Arab J. Nucl. Sci. Appl., Vol. 56, 3, 95-104 (2023)

ISSN 1110-0451

Arab Journal of Nuclear Sciences and Applications

Web site: ajnsa.journals.ekb.eg

(E S N S A)

Influence of Symmetry Energy on Neutron Skin Thickness of ^{40}Ca, ^{48}Ca, ^{90}Zr, ^{116}Sn, ^{144}Sm and ^{208}Pb

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ARTICLE INFO

Article history:

Received: 8th Sept. 2022

Accepted: 7th Jan. 2023

Keywords:

Hartree-Fock HF;

Skyrme interaction;

Neutron Skin Thickness.

ABSTRACT

In the present work, the neutron skin thickness of neutron-rich closed-shell nuclei ^{40}Ca, ^{48}Ca, ^{90}Zr, ^{116}Sn, ^{144}Sm and ^{208}Pb, has been calculated using self-consistent Hartree-Fock (HF) with 20 types of Skyrme interactions: SKM*, SQMC650, SV-K218, SQMC700, SkO, NRAPR, KDE0, T31, Skχm*, v075, SkSC10, RATP, SAMi, SK255, SkD, GS2, SIV, QMC2 and SIII of deferent nuclear matter properties. Neutron skin thickness (R_{skin}) can be obtained through the proton and neutron density distribution which is the difference between the neutron and proton root-mean-square radii, and depends on the properties of the nuclear matter. The authors examined the sensitivity of neutron skin thickness (R_{skin}) to nuclear matter properties, such as symmetry energy density J (MeV) and the symmetry energy coefficient L (MeV), and deduced the value of neutron skin thickness (R_{skin}) as function of isospin asymmetric ratio $I = (N - Z)/A$ as well. The present work was supported by statistical calculations and determining and discussion the Pearson linear correlation coefficient.

1. INTRODUCTION

Theoretical nuclear physics aims to study the structure of atomic nuclei (which considered as many-body system) which has been founded since the formulation of quantum mechanics [1], where the many-body systems (in all fields of physics) is one of the most challenging tasks of theoretical physics [2] and the nature of the nuclear force facing theoretical nuclear physicists. The distribution of protons and neutrons inside a nucleus is one of the basic and fundamental problems in nuclear structure to demonstrate the success of the theory [3, 4].

In stable nuclei, charge distributions can be measured experimentally by electron elastic scattering and muonic X-ray (charge-sensitive) [5] and information about the distributions of charge can be obtained with high accuracy. The density distributions of neutron and proton have a similar shape in stable nuclei, while it has a different shape in some unstable or neutron-rich nuclei [6]. Information of electromagnetic interaction about neutron density distributions is little; therefore, neutron density distributions is difficult to deduce. Neutron density distributions is closely related to the symmetry terms of the equation of state (EOS) [7, 8].

Experimentally, also nuclear charge radii can be obtained using the electromagnetic interaction between the nucleus and electrons or muons, which describes the impact of the effective interactions on nuclear structure [9, 10]. Neutron skin thickness (R_{skin}) can be expressed through the proton and neutron density distribution and radii which definitely depends on the properties of the nuclear surface.

Neutron skin in nuclei has received a considerable attention in from many groups of research [11-13], it depends on the balance among the various nuclear

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DOI: 10.21608/ajnsa.2023.161509.1635

matter properties [4]. As neutron number increases, the radius of the neutron density distribution becomes larger than that of the protons, reflecting the pressure of the symmetry energy. A relationship between the symmetry energy and the neutron-skin thickness has actively been discussed by several theoretical works [14–19]. The determination of the neutron skin thickness of neutron-rich nuclei is one of the main science drivers of the Facility for Rare Isotope Beams (FRIB) [20].

This work aims to examine the sensitivity of neutron skin *(R_{skin})* to symmetry energy such as symmetry energy density J (MeV) and the symmetry energy coefficient L (MeV). The calculations is performed based on the self-consistent Hartree-Fock (HF) method with 20 Skyrme-type interactions.

