



Egyptian Knowledge Bank

International Journal of Advances in Structural and Geotechnical Engineering

https://asge.journals.ekb.eg/

Print ISSN 2785-9509

Online ISSN 2812-5142

Special Issue for ICASGE'19

Shear Wave Velocity Measurements of Granular Soils using the P-Rat

Sarah Abdelrahman, Mona Mansour and Mohamed Rabie

ASGE Vol. 06 (02), pp. 40-51, 2022

ICASGE'21 29 March – 1 April 2021, Hurghada, Egypt



Shear Wave Velocity Measurements of Granular Soils using the P-Rat

Sarah Abdelrahman¹, Mona Mansour² and Mohamed Rabie³

 ¹M.Sc. Candidate, Faculty of Engineering, Helwan University, Egypt E-mail: <u>sarahahmoussa@gmail.com</u>
²Professor, Faculty of Engineering, Helwan University, Egypt E-mail: <u>monamansor@m-eng.helwan.edu.eg</u>
³Dean, Faculty of Engineering, Helwan University, Egypt E-mail: <u>m.rabie@talk21.com</u>

ABSTRACT

The small strain shear modulus, G_{max} is considered a fundamental design parameter in many geotechnical applications for soils under dynamic loads. G_{max} can be estimated from both in situ and laboratory techniques. The piezoelectric ring-actuator technique (*P-RAT*) was developed at the geotechnical laboratory of Université de Sherbrooke (QC, Canada) to measure the shear wave velocity (V_s) and accordingly G_{max} . This technique can be incorporated into different conventional apparatuses (e.g., triaxial, oedometer). This paper represents a description of the development of *P-RAT* to estimate V_s with a suitable accuracy as well as the developed interpretation method of output signals during V_s measurements. In addition, the V_s results of three granular soils were measured and a correlation between the void ratio (*e*) and V_s was then established. The obtained results showed compatibility with different V_s -e correlations proposed in the literature.

Keywords: Oedometer, P-RAT, Shear wave velocity, Small strain stiffness modulus.

INTRODUCTION

The small strain stiffness modulus, G_{max} , can be evaluated either from in-situ [1-2] or laboratory methods [3-4-5]. Given the theoretical relation between V_s and G_{max} , V_s also attracted many researchers and has been studied for years. It imposes only elastic shear deformation without volume change. Consequently, evaluating V_s can be considered a direct method to estimate material stiffness as it is directly related to the soil skeleton and not strongly affected by the presence of gas or water. The relation between G_{max} and V_s is usually represented as follow:

 $G_{\rm max} = \rho V_s^2$

where ρ is the density of soil.

Simultaneously, V_s also has a great role in soil characterization. It is considered a major design parameter in many geotechnical applications. Measuring V_s accurately makes it possible to analyze and design geotechnical structures, predict layer structure, bedrock position and degree of compaction, investigate ground reaction to earthquake, clarify geotechnical and mechanical characteristics of soil, estimate in situ soil density and assessment of liquefaction potential [6-7-8].

While V_s can be measured in laboratory using different testing methods such as Bender element (*BE*) and Resonant column (*RC*) tests. Some may question the accuracy of the results obtained from these tests. Several facts may affect the results of *BE* including:

- The soil is penetrated causing disturbance and the received waves are combination of shear wave, primary wave and reflections (Arulnathan et al. 1998, Lee and Santamarina 2005).
- It is hard to accurately know travel time and distance (which is related directly to shear wave) as it is difficult to detect the first arrival of shear wave and so, the exact travel time between input and output signals. There were also disagreements about the right distance to take as travel distance and whether to include the bender element or take the distance between them [11].
- It is hard to test dense or saturated materials in the bender element as it causes short circuits giving non-reliable results (BE must be waterproofed).
- Despite the importance of the BE method and the number of researchers using it, there is no accepted standard procedure for evaluating shear wave velocity nor an interpretation technique with a sufficient level of precision that can be adopted as a standard [12].

Moreover, several literature studies highlighted various sources of error in *RC* related to the interpretation method, equipment and specimen compliance, and non-uniform stress/strain distribution [13-14-15-16-17-18], also RC measures dynamic properties of soil only when shear strain amplitude is < 0.05% and can not be used with stiff specimens.

