates may diverge causing track loss. The divergence is due to incorrect modeling of the target dynamics and is the chief difficulty in manoeuvring target tracking. There are many approaches and techniques as solutions to track manoeuvring targets [1, 4, 5].
The variable dimension Kalman filter is the manoeuvring target tracking technique witch use the constant velocity model during the non-manoeuvring tracking periods. Once a maneuver has been detected the constant velocity model is abandoned (dropped) and the algorithm "switches" to an augmented model such as the constant acceleration model [2].
As noted by [3, 4, 5], the variable dimension filter is performing better on average than other techniques for manoeuvring targets as the input estimation manoeuvring target tracking technique, despite the latter being considerably more complex and as the two level white noise model, the simplest algorithm of the three, witch is performing better than the input estimation technique and only slightly worse than the variable dimension filter. It is also noted, that the variable dimension filter, strictly speaking, is not a filter due to the requirement that for a manoeuvre detector with (effective) window length s declaring a manoeuvre at time k, the augmented model must be initialized at time $k-s$, that is, retrospectively.

This undesired requirement is not needed using the proposed real time adaptive variable dimension filter, represented in this work, as indicated in Fig. 3.
By using the sample mean estimate of the conventional variable dimension filter and the constant acceleration filter added in parallel with it as described in Fig. 3, it would be possible to get real time and more exact state vector estimate $\hat{\boldsymbol{X}}_k$ from the measurement vector $\boldsymbol{Z}_k$ with less divergence level when the target experiences new model at the expense of some estimate errors added when the target experiences a constant velocity model. The vector estimate $\hat{\boldsymbol{X}}_k$ is real time because the states, through the period from the time point when the new model started until this new model is declared, are not recomputed, so the proposed filter is therefore suitable for use in a practical environment.

DYNAMIC MODEL

In general, the position of the target is assumed to be described by the following equation of motion [6]:

$$\begin{aligned} x_{k+1} &= x_k + \dot{x}_k T + \ddot{x}_k T^2 / 2 \\ \dot{x}_{k+1} &= \dot{x}_k + \ddot{x}_k T \\ \ddot{x}_{k+1} &= \ddot{x}_k + a_k \end{aligned} \tag{1}$$

a_k is assumed to be zero mean and of constant variance σ_a^2 and also uncorrelated with its values at other intervals.
In vector matrix form, equation (1) can be written as

$$\boldsymbol{X}_{k+1} = \boldsymbol{F}\boldsymbol{X}_k + \boldsymbol{A}_k \tag{2}$$

Where

$$\boldsymbol{X}_k = \begin{bmatrix} x_k \\ \dot{x}_k \\ \ddot{x}_k \end{bmatrix} \quad ; \quad \boldsymbol{A}_k = \begin{bmatrix} 0 \\ 0 \\ a_k \end{bmatrix} \tag{3}$$

$$\boldsymbol{F} = \boldsymbol{F}_{acc.} = \begin{bmatrix} 1 & T & T^2/2 \\ 0 & 1 & T \\ 0 & 0 & 1 \end{bmatrix} \text{ - for constant acceleration model} \tag{4}$$

$$F = F_{vel.} = \begin{bmatrix} 1 & T & T^2/2 \\ 0 & 1 & T \\ 0 & 0 & 0 \end{bmatrix} \text{ - for constant velocity model} \quad (5)$$

MEASUREMENT EQUATION

The measurement equation may be written as

$$z_k = \boldsymbol{HX}_k + v_k \quad (6)$$

where

$$\boldsymbol{H} = [1 \quad 0 \quad 0] \quad (7)$$

The measurement noise v_k is assumed to be zero mean and of constant variance σ_z^2 and also uncorrelated with its values at other intervals.