2. THEORETICAL FRAMEWORK

The Skyrme effective interaction (of parameters $t_{\circ}.t_1.t_2.t_3.x_{\circ}.x_1.x_2.x_3$, W_o and α) is one of the most convenient forces used in HF calculations to describe the nuclear probertite in ground-state. [21-25]:

$$V(r_1,r_2) = t_0(1+ x_0\, P_\sigma)\delta(r_1 - r_2)$$

$$+\tfrac{1}{2}t_1(1 + x_1 P_{12}^{\alpha}) \times \left[\overleftarrow{k}_{12}^{2}\delta(r_1 - r_2) + \delta(r_1 - r_2)\vec{k}_{12}^{2}\right] \text{central term}$$

$$+ t_2(1 + x_2 P_{12}^{\alpha})\overleftarrow{k}_{12}\delta(r_1 - r_2)\vec{k}_{12} \qquad \text{non-local term}$$

$$+\tfrac{1}{6}t_3\,(1 + x_3 P_{12}^{\alpha})\rho^{\alpha}(R)\delta(r_1 - r_2) \quad \text{density-dependent term}$$

$$+i\,W_0\overleftarrow{k}_{12}\delta(r_1 - r_2)(\vec{\sigma}_1 + \vec{\sigma}_2)\,\vec{k}_{12} \qquad \textbf{(1)}$$

where $\vec{\sigma}_i$ is the Pauli spin operator $\overleftarrow{k}_{12} = -\frac{i(\vec{\nabla}_1 - \vec{\nabla}_2)}{2}$, $\vec{k}_{12} = -\frac{i(\overleftarrow{\nabla}_1 - \overleftarrow{\nabla}_2)}{2}$, P_{12}^{α} is the spin-exchange operator. [26]

The total energy E of HF equations based on Skyrme's interaction, can be calculated through applying the variational principle to an ansatz (trial wave function) as a product of single-particle functions φ [27],

$$\langle \delta\varphi | H(r) | \varphi \rangle = 0 \qquad \textbf{(2)}$$

where the Skyrme energy-density functional $H(r)$ (in terms of a kinetic-energy K term, a zero-range H_0 term, a density-dependent H_3 term, an effective-mass H_{eff} term, a finite-range H_{fin} term, a spin-orbit term H_{so}, a tensor coupling with spin and gradient H_{sg} term and Coulomb interaction H_{coul} term) is given by [27, 28],

$$H = K + H_0 + H_3 + H_{eff} + H_{fin} + H_{so} + H_{sg} + H_{coul} \quad (\mathbf{3})$$

The charge density distribution (in terms of Skyrme HF radial wave function and occupation probability η) of the ground state of a nucleus can be obtained [29-31],

$$\rho_q\,(r) = \frac{1}{4\pi}\sum_{n\ell j}\eta_q\,(2J+1)\left|\frac{u(n\ell j)}{r}\right|^2 \qquad (\mathbf{4})$$

Where q represents neutron or proton of the state $n\ell j$. The root-mean-square (rms) radii is defined as,

$$r_q = \langle r_q^2\rangle^{1/2} = \left[\frac{\int r^2\,\rho_q(r)dr}{\int \rho_q\,(r)dr}\right]^{1/2} \qquad (\mathbf{5})$$

$$\langle r_{ch}^2 \rangle = \langle r_p^2\rangle + \langle r\rangle_p^2 \;;\; \text{rms radius } \langle \mathrm{r}\rangle_{\mathrm{p}} = 0.8 \text{ fm} \qquad (\mathbf{6})$$

The neutron skin thickness (*R_{skin}*) is obtained by,

$$R_{skin} \equiv r_n - r_p \qquad \textbf{(7)}$$

3- RESULT AND DISCUSSION

There are various iteration techniques used in solving Hartree problems [32]. Here, the equations of the static HF were solved numerically within a model space based on 20 Skyrme-type interactions by using the Numerov method. Nuclear matter properties of the investigated Skyrme interactions such as symmetry energy density J (MeV) and the symmetry energy coefficient L (MeV), are presented in Table (1)

Table (1): Nuclear matter properties of the investigated Skyrme interactions [33]