Therefore, this paper represents the development of P-RAT in order to reach more accurate results and to reduce the complications related to other techniques that measure V_s . Moreover, details of the developed interpretation method of output signals during V_s measurements were described.

PIEZOELECTRIC RING-ACTUATOR TECHNIQUE

The development of *P-RAT* in the geotechnical laboratory of Université de Sherbrooke was to reduce the complications related to other techniques that measure V_s (e.g., *BE* and *RC*) [19-20-21-22-23].

P-RAT contains two piezoelectric elements; one acts as an emitter and the other as a receiver. Both are fixed by a layer of silicone at the bottom and top heads of different conventional geotechnical apparatus. Inside the piezoelectric rings, a porous stone is set using special epoxy, as shown in Fig. 1. Transmitting different electrical voltage pulses to the emitter causes deformations in the inner stone, creating vibrations in the radial direction producing practically pure shear waves. The receiver diffuses data to the acquisition card in order to estimate the V_s . *P-RAT* was developed through the years to improve signal quality and interpretation method of output signals. Originally, the porous stone was not divided. However, with the continuous researches to generate a pure shear wave, it was divided into four pieces. With more investigations, the four pieces were kept radially in contact at their outer edges, as shown in Fig. 2, restraining the generation of any primary wave associated with radial deformation, reducing magnetic field and eliminating any longitudinal displacement [23]. More details about the methodology and the scientific development of *P-RAT* were provided by (Karray et al. 2015) [5]. The main reasons to believe that *P-RAT* is a powerful tool for measuring V_s are:

- *P-RAT* can be incorporated in traditional geotechnical apparatus (e.g., triaxial, RC, simple shear cell and oedometer cells) and could be used for testing different types of soils (e.g., granular, cohesive soils, rock fills).
- The *P-RAT* does not penetrate the soil, it minimizes the compression waves' energy, the near field effect is avoided, and it overcomes wave reflections at the top and bottom caps of the device.
- It minimizes apparatus effect by doing tip to tip test to evaluate the sensors and know how they interact with samples and applied pressure, generating a practically pure shear wave.
- The sensor's design with a large area of contact with the tested specimen provides results that represent the soil well and ensures axisymmetric distribution without favoring specific direction. Also, the stainless-steel encapsulation of the sensors absorbs the longitudinal expansion of the piezoelectric ring and offers a better distribution of the stresses onto the sample.



Fig. 1: Piezoelectric ring-actuator sensors (emitter or receiver) (after[5])

INTERPRETATION METHOD

A frequency domain method that could be used with different piezoelectric techniques (e.g., *P*-*RAT*, *BE*) was developed at the University of Sherbrooke [19-20-25-5-22-23]. The velocity that is determined from the travel time between emitted and received signals does not directly represent the shear wave velocity of soil but rather the phase velocity (V_{ph}) as it differs with the frequency. Any generated signal usually changes going through the dynamic system used. The dynamic system can be evaluated on the basis of the system's transfer function (*TF*), which is defined by its amplitude $A(\omega)$ and phase $\theta(\omega)$; where ω is the angular frequency.





Fig. 2: Improvement of porous stone of P-RAT (modified after [23-24])

In order to estimate accurate shear wave velocity of tested soil, the dynamic system used should not cause phase shift or amplification, but this is not possible. Theoretically, in a perfect dynamic system, $V_{ph} = V_s$ = constant for a chosen soil under the same testing conditions, applied stress and different frequency input. The interpretation method used depends on investigating system characteristics and minimizing dynamic system error in order to estimate accurate shear wave velocity of the tested sample.

It was assumed that the used dynamic system has a fixed phase-shift error function that does not depend on or get affected by the soil tested nor the signal transmitted. To estimate the sensor characteristics (resonant frequency and phase shift), a tip to tip test can be easily done estimating phase shift error as a function of the frequency (TF) that can be added or subtracted giving true V_s of soil where V_s , is one of the soil characteristics that should not vary with frequency [5].

A brief description of how the signal is usually being processed as shown in Fig. 3 [5-21-24]

- The received signal in the time domain, Fig. 3-a, is converted into frequency domain, Fig. 3-b, showing where the energy is located.
- Theoretical phase shift caused by each sensor behaves as a single degree of freedom system (*SDOF*), (solid line) Fig. 3-c, the characteristics of the sensor is estimated by doing tip to tip test (test with no soil sample only sensor on sensor) reaching the theoretical phase shift curve, (Red plotted curve) Fig. 3-c, the corrected phase shift, black plotted curve Fig. 3-c, is obtained by canceling phase shift generated by the sensor.
- The experimental dispersion curve returns to a constant value referring to the speed of the shear wave, *V*_s is constant and independent of the phase or the frequency content of the transmitted signal, Fig. 3-d.