FILTERING EQUATIONS

Given equations (2) and (7), it would be possible to use the Kalman filtering algorithm for finding the optimal estimate of the state vector after the measurement is processed:

$$\hat{\boldsymbol{X}}_k = \tilde{\boldsymbol{X}}_k + \boldsymbol{G}_k [\boldsymbol{Z}_k - \boldsymbol{H}\tilde{\boldsymbol{X}}_k] \quad (8)$$

The state vector $\tilde{\boldsymbol{X}}_k$ before the measurement is processed is given by

$$\tilde{\boldsymbol{X}}_k = \boldsymbol{F}\hat{\boldsymbol{X}}_{k-1} \quad (9)$$

The Kalman gain matrix is given by

$$\boldsymbol{G}_k = \tilde{\boldsymbol{P}}_k \boldsymbol{H}^T (\boldsymbol{H}\tilde{\boldsymbol{P}}_k \boldsymbol{H}^T + \boldsymbol{R})^{-1} \quad (10)$$

The predicted covariance matrix $\tilde{\boldsymbol{P}}_k$ is given by

$$\tilde{\boldsymbol{P}}_{k+1} = \boldsymbol{F}\tilde{\boldsymbol{P}}_k \boldsymbol{F}^T + \boldsymbol{Q} \quad (11)$$

and the filtered covariance matrix is given by

$$\tilde{\boldsymbol{P}}_k = (I - \boldsymbol{G}_k \boldsymbol{H})\tilde{\boldsymbol{P}}_k \quad (12)$$

Q is the covariance matrix of the plant noise and is given by

$$\boldsymbol{Q} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \sigma_a^2 \end{bmatrix} \quad (13)$$

REAL TIME ADAPTIVE VARIABLE DIMENSION FILTER SIMULATION

The conventional variable dimension filter is presented in Fig.1. By using this filter, it is necessary for several state vector estimates, before the time point of new target model is declared, to be recomputed. So, therefore such a filter could not work in real time through several scans needed to detect and declare the new target model. This undesired property may cause some risk in tracking process especially when the new target model rate becomes more and more. So, this filter in fact is not a practical filter although it is performing well as indicated in Fig.2. In this resultant simulating figure, as in other following simulating figures, it is assumed that the target starts its motion with a constant velocity model using (2) and (5) until the scanning sample 400, then after sample 400, it experiences a constant acceleration model using (2) and (4) until the final scanning simulated sample 1000 with $\sigma_a = 3\ m/s^2$. Also it was assumed that $\sigma_z = 800\,m;\ T = 0.1s$.

In this case the simulating resultant standard deviation of state vector estimate error was found as:

$\sigma_{conv.(0-400)} = 78\,m$ - where the constant velocity model is handled through the first 400 samples (until divergence period).

$\sigma_{conv.(400-600)} = 97\,m$ - where the target starts the new constant acceleration model and the detection process of this model is finished (divergence period).

$\sigma_{conv.(600-1000)} = 105\,m$ - where the new constant acceleration model is declared and the filter switches to this model (after divergence period).

The proposed real time adaptive variable dimension filter is presented in Fig.3. The proposed filter output is the average (sample mean) of the outputs of two filters.

The first one is the conventional variable dimension filter without the previous state vector estimates to be recomputed so as to avoid the filter divergence when the target experiences a new motion model. Later, this filter will be called a real time conventional variable dimension filter. This filter could work separately in real time but with undesired estimate divergence as shown in Fig.4.

The second one is the constant acceleration filter. This filter is a simple filter and also could work separately in real time with no estimate divergence when the target experiences a new motion model, but at the expense of some increasing in the state vector estimates error level when the target experiences a constant velocity model as shown in Fig.5. So, such a filter could be the better filter when the target frequently experiences the constant acceleration model (high manoeuvring target).

The proposed filter presented in Fig.3., gives the average of the two state vector estimates: $\hat{\boldsymbol{X}}1_k = \boldsymbol{X}_k + \Delta\boldsymbol{X}1_k$ and $\hat{\boldsymbol{X}}2_k = \boldsymbol{X}_k + \Delta\boldsymbol{X}2_k$.
So, the proposed filter output is the state vector estimate:

$$\hat{\boldsymbol{X}}_k = \frac{\hat{\boldsymbol{X}}1_k + \hat{\boldsymbol{X}}2_k}{2} = \boldsymbol{X}_k + \frac{\Delta\boldsymbol{X}1_k + \Delta\boldsymbol{X}2_k}{2} = \boldsymbol{X}_k + \Delta\boldsymbol{X}_k \tag{14}$$

