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EMERGENCY LOCATIONS PROBLEMS WITH STOCHASTIC PARAMETERS

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Abstract *The problem of determining the optimum locations for the emergency facility services in the case in which three possible routes between every pair of customers and service center was discussed assuming deterministic values for the parameters of the problem. In this paper, we introduce a description and solution algorithm for this problem focusing on the case in which the values of the spatial separations among the service centres and the customers are not deterministic but stochastic. The introduced solution is depending on hybridizing the Monte-Carlo Simulation technique with the Max-Separable optimization technique*

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1. Introduction

Finding the optimum locations of the Emergency facility centres as ambulance, police stations, fire stations, etc is one of these important optimization problems, there are several previous treatments and solutions for different cases of this problem e.g. Cunninghame-Green [2], Hakimi [4], Tharwat and al.[9, 10], Gavalec and Hudec [3, 5], Minika [6] Zimmermann [9, 11, 12], most of these treatments assumed that the data of the problem (e.g. the time needed to transport between a center and a customer) is deterministic but that contradicts what everybody knows in the real life.

One of the most important forms to formulate nondeterministic data is the stochastic form in which every input is given by a probability distribution. This paper suggests a solution to this situation to problem assuming the existence of three possible routes from every service center to any customer which is solved in the deterministic case in [7], the suggested solution is depending on the Monte-Carlo Simulation technique which gives a practical approximation of the stochastic data to deterministic data to allow the algorithm used in the mentioned reference to solve it. In the next sections, we give the formulation of the problem and a brief sketch of the algorithm which solves it in the deterministic situation, also a description of Monte Simulation and how to mix it with the mentioned algorithm.

2. 2. The Deterministic Problem:

This section introduces the problem that we will solve, first we introduce the original problem with deterministic factors as it was introduced in [7], after that the modified problem with stochastic data will be introduced.

2.1 Basic Notations:

Following we introduce the formulation of the problem that we aim to solve (see figure 1)

Assume that there exist *m*-fixed points (places) (Y_i , i = 1, ... , m) which are given. These points are to be served by n-service centers (S_j , *j*=1, ... , n). Each service centre S_j has to be placed on a segment (link) $\overline{A_j B_j}$, where A_j and B_j are given points, assume also that for each line segment $\overline{A_j B_j}$ there exists a fixed point $C_j \in \overline{A_j B_j}$. Let the spatial separation (distance, cost, cost) $Y_i A_j$, $Y_i B_j$ and $Y_i C_j$ are known for all $i \in \{1, 2, ..., n\}$ and $j \in \{1, 2, ..., m\}$. The given spatial separations will be denoted as follows:

- $d_j = |A_j B_j|$
- $z_j = |A_j C_j|$
- $a_{ij} = |Y_i A_j|$
- $b_{ij} = |Y_i B_j|$
- $c_{ij} = |Y_i C_j|$

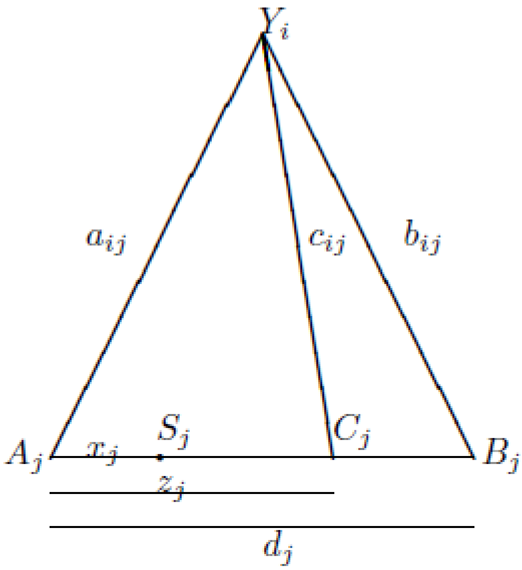

Figure1(Parameters of the the problem)

One variable x_j which denotes the spatial separation between the service center S_j and the point A_j will be involved in our problem, (i.e. $x_j = |A_j S_j|$).

So we can conclude that:

- $S_j B_j = d_j - x_j$
- $|S_j C_j| = |d_j - x_j|$ (The absolute value of $d_j - x_j$)

From the above definition of spatial separations we can see that each service center S_j could be reached from each customer Y_i, by any of the following three routes:

- $Y_i A_j S_j$ *Sy* with spatial separation $a_{ij} + x_j$.
- $Y_i B_j S_j$ *Sy* with spatial separation $b_{ij} + d_j - x_j$.
- $Y_i C_j S_j$ *Sy* with spatial separation $c_{ij} + |x_j - z_j|$.