The interpretation method gives clear shear waves as it minimizes near field effects, boundary effects and primary waves making it easy to determine the first arrival of the shear wave.



Fig. 3: Example of signal processing using P-RAT interpretation technique (modified after [21])

EXPERIMENTAL RESULTS AND DISCUSSION

Three granular soils were tested using P-RAT incorporated in an oedometer cell. The design of the cell used and the procedure are almost identical to the standard oedometer test Fig. 4. The diameter and height of the oedometer ring used are 63.5 mm and 18 mm respectively.

In the current study, four input signals with different time and frequency (Table 1) were used covering frequency bands below and above the fundamental frequency of the system and characterizing the emitter-receiver dynamic system in terms of resonance frequency and damping ratio. These signals were produced by an arbitrary waveform generator card by making a digital representation of the waveform. The input wave used is a sinusoidal signal to avoid the complications of square signals [26-27-28-29]. Fig. 5 shows a sample of input and output signals obtained during the P-RAT test. The frequency domain method which is developed at the University of Sherbrooke [5], was used in this research to analyze the P-RAT results.



Fig. 4: The oedometer apparatus equipped with P-RAT sensors (modified after [5])



Table 1: Input signals used in P-RAT tests, modified after [5]



Each soil is tested under eleven different stress conditions. After ensuring full consolidation of each increment, the four different input signals are generated, giving four received waves that can be analyzed using the interpretation method to obtain the V_s at every stress condition and subsequently, the G_{max} can be calculated using Eq.1.

The three granular materials tested were obtained from different regions of Quebec, Canada. The characteristics of the tested soils were estimated in the geotechnical laboratory at the University de Sherbrooke. The specific gravity (G_s), maximum void ratio(e_{max}), minimum void ratio (e_{min}), mean grain size (D_{50}), uniformity coefficient (C_u) and fine content (FC) were obtained (Table. 2) following the ASTM specification guidelines [30-31-32-33]. The tested soils' particle size distribution curves are shown in Fig. 6.

	B00-CF2	B00-CF5	Péribonka
Gs	2.77	2.82	2.82
D ₅₀ (mm)	0.220	0.165	0.285
C_u	3.38	2.60	4.71
e _{max}	1.136	1.290	0.702
e _{min}	0.530	0.626	0.369
FC (%)	9.80	13.00	4.81
Particle Shape	SA-A	SA	SA
Note: SA= Subangular & SA–A = Subangular to Angular			

Table 2: Characteristics of the used granular samples



Fig. 6: Particle size distribution curves

Various tests were carried out with different initial relative density values, deformation of tested soil and the corresponding V_s were estimated under different stresses using *P-RAT* incorporated in oedometer cell. Since V_s is a function of $\sigma'_{v'}$, where $n \approx 0.25$, in agreement with a majority of research in the previous studies, [4-34-35], V_s values were routinely stress normalized following numerous researchers [7-36-37-38] as:

 $V_{s1} = V_s \left(\frac{P_a}{\sigma_v}\right)^c$

where V_{s1} is the stress-normalized shear wave velocity, and P_a is the atmospheric reference pressure of 100 kPa. Fig. 7 represents the relation between the void ratio and the corresponding V_{s1} of tested soil. It is clearly observed that the void ratio is the major parameter that affects V_{s1} in agreement with previous studies [4-39-40-41-42].



Fig. 7: Normalized shear wave velocity (V_{s1}) as a function of void ratio for tested soils.

Figure 7 shows that the range of V_{S1} between the highest and lowest density state variance does not exceed 100 m/s which agrees with the field measurements of V_{S1} [38-43-44].