Type	Symmetry Energy Density J (MeV)	Symmetry Energy Coefficient L (MeV)
SkM* [34]	30.03	45.78
SQMC650 [35]	33.65	52.92
SV-K218 [36]	30.00	34.62
SQMC700 [35]	33.47	59.06
SkO [37]	31.97	79.14
NRAPR [38]	32.78	59.63
KDE0 [39]	33.00	45.22
T31 [40]	32.00	49.75
Skχm* [41]	30.94	45.6
v075 [42]	28.00	-0.31
SkSC10 [43]	22.83	19.13
RATP [44]	29.26	32.39
SAMi [45]	28.00	44.00
SK255 [46]	37.40	95.05
SkD [47]	30.81	78.15
GS2 [48]	25.96	30.27
SIV [49]	31.22	63.50
QMC2 [50]	28.70	8.67
SIII [48]	28.16	9.91
SVII [51]	26.96	-10.16

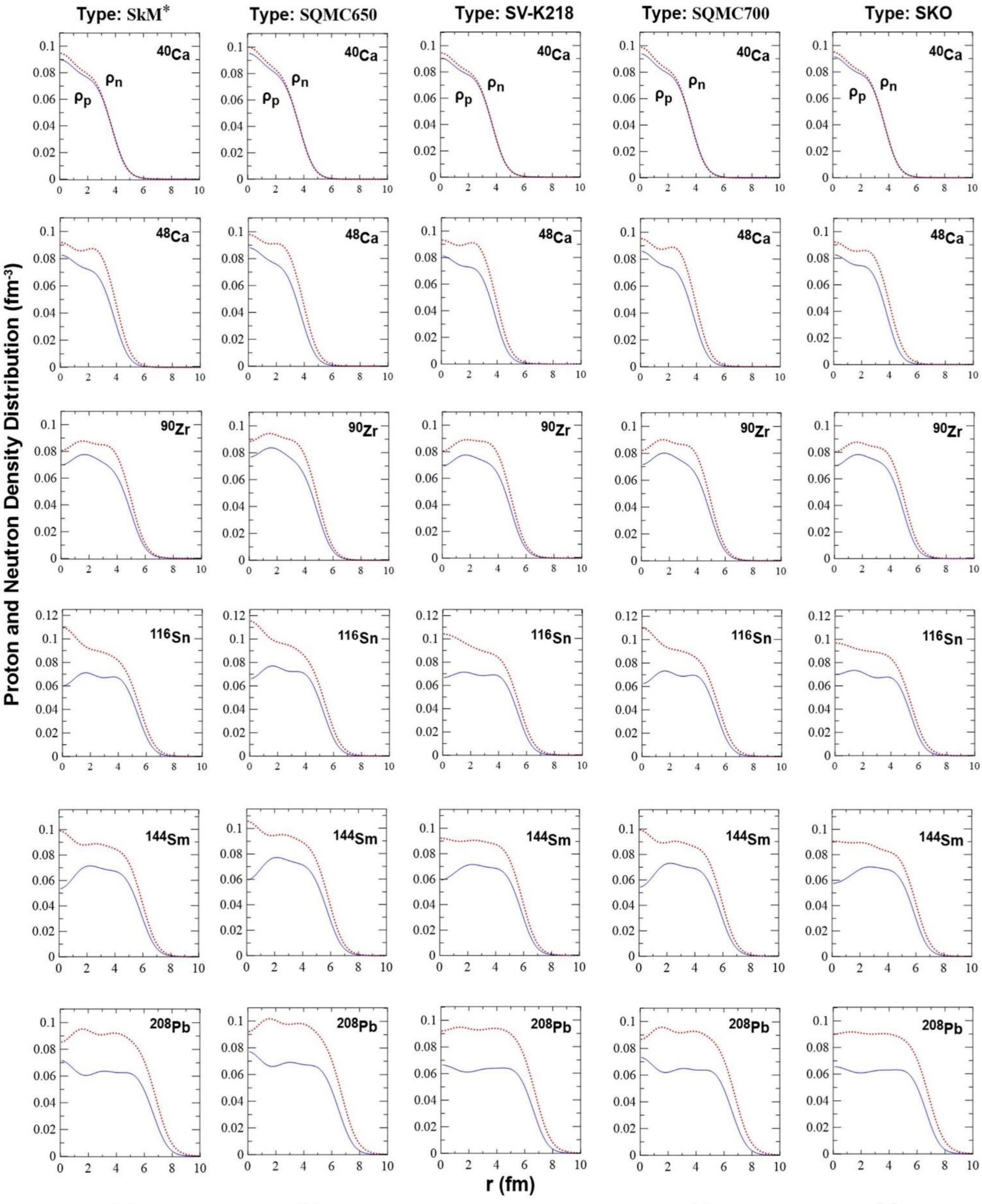
In this work, the HF calculations have been implemented for ^{40}Ca, ^{48}Ca, ^{90}Zr, ^{116}Sn, ^{144}Sm and ^{208}Pb nuclei. The behaviors and differences between neutron and proton density distributions are depicted in Figs. (1-4) for the investigated nuclei with 20 types of Skyrme interaction, the neutron densities are larger than the proton densities with some regularity. The distribution of neutrons increases differently from the distribution of protons as the difference between neutrons and protons increases towards heavy nuclei, especially in the interior of the nuclei, a similar behavior can be found in several previous publications[52-53].