Assuming that $\Delta\boldsymbol{X}1_k$ and $\Delta\boldsymbol{X}2_k$ are Gaussian processes and

$$E[\Delta\boldsymbol{X}1_k] = E[\Delta\boldsymbol{X}2_k] = 0; \quad E\{[\Delta\boldsymbol{X}1_k]^2\} = \sigma^2_{\Delta\boldsymbol{X}1_k}; \quad E\{[\Delta\boldsymbol{X}2_k]^2\} = \sigma^2_{\Delta\boldsymbol{X}2_k}, \tag{15}$$

the standard deviation of state vector error when the proposed filter is used, will be:

$$\sigma_{\Delta\boldsymbol{X}_k} = \frac{\sqrt{\sigma^2_{\Delta\boldsymbol{X}1_k} + \sigma^2_{\Delta\boldsymbol{X}2_k}}}{2} \tag{16}$$

In this case and for the same simulated target motion, the simulating resultant standard deviation of state vector estimate error was found as shown in Fig.6.:

$\sigma_{prop.(0-400)} = 89\,m$

$\sigma_{prop.(400-600)} = 122\,m$

$\sigma_{prop.(600-1000)} = 100\,m$

By comparing these simulation results with the simulation results obtained for the conventional variable dimension filter, we could note that, the error variance, where the constant velocity model is executed will be slightly larger than the one obtained by conventional variable dimension filter.

$$\sigma_{prop.(0-400)} = 89\,m > \sigma_{conv.(0-400)} = 78\,m$$

the error variance, where the divergence is executed will be somewhat larger than the one obtained by conventional variable dimension filter.

$$\sigma_{prop.(400-600)} = 122\,m > \sigma_{conv.(400-600)} = 97\,m$$

the error variance, where the constant acceleration model is executed, is less than the error variance when the conventional or the constant acceleration filter is used separately:

$$\sigma_{prop.(600-1000)} = 100\,m < \sigma_{conv.(600-1000)} = 105\,m$$

So, the proposed filter is nearly performing as the conventional in average, and the difference is: the proposed is a real time and the conventional is not a real time filter.

CONCLUSION

1. The conventional variable dimension filter is not a real time and therefore is not a practical filter once the previous state vector estimates for several scans must be recomputed when a new target model is detected so as to avoid the estimates divergence arised in this case.
2. It is better to strongly avoid the filter divergence by using the simplest real time constant acceleration filter which is a part of the proposed filter, instead of using the conventional variable dimension filter, especially when the constant acceleration model is more frequently occurred through the hole target motion.
3. It is better to acceptably avoid the filter divergence by using the proposed real time adaptive variable dimension filter without the need to recompute the previous estimates, instead of using the conventional variable dimension filter and also instead of using the constant acceleration filter, especially when the constant acceleration model is rarely occurred through the hole target motion compared with the constant velocity model.

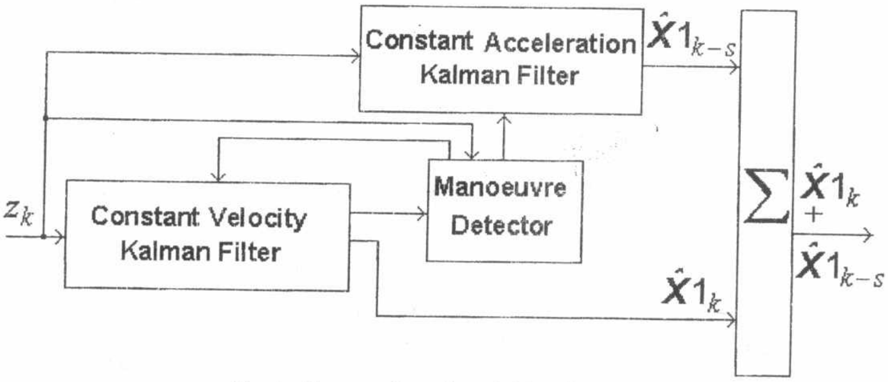

Fig.1. Conventional variable dimension filter.