The obtained V_{s1} value was compared with the correlations proposed by Hussien and Karray [38]-Lashin et al. [42]. Hussien and Karray [38] proposed a correlation between V_{s1} and relative density for granular soils with (0.2 mm $\leq D_{50} \leq$ 10mm) based on V_{s1} values available in the literature as:

$$V_{s1} = 5.68[\ln(D_{50}) + 4.84]\sqrt{I_d + 25}$$

Also, Lashin et al. [42] correlated the V_{s1} to the void ratio based on P-RAT testes carried out on twenty-two different granular soil with (0.217 mm $\leq D_{50} \leq$ 0.6 mm) and (1.5 mm $\leq C_u \leq$ 250 mm) as:

$$V_{s1} = 370 D_{50}^{0.025} C_u^{-0.065} \exp^{\left[\frac{2}{10000} A_{2D}\right]} \exp^{\left[-1.035e\right]}$$

where A_{2D} is the two-dimensional angularity of the tested soils. The experimental results show good compatibility with the previous proposed correlations as shown in Fig. 8.

The two-dimensional angularity (A_{2D}), which represents the shape characteristics of soil, was determined based on the method developed by (Ghali et al. 2018, 2020) [45-46]. A sieve analysis test was done to segregate the soil samples, six specimens (20 grams each) were obtained arbitrary from every sieve size and scaled images of the chosen soil were taken by light stereomicroscope (Leica MZFL-III fluorescence stereo zoom microscope), resulting in more than 25 photographs for each soil sample. A sample of the digital images of tested soils is shown in Fig. 9. A_{2D} for each soil was then obtained based on the chart, represented in Fig. 10 proposed by Lees (1964a & 1964b) [47-48]. The mean A_{2D} of the tested soil sample was calculated from the percentage of particle angularities in each grain size range resulting in A_{2D} =750,675,620 for B00-CF2, B00-CF5 and Peribonka, respectively.



Fig. 8: Comparison between V_{s1} obtained from the experimental results, the correlation proposed by Hussien and Karray [38], and the correlation proposed by Lashin et al. [42].



B00-CF2



Peribonka







CONCLUSION

This paper represents one of the most simple and powerful techniques to estimate shear wave velocity, P-RAT. The development of this technique, as well as the new interpretation method, were explained in detail. Also, this study demonstrates the superiority of determining the small-strain shear modulus using the P-RAT over the other conventional techniques. Moreover, the V_s of three granular soil were obtained using the P-RAT incorporated in the conventional oedometer cell, then the relationship between the void ratio (e) and V_{s1} was plotted for the tested soil. Finally, the V_{s1} results were compared with several V_{s1} - e proposed correlations, available in the literature, which showed acceptable compatibility.

REFERENCES

- 1. Campanella, R. G., Robertson, P. K., & Gillespie, D. (1986), "Seismic cone penetration test. In Use of in situ tests in geotechnical engineering", ASCE, pp. 116-130.
- 2. Hryciw, R.D. (1990), "Small-strain-shear modulus of soil by dilatometer", Journal of Geotechnical Engineering, ASCE, Vol. 116(11), pp. 1700-1716.
- Brignoli, E.G.M., M Gotti, & K.H. Stokoe. (1996), "Measurement of Shear Waves in Laboratory Specimens by Means of Piezoelectric Transducers", *Geotechnical Testing Journal*, Vol. 19(4), pp. 384–97.
- 4. Hardin, B. O., & Jr F E Richart. (1963), "Elastic Wave Velocities in Granular Soils", *Journal of the Soil Mechanics and Foundations, Vol.* 89(SM1), pp. 33–63.
- 5. Karray, M., Ben Romdhan, M., Hussien, M. N., and Éthier, Y. (2015), "Measuring shear wave velocity of granular material using the piezoelectric ring-actuator technique (P-RAT)", *Canadian Geotechnical Journal, Vol. 52*(9), pp. 1302-1317.
- 6. Kramer, Steven L. (1996), "Geotechnical Earthquake Engineering", Environmental & Engineering Geoscience.
- Youd, T. L., & Idriss, I. M. (2001), "Liquefaction resistance of soils: summary report from the 1996 NCEER and 1998 NCEER/NSF workshops on evaluation of liquefaction resistance of soils", *Journal of geotechnical and geoenvironmental engineering*, Vol. 127(4), pp. 297-313.
- Andrus, R. D., & Stokoe II, K. H. (2000), "Liquefaction resistance of soils from shear-wave velocity", *Journal of geotechnical and geoenvironmental engineering, Vol.* 126(11), pp. 1015-1025.
- 9. Arulnathan, R., R. W. Boulanger, and M. F. Riemer. (1998), "Analysis of Bender Element Tests", *ASTM geotechnical testing journal, Vol.* 21(2), pp. 120–31.
- Lee, Jong Sub, and J. Carlos Santamarina, 2005, "Bender Elements: Performance and Signal Interpretation", *Journal of Geotechnical and Geoenvironmental Engineering*, Vol. 131(9), pp. 1063–70.
- 11. Dyvik, R., and C. Madshus. (1985), "Lab Measurements of Gmax Using Bender Elements", In Advances in the Art of Testing Soils Under Cyclic Conditions, Michigan, pp.186–96.
- 12. Camacho-Tauta, Javier et al. 2011, "Measurements of Shear Wave Velocity by Resonant-Column Test, Bender Element Test and Miniature Accelerometers", *Proceedings of the 2011 Pan-Am Geotechnical Conference* (1998), pp. 1–9.
- 13. Richart, F. E., Hall, J. R., and Woods, R. D. (1970), "Vibrations of soils and foundations", Prentice-Hall, Englewood Cliffs, N.J., 414.
- 14. Drnevich, V. P., Hardin, B. O., & Shippy, D. J. (1978), "Modulus and damping of soils by the resonant-column method', In *Dynamic geotechnical testing*, ASTM International.
- 15. Isenhower, W. M. (1979), "Torsional simple shear/resonant column properties of San Francisco Bay mud", PhD thesis, *The Univ. of Texas at Austin.*
- 16. Ishihara, K. (1996), "Soil behavior in earthquake geotechnics", Clarendon Press, Oxford University Press Inc., New York.
- 17. Sasanakul, Inthuorn, and James A. Bay. (2008), "Stress Integration Approach in Resonant Column and Torsional Shear Testing for Soils", *Journal of Geotechnical and Geoenvironmental Engineering, Vol.* 134(12), pp. 1757–62.
- 18. Bui, Man T. (2009), "Influence of Some Particle Characteristics on the Small Strain Response of Granular Materials", PhD thesis, *University of Southampton*.