The neutron skin thickness R_{skin} is useful observable for providing information about the nuclear shape and analyzing nuclear structure [32], also, it is a good indicator of nuclear properties and depends on neutron number to proton number and the nuclear symmetry energy that characterizes the equation of state of neutron-rich matter [17]. Additionally, it was found to strongly correlate with other observables in nuclei [55-62].

Each nucleus has its specific structure which is more important than any general trend. In the present study, it was observed that the size of neutron skin thickness regularly increases when the value of isospin asymmetric ratio $\mathrm{I} = (\mathrm{N} - \mathrm{Z})/\mathrm{A}$ ratio increases with some exception at ^{144}Sm as presented in Fig. (5), also it can be seen that the skin has a good correlation with I and the Pearson linear correlation reach C= 0.794**.

Fig. (6). illustrates R_{skin} as a function of symmetry energy density J and symmetry energy slope L for the investigated nuclei. The linear correlation coefficients of R_{skin} versus J within the skyrme forces reaches C= 0.633**, C= 0.684**, C= 0.733**, C= 0.897**, C= 0.592** and C=0.373 for ^{40}Ca, ^{48}Ca, ^{90}Zr, ^{116}Sn, ^{144}Sm and ^{208}Pb, respectively. From relation between J and R_{skin}, it was found that the linear correlation coefficients of R_{skin} versus J are strong in ^{40}Ca, ^{48}Ca, ^{90}Zr, ^{116}Sn, ^{144}Sm, respectively and weak in ^{208}Pb. This illustrates that the quantities J and R_{skin} are strongly coupled by the Skyrme forces in ^{40}Ca, ^{48}Ca, ^{90}Zr, ^{116}Sn and ^{144}Sm, but weak for ^{208}Pb.

The linear correlation coefficients of R_{skin} versus L within the Skyrme forces reach C= 0.372, C= 0.841**, C= 0.837**, C= 0.873**, C= 0.688** and C=0.487* for ^{40}Ca, ^{48}Ca, ^{90}Zr, ^{116}Sn, ^{144}Sm and ^{208}Pb, respectively. From the relation between L and R_{skin}, it was found that the linear correlation coefficients of R_{skin} versus L are strong in ^{48}Ca, ^{90}Zr, ^{116}Sn and ^{144}Sm, and weak in ^{40}Ca and ^{208}Pb. This illustrates that the quantities L and R_{skin} are strongly coupled by the Skyrme forces in ^{48}Ca, ^{90}Zr, ^{116}Sn and ^{144}Sm ,but weak for ^{40}Ca and ^{208}Pb.



(a) (b) (c) (d) (e)

Fig. (1): The calculated proton and neutron density distribution (fm^{-3}) of the investigated nuclei with Skyrme interactions: SkM*, SQMC650, SV-K218, SQMC700 and SKO. Proton densities are plotted by blue solid line and the red dotted line shows neutron densities