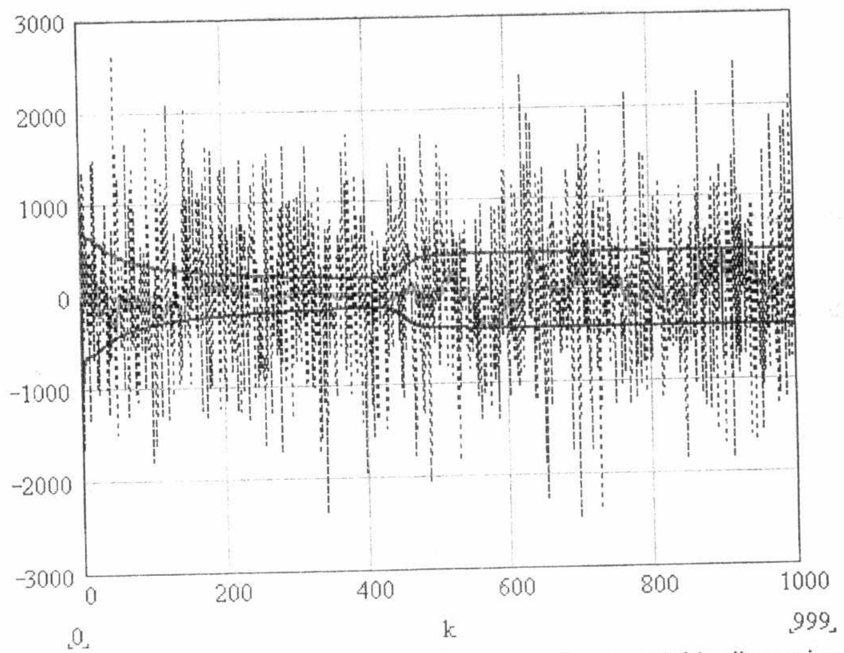

Fig.2. State vector estimate error by using the conventional variable dimension filter.

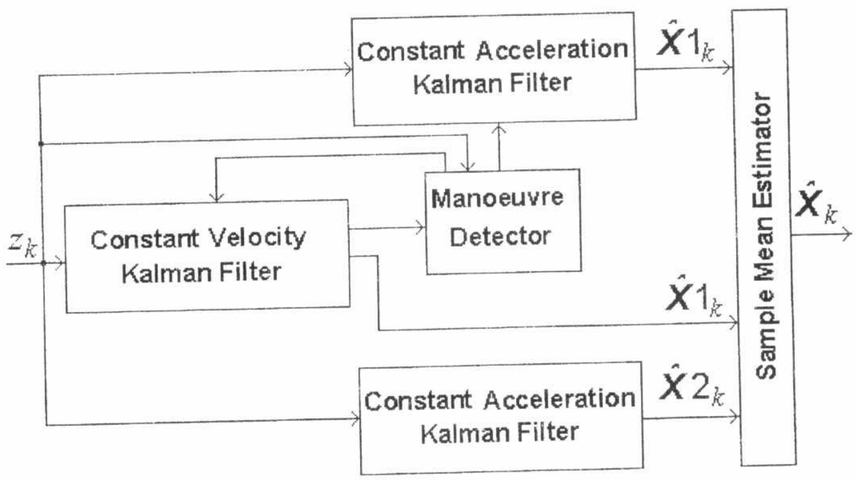

Fig.3. Proposed real time adaptive variable dimension filter.

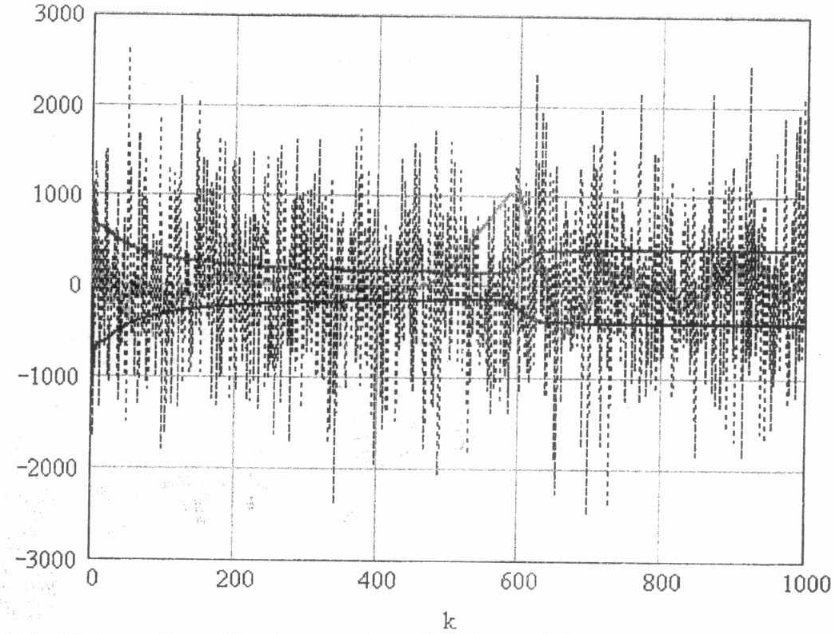

Fig.4. State vector estimate error by using the real time conventional variable dimension filter.