- 19. GamalEI-Dean, (2007), "Development of a New Piezoelectric Pulse Testing Device and Soil Characterization Using Shear Waves", Université de Sherbrooke.
- 20. Ethier, Yannic A. (2009), "La mesure en laboratoire de la vitesse de propagation des ondes de cisaillement", PhD thesis, *Université de Sherbrooke*.
- 21. Karray, Mourad et al. (2019), "The Piezo-Electric Ring Actuator Technique (P-RAT) 16 Years of Progress", In *Geo st'Johns conference* St. Johns, NF, Canada.
- Mhenni, A., Hussien, M. N., & Karray, M. (2015), "Improvement of the Piezo-electric Ring Actuator technique (P-RAT) using 3D numerical simulations", In 68e Conférence Canadienne de Géotechnique et 7e Conférence Canadienne sur le Pergélisol, Vol. 9, pp. 2015.
- 23. Mhenni, Ahmed, Mahmoud Nasser Hussien, M. Karray, & Y.A. Ethier. (2016), "Versatility of the P-RAT for Shear Wave Velocity Measurement", *Geo Vencouver*.
- 24. Hussien, M. N., & Karray, M. (2020), "Piezoelectric ring-actuator technique (P-RAT): in-depth scrutiny of interpretation method", ASTM Geotechnical Testing Journal. doi:10.1017/S0007114518003884.
- 25. Ben Romdhan, M., Hussien, M.N., and Karray, M. (2014), "The Use of Piezoelectric Ring-Actuator Technique (P-RAT) in Shear Wave Velocity Measurement in Granular Media", GeoRegina2014, (SEPTEMBER). doi:10.13140/2.1.5060.3526.
- 26. Leong, E. C., Yeo, S. H., & Rahardjo, H. (2005), "Measuring shear wave velocity using bender elements", Geotechnical Testing Journal, Vol. 28(5), pp. 488-498.
- 27. Jovičić, Vojkan, M. R. Coop, and M. Simić, "Objective criteria for determining G max from bender element tests", *Geotechnique* 46, Vo. 2 (1996), pp. 357-362.
- 28. Viggiani, G., and Atkinson, J. H. (1995), "Interpretation of bender element tests", Ge^otechnique, London, Vol. 45(1), pp. 149–154.
- 29. Blewett, J., Blewett, I. J., & Woodward, P. K. (2000), "Phase and amplitude responses associated with the measurement of shear-wave velocity in sand by bender elements", *Canadian Geotechnical Journal, Vol.* 37(6), pp. 1348-1357.
- 30. ASTM D2487-11 (2011), "Standard Practice for Classification of Soils for Engineering Purposes (Unified Soil Classification System)", ASTM International, West Conshohocken, PA,
- 31. ASTM D4253-14 (2014), "Standard Test Methods for Maximum Index Density and Unit Weight of Soils Using a Vibratory Table", ASTM International, West Conshohocken, PA,
- 32. ASTM D4254-14 (2014), "Standard Test Methods for Minimum Index Density and Unit Weight of Soils and Calculation of Relative Density", ASTM International, West Conshohocken, PA,
- 33. ASTM D854-14 (2014), "Standard Test Methods for Specific Gravity of Soil Solids by Water Pycnometer", ASTM International, West Conshohocken, PA.