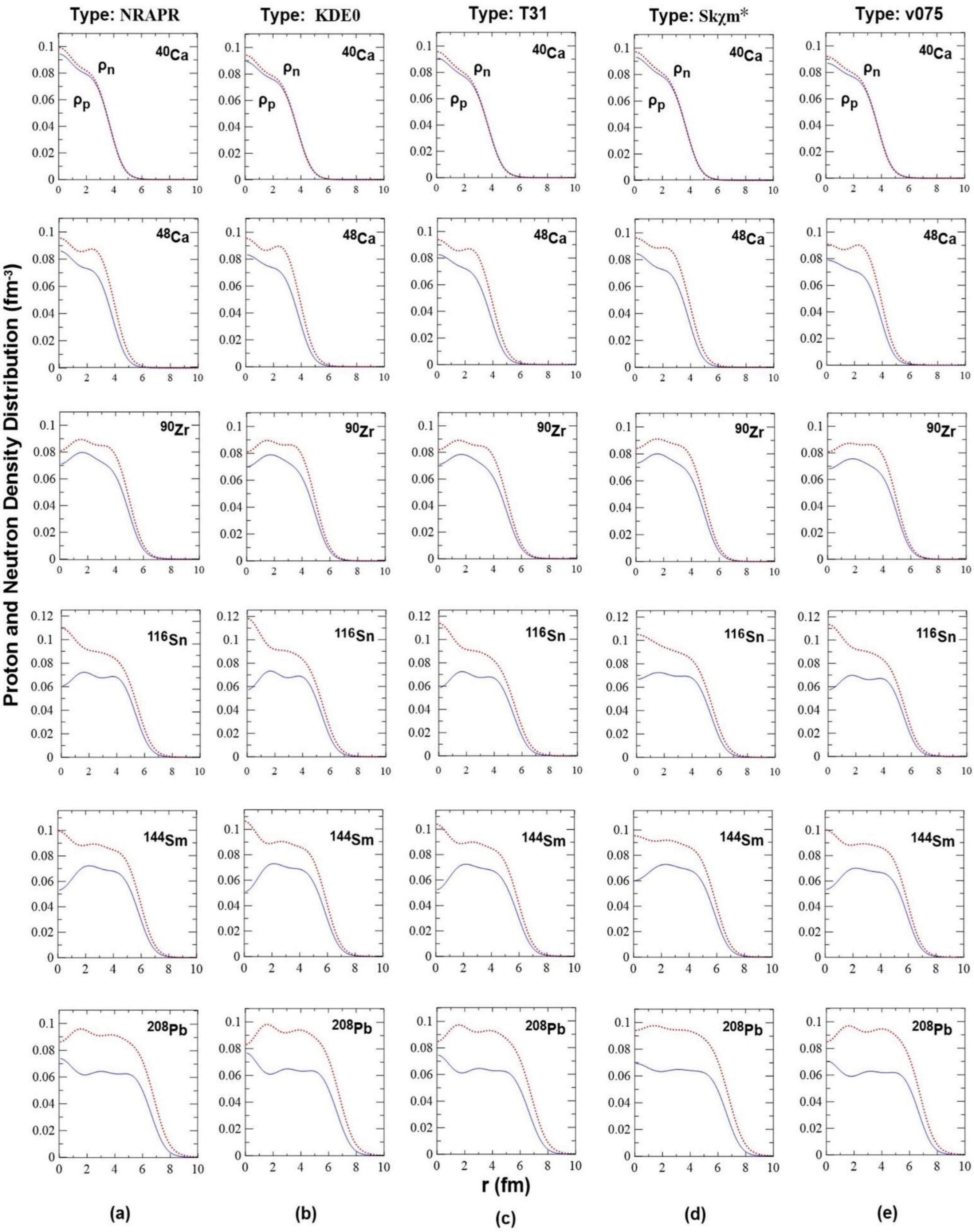


Fig. (2): The calculated proton and neutron density distribution (fm^{-3}) of the investigated nuclei with Skyrme interactions: NRAPER, KDE0, T31, Skχm* and v075. Proton densities are plotted by blue solid line and the red dotted line shows neutron densities