It is obvious that the route of the minimum spatial separation (r_{ij}) of the above three will used connect Y_i, and S_j i.e.

$$r_{ij}(x_j) = \min\{a_{ij} + x_j, d_{ij} + d_j - x_j, c_{ij} + |x_j - z_j|\} \qquad (2.1)$$

2.2 The Nearest service center:

If some customer Y_i, is calling for a service it is obvious that this customer will be served by the nearest service center to its position. So we can define the spatial separation between Y_i, and its nearest center as:

$$\forall i \in \{1,2,...m\},\ f(Y_i) = \min_{x\in\{1,2,...,n\}} r_{ij}(x_j) \qquad (2.2)$$

The objective of our problem is to determine the positions x_i where $i \in \{1,2,...,n\}$ of the service centers S_i which minimizes the worst spatial separation between the customers and their corresponding service center

The definition of this worst-case function can be formulated as:

$$\text{Let } Y = (Y_1, Y_2, ..., Y_n) \text{ and } X = (x_1, x_2, ..., x_m)$$
$$\text{then } F(Y) = \max_{i\in\{1,2,...m\}}\{f(Y_i)\} \qquad (2.3)$$

Assume also that the choice of X is constrained by that the spatial separation between any customer and its service center must not exceed the given positive number α i.e:

$$f(Y_i) \le \alpha, \forall i \in \{1,2,...,n\} \qquad (2.4)$$

Consequently

$$F(Y) \le \alpha \qquad (2.5)$$

According to the above discussion the problem can be formulated by:

Definition 2.1 (Problem P1):

Objective function: $F(Y) \to min$

Subject to: $F(Y) \le \alpha$

$0 \le x_j \le d_j$

2.3 Adding Ordering Assumption:

The above problem had been proved to be NP-hard, see [5], also it is non-linear and non-differentiable. To treat this situation Zimmermann [12] assumed ordering assumptions to the problem in the case existence of two routes, these assumptions change the problem to be solvable with an efficient algorithm, for example, in [10] Tharwat and El-Khodary used these assumptions to solve the problem in the case of two possible routes.

Here we extend these assumptions to our problem (the case of three routes) and reformulate the problem.

The first assumption states that for each $j^* \in \{1, 2, ..., n\}$ there exists a permutation $\pi^* \in \{1^*, 2^*, ..., m^*\}$ of the indices $\{1, 2, ..., m\}$ such that:

$$(a_{j_{1^*}}, b_{j_{1^*}}, c_{j_{1^*}}) \le (a_{j_{2^*}}, b_{j_{2^*}}, c_{j_{12}}) \le ... \le (a_{j_{n^*}}, b_{j_{n^*}}, c_{j_{1n}})$$

That means that there is an ordering for the set $Y_i, i \in \{1, 2, ..., m\}$ according to the spatial separation between each of them and any segment $\overline{A_j B_j}$.

The above assumptions lead to the following one see [12]:

For each $j^* \in \{1, 2, ..., n\}$ there exists a permutation $\pi^* \in \{1^*, 2^*, ..., m^*\}$ of the indices $\{1, 2, ..., m\}$ such that:

$$\mathrm{V}_{1^* j^*}(\alpha) \supseteq \mathrm{V}_{2^* j^*}(\alpha) \supseteq .... \supseteq \mathrm{V}_{m^* j^*}(\alpha) \qquad (2.6)$$

where $\mathrm{V}_{ij}(\alpha) = \{x_j : 0 \le x_j \le d_j, r_{ij}(x_j) \le \alpha\}$ (2.7)

This assumption can be illustrated in figure (2):

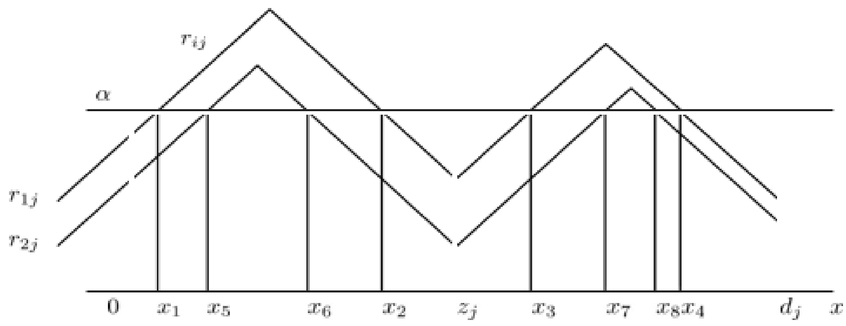

Figure (2) (The ordering assumptions)