- 34. Hardin, B. O., & Black, W. L. (1966), "Sand Stiffness Under Various Triaxial Stresses", Journal of the Soil Mechanics and Foundations, Vol. 92(2), pp. 27-42.
- 35. Hardin, B.O., & V.P. Drnevich. (1972), "Shear Modulus and Damping in Soils: Measurement and Parameter Effect", *Journal of the Soil Mechanics and Foundations, Vol.* 96(SM7), pp. 667–92.
- 36. Robertson, P.K., Woeller, D.J., Kokan, M., Hunter, J., & Luternaur, J. (1992), "Seismic techniques to evaluate liquefaction potential", In *Proceedings of the 45th Canadian geotechnical conference*, Toronto, Ont., pp. 5-1–5-9.
- Karray, Mourad, Guy Lefebvre, Yannic Ethier, & Annick Bigras. (2011). Influence of Particle Size on the Correlation between Shear Wave Velocity and Cone Tip Resistance. Canadian Geotechnical Journal 48(4): 599–615.
- 38. Hussien, M. N., & Karray, M. (2016), "Shear Wave Velocity as a Geotechnical Parameter: An Overview", *Canadian Geotechnical Journal, Vol.* 53(2), pp. 252–72.
- Iwasaki, T., & F. Tatsuoka. (1977), "Effect of Grain Size and Grading on Dynamic Shear Moduli of Sand", Soils and Foundations, Vol. 17(3), pp. 19–35.
- 40. Hussien, M. N, & Karray, M. (2013), "Grain-Size Distribution Effects on the Mechanical Behaviour of Granular Materials", *GeoMontreal 2013*.
- 41. Lashin, I., Ghali, M, & Karray, M Mourad Karray. (2019), "A Laboratory-Based Study Correlating the Large-Strain Static Constrained Modulus to the Small-Strain Shear Modulus for Clean Sands", *Geo St.John's* 2019, St.John's, NL, Canada.
- Lashin, I., Ghali, M., Hussien, M. N., Chekired, M., & Karray, M. (2021), "Investigation of smallto large-strain moduli correlations of normally consolidated granular soils", *Canadian Geotechnical Journal*, Vol. 58(1), pp.1-22. DOI: 10.1139/cgj-2019-0741.

- 43. Lefebvre, G., & Karray, M. (1998) "New developments in in-situ characterization using Rayleigh waves", In *Proceedings of the 51st Canadian Geotechnical Conference, Edmonton, Alta*, pp. 4-7.
- 44. Karray, M., & Lefebvre, G. (2008), "Significance and evaluation of Poisson's ratio in Rayleigh wave testing", *Canadian Geotechnical Journal*, Vol. *45*(5), pp. 624-635.
- 45. Ghali, M., Chekired, M., & Karray, M. (2018), "Framework to improve the correlation of SPT-N and geotechnical parameters in sand", *Acta Geotechnical*, pp. 1-25.
- 46. Ghali, M., Chekired, M., & Karray, M. (2020), "Laboratory Simulator for Geotechnical Penetration Tests", *Geotechnical Testing Journal*, Vol. 43(1), pp. 211-234.
- 47. Lees, G., (1964) a, "A new method for determining the angularity of particles", *Sedimentology*, Vol., 3, pp. 2-21.
- 48. Lees, G., (1964) b, "The measurement of particle shape and its influence in engineering materials", *British Granite Whinstone Federation*, Vol. 4, No. 2, pp. 17-38.