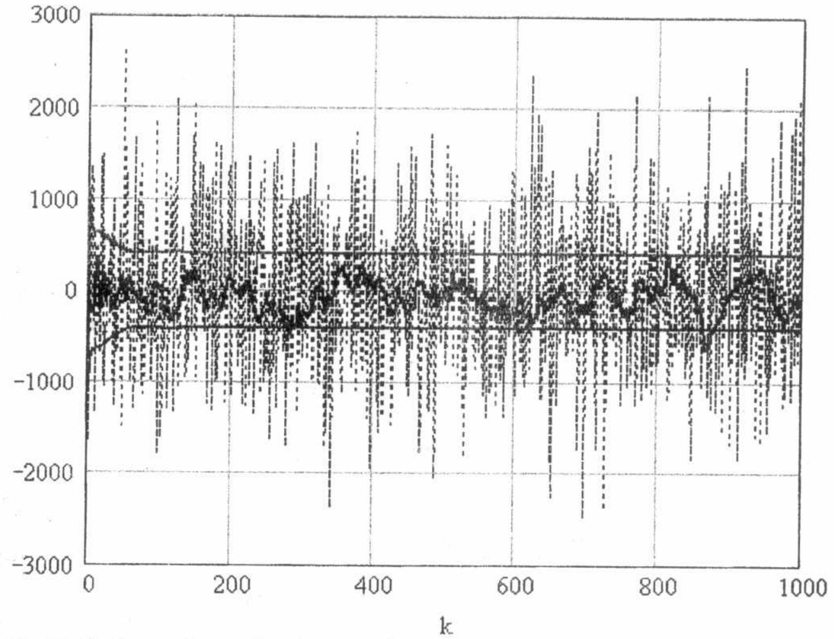

Fig.5. State vector estimate error by using the constant acceleration filter.

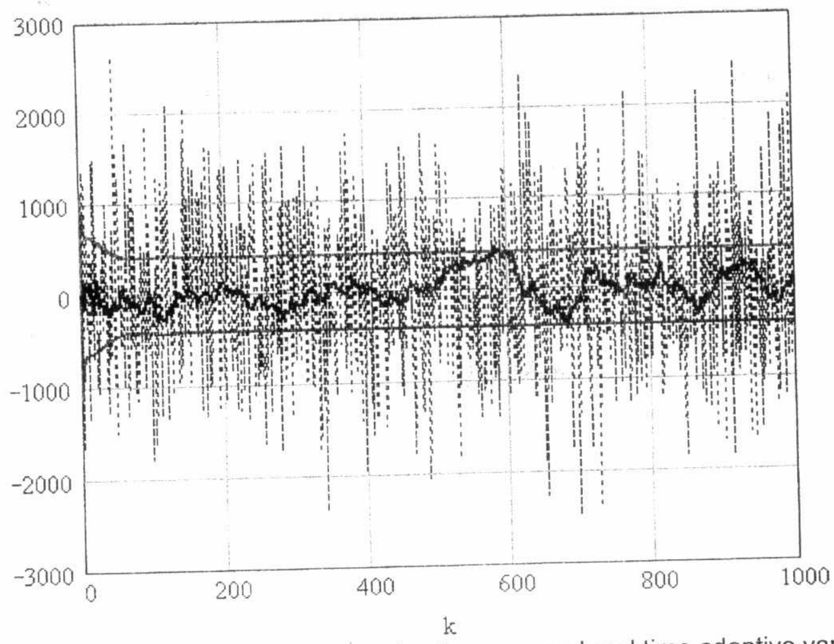

Fig.6. State vector estimate error by using the proposed real time adaptive variable dimension filter.

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