From the figure we caeasily see that:

$V_{1j}(\alpha) = [0, x_1] \cup [x_2, x_3] \cup [x_4, d_j]$

And

$V_{2j}(\alpha) = [0, x_5] \cup [x_6, x_7] \cup [x_8, d_j]$

And

$V_{1j}(\alpha) \subset V_{2j}(\alpha)$

The above assumptions allows us to define the following set (see [12]):

$$\forall \alpha \in R^{+} : M(\alpha_j) = \{x_j : 0 \le x_j \le d_j, F(x_j) \le \alpha\} \qquad (2.8)$$

So we can rewrite the problem (P1) as:

Definition (Problem P2):

Objective function: $\alpha \to \min$

Subject to: $M(\alpha) \neq \phi$

Problem with Stochastic Data:

To use the above algorithm to solve the problem in the case of stochastic data we use the Monte-Car lo simulation technique to transform the stochastic data into deterministic, in this section the problem with stochastic data will be presented.

3.1 Stochastic Spatial Separation:

The problem formulated in the previous section is a very important optimization problem, but in real life, we can easily see that the spatial separations a_{ij}, b_{ij} and c_{ij}, can't be assumed always as deterministic values, so we need to suppose that each one of this spatial separation is a stochastic according to a given probability distribution. To apply the mentioned above algorithm see [7] which solves the problem in the case of deterministic data we have to find a way to determine an approximated deterministic value for each spatial separation. the usual statistical methods (like the chance constraints method) are not practical to do this task because the problem is not linear and also neither differentiable nor convex, so we will use the experimental method (Monte-Carlo simulation) to change the problem from its stochastic nature into the deterministic case.

The Monte-Carlo simulation method sees [1] as an approximate solution to a variety of problems that can be described in terms of probability distribution functions (pdf's). The goal of the Monte-Carlo method is to simulate the Physical system by random sampling from the given pdf's and then performing the necessary supplementary computations needed to describe the system evolution. Replacing Mathematics and Physics by random sampling of possible states from pdf's that can describe the system e actively if the given pdf's is suitable to represent the actual behaviour of the physical or mathematical model. Monte Carlo can be considered as a statistical method, for more details see [1]. The major components of the Monte-Carlo algorithm are defined as follows:

- Probability distribution functions (pdf's): the physical (or mathematical) system must be described by a set of suitable pdf's.
- Random number generator: a source of random numbers uniformly distributed on the unit interval [0, 1] must be available.
- Sampling rule: a description for sampling from the specified pdf's as- summing the availability of random numbers in the unit interval.
- Scoring (or tallying): the outcomes must be accumulated into overall tallies or scores for the quantities of interest.

The first stage of Monte-Carlo methods execution is to find the cumulative probability distribution *F(x)* for every density function *f(x).* The cumulative probability distribution function is defined by [1] as:

$$F(x) = \int_{x_{\min}}^{x} f(t)dt \qquad (3.9)$$

In our emergency location problem Monte Carlo simulation is used to treat the randomness of the spatial separations a_{ij}, b_{ij} and c_{ij} . The methodology followed includes 10^K trials for some $k \in \{1, 2, ...\}$, in each trial:

1. Determine the probability distribution of each spatial separation.
2. Find the cumulative function of each of the mentioned above distribution.
3. Generate random numbers I between 0 and 1 with uniform distribution using the Monte Carlo method to satisfy the randomness of the spatial separation.
4. Substitute the generated random number in the inverse function of the cumulative distribution function.

5. Execute the algorithm which was introduced in section 3.5 to solve the three routes emergency locations problem using the result of the previous step as the spatial separations a_{ij}, b_{ij} and c_{ij} .

For each trial $q, 1 \le q \le 10^k$, by the execution of the above steps, we get a solution set $x_{j_q} \in [0, d_j], j \in \{1, 2, ..., n\}$. The final solution can be obtained by finding the average of the results of all these trials, i.e the final solution will be: $x_j^{opt} = \sum_1^{10^k} \frac{x_{j_q}^{opt}}{10^k}$.

Remark 3.1: The above-mentioned average can be obtained only if the solution set for each line segment $\overline{A_j B_j}, j \in \{1, 2, ..., n\}$ and for each trial q is a singleton, but we can see that this result is not always guaranteed. To avoid this complication we will apply the following steps:

1. If the solution set consists of a finite number of points take the average of them as the approximated solution set for this segment in the trial in hand.
2. If the solution set contains some sub-interval of $[0, d_j]$ then for each sub-interval take the mid-point of it as representative and apply the previous step

This modification gives a practical approximation (not exact) solution for the solution set.