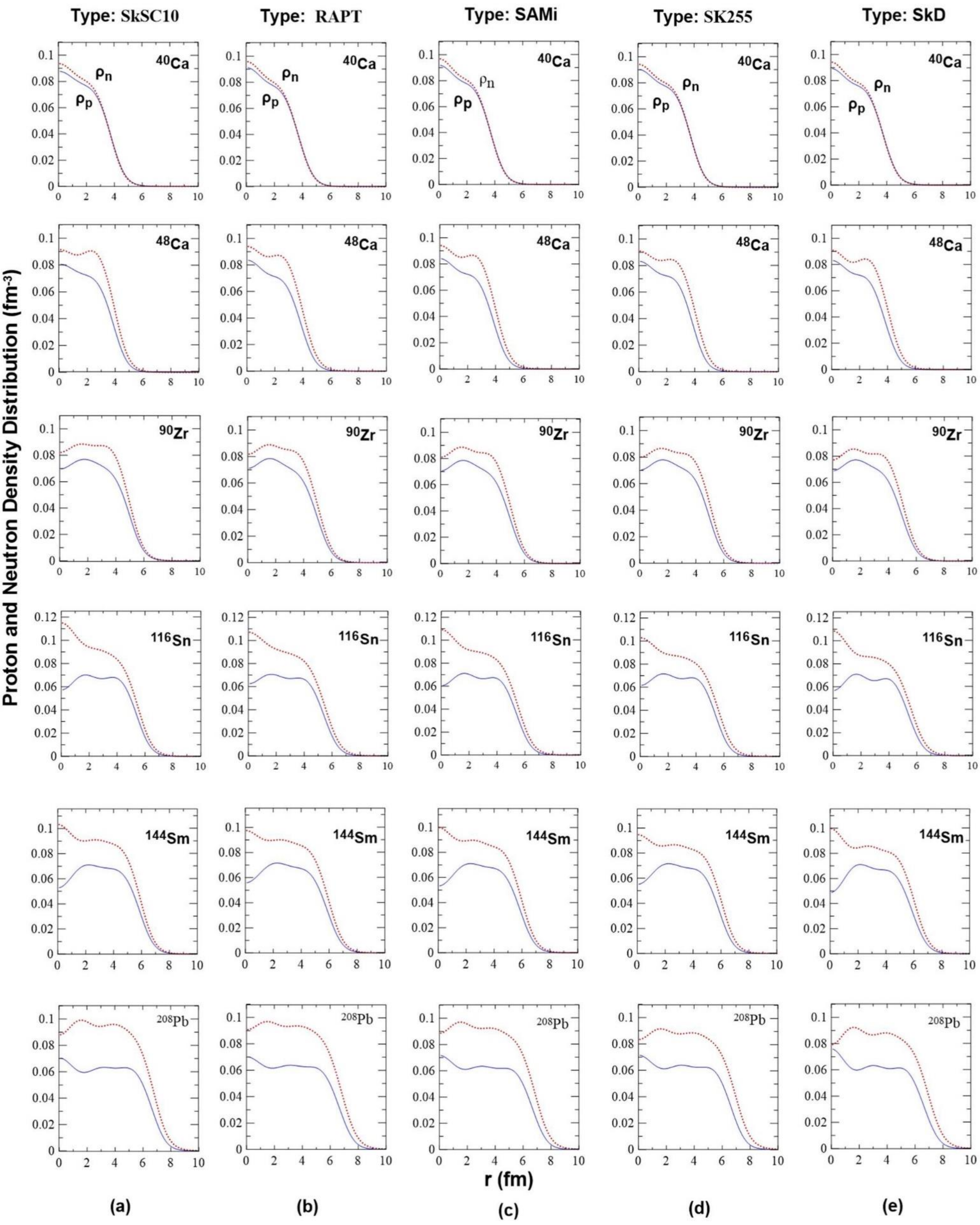


Fig. (3): The calculated proton and neutron density distribution (fm^{-3}) of the investigated nuclei with Skyrme interactions: SKSC10, RATP, SAMi, SK255 and SKD. Proton densities are plotted by blue solid line and the red dotted line shows neutron densities

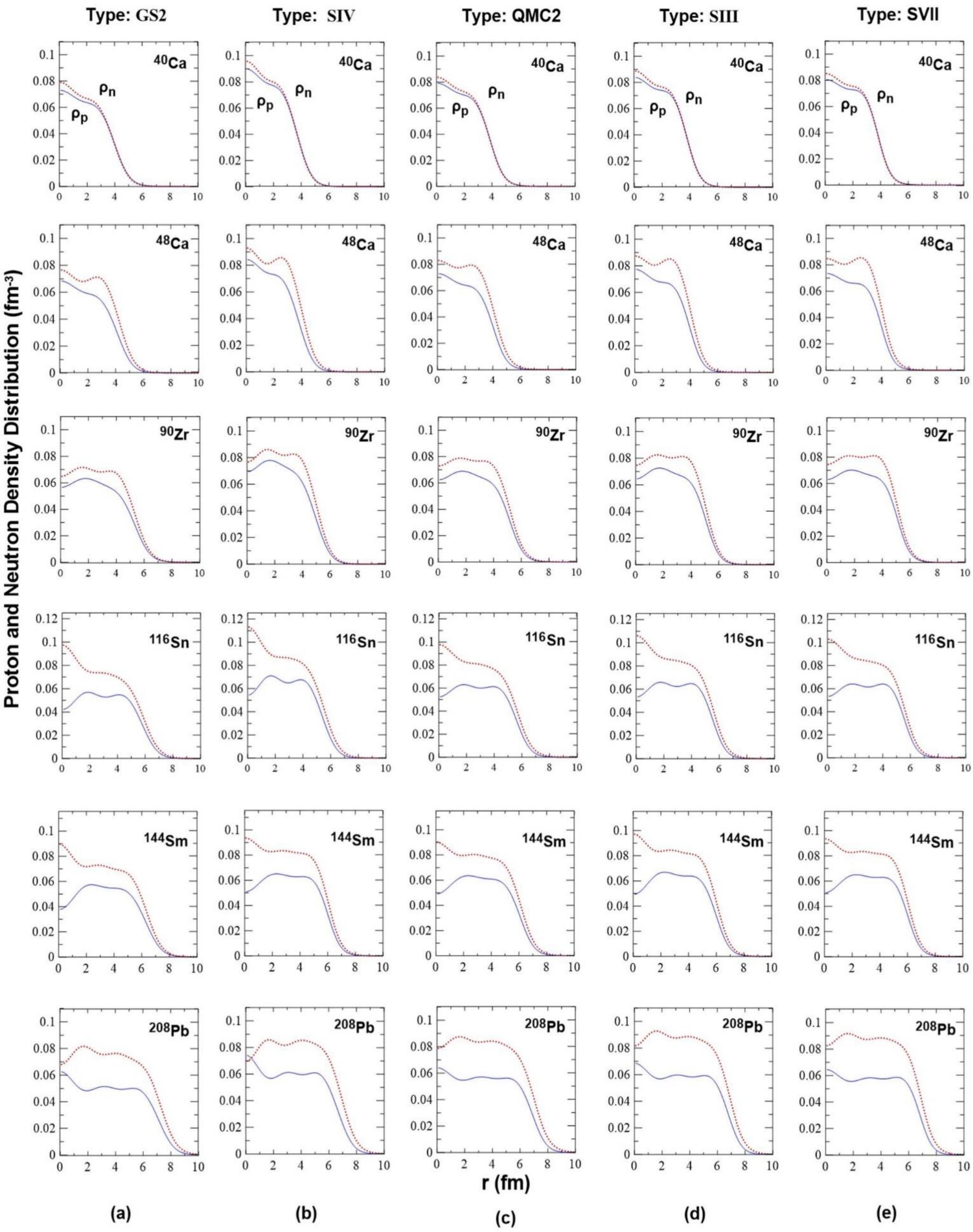


Fig. (4): The calculated proton and neutron density distribution (fm^{-3}) of the investigated nuclei with Skyrme interactions: GS2, SIV, QMC2, SIII and SVII. Proton densities are plotted by blue solid line and the red dotted line shows neutron densities

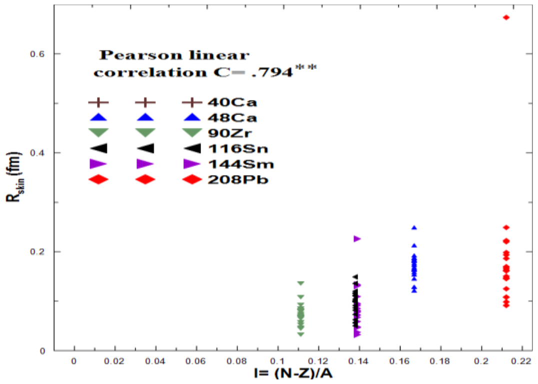


Fig. (5): R_{skin} deduced in this work for the investigated nuclei with (20) Skyrme-type parameter vs isospin asymmetric ratio $I = (N - Z)/A$