4. The Algorithm:

According to the above discussion, we can introduce the algorithm which solves problems P1, P2 as follows:

1. Input the following parameters:
   - The number of customers m
   - The number of emergency service center n

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- The spatial parameters (as probability distributions) $a_{ij}, b_{ij}, c_{ij}, i \in \{1, 2, ..., m\},\ j \in \{1, 2, ..., n\}$
- The inverse cumulative probability functions $F_{a_{ij}}^{-1}, F_{b_{ij}}^{-1}, F_{c_{ij}}^{-1}, i \in \{1, 2, ..., m\},\ j \in \{1, 2, ..., n\}$
- The deterministic spatial parameters (as scalars) $d_j, z_j, \forall j \in \{1, 2, ..., n\}$
- The number of trials 10^K
- maximum threshold α (optional)

2. Put q = 1 (q is the number of the current trial)
3. Generate the random Numbers $\chi_{a_{ij}}^q, \varphi_{b_{ij}}^q, \omega_{c_{ij}}^q \in [0,1], i \in \{1, 2, ..., m\},\ j \in \{1, 2, ..., n\}$
4. Substitute the generated random numbers $\chi_{a_{ij}}^q, \varphi_{b_{ij}}^q, \omega_{c_{ij}}^q$ in the inverse function $F_{a_{ij}}^{-1}, F_{b_{ij}}^{-1}, F_{c_{ij}}^{-1}$ of the cumulative distribution functions finding a_{ij}, b_{ij}, c_{ij} respectively
5. Test the satisfaction of the ordering assumption. If the ordering assumption is satisfied go to step 6, If the ordering assumption is not satisfied go to step 3
6. $\forall i \in \{1, 2, ..., m\},\ j \in \{1, 2, ..., n\}$, calculate the value $\delta_{ij} = \min\{a_{ij}, b_{ij}, c_{ij}\}$
7. $\forall i \in \{1, 2, ..., m\}$ calculate $\alpha_i = \min\{\delta_{ij}, j \in \{1, 2, ..., n\}\}$,
8. If the maximum threshold α is imposed then update α_i as follows: $\alpha_i = \alpha^*$ if $\alpha^* < \alpha_i$ and $\alpha_i = \alpha_i$ if $\alpha^* \geq \alpha_i$.
9. Calculate the maximum threshold value $\alpha = \max\{\alpha_i, i \in \{1, 2, ..., m\}\}$
10. Calculate the matrix $V_{ij}(\alpha)$
11. $\forall i \in \{1, 2, ..., m\}$, determine an index *j(i)* such that: $V_{ij}(\alpha) \neq \phi$.
    - If there are more than one such index, then break the tie arbitrary.

- If $\forall i \in \{1, 2, ..., m\}, \forall j \in \{1, 2, ..., n\}, V_{ij} = \phi$ then go to step 15.

12. $\forall j \in \{1, 2, ..., n\}$ determine: $P_j(\alpha) = \{i \in \{1, 2, ..., m\}, V_{ij} \neq \phi\}$
13. $\forall j \in \{1, 2, ..., n\}$ determine: $V_j(\alpha) = \bigcap_{i \in P_j(\alpha)} V_{ij}$
14. $\forall j \in \{1, 2, ..., n\}$ determine: $\begin{cases} V_{ij}(\alpha) \text{ if } P_j(\alpha) \neq \phi \\ [0, d_j] \text{ if } P_j(\alpha) = \phi \end{cases}$

    Go to step 16.
15. Print "there is no solution" and stop.
16. calculate $(x_1^{opt_q}, x_2^{opt_q}, ..., x_2^{opt_q})$ as mentioned in the above section.

17. If $q < 10^k$ put $q = q + 1$ and go to step 3
    If $q = 10^k$ go to sep 18
18. print the solution is $\left(\frac{\sum_{q=1}^{10^k} x_1^{opt_q}}{10^k}, \frac{\sum_{q=1}^{10^k} x_2^{opt_q}}{10^k}, ..., \frac{\sum_{q=1}^{10^k} x_n^{opt_q}}{10^k} \right)$

5. Conclusion:

In this paper, the Monte-Carlo method is used to handle the stochastic data of the spatial separations among the customers and the emergency centers to allow a pre-developed algorithm to find the optimum location of every emergency center. Monte-Carlo method in the emergency locations problem.

For further research, we suggest studying the case in which the spatial separations are fuzzy numbers.

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