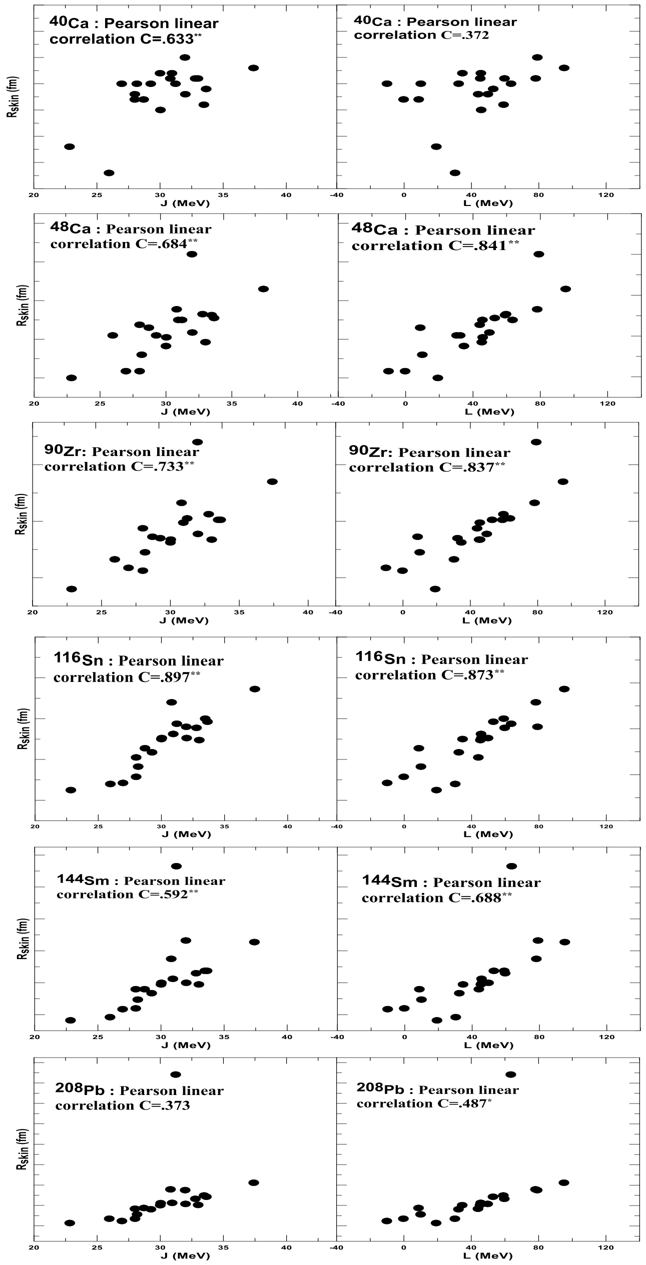


Fig. (6): R_{skin} (fm) vs. symmetry energy density J and symmetry energy slope L for ^{40}Ca, ^{48}Ca, ^{90}Zr, ^{116}Sn, ^{144}Sm and ^{208}Pb nuclei

4. CONCLUSIONS

In the present study, the R_{skin} of neutron-rich closed shell nuclei ^{40}Ca, ^{48}Ca, ^{90}Zr, ^{116}Sn, ^{144}Sm and ^{208}Pb have been studied using the self-consistent Hartree-Fock (HF) method with 20 Skyrme type effective nucleon-nucleon interaction, where its sensitivity to nuclear matter properties, such as symmetry energy density J (MeV) and the symmetry energy coefficient L (MeV) was examined by determining the Pearson linear correlation coefficient between the calculated neutron skin (R_{skin}) and the mentioned NM properties. The neutron skin thicknesses were increased regularly when the value of isospin asymmetric $I = (N - Z)/A$ isospin asymmetric ratio increases with some exception at ^{144}Sm. The correlation ($R_{skin} \leftrightarrow J$) is strongly coupled by the Skyrme forces in ^{40}Ca, ^{48}Ca, ^{90}Zr, ^{116}Sn and ^{144}Sm, but it was weak for ^{208}Pb. On the other hand, the correlation ($R_{skin} \leftrightarrow L$) is strongly coupled by the Skyrme forces in ^{48}Ca, ^{90}Zr, ^{116}Sn and ^{144}Sm, but it was weak for ^{40}Ca and ^{208}Pb.

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