# NEW UNIFIED INTEGRALS INVOLVING A SRIVASTAVA POLYNOMIALS AND $\bar{H}$-FUNCTION 

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#### Abstract

The aim of the present paper is to study some new unified integral formulas (IF) associated with the $\bar{H}$-function, which was introduced by InayatHussain. Each of these formula involves a product of the $\bar{H}$-function and Srivastava polynomials with essentially arbitrary coefficients and the value of the formulas are obtained in terms of $\psi(z)$ (the logarithmic derivative of $\Gamma(z)$ ). By assigning suitably special values to these coefficients, the main results can be reduced to the corresponding integral formulas involving the classical orthogonal polynomials including, for example, Hermite, Jacobi, Legendre and Laguerre polynomials. Furthermore, the $\bar{H}$-function occurring in each of our main results can be reduced, under various special cases, to such simpler functions as the generalized Wright hypergeometric function and generalized WrightBessal function. A specimen of some of these interesting applications of our main integral formulas is presented briefly.


## 1. Introduction

The Srivastava polynomials $S_{n}^{m}[\mathrm{x}]$ will be defined and represented as follows ([1], p.1, Eq.1):

$$
\begin{equation*}
S_{n}^{m}[x]=\sum_{k=0}^{[n / m]} \frac{(-n)_{m k}}{k!} A_{n, k} x^{k},(n=0,1,2 \ldots) \tag{1}
\end{equation*}
$$

where m is an arbitrary positive integers and the coefficients $A_{n, k}(n, k \geq 0)$ are arbitrary constants, real or complex. On suitably specializing the coefficients $A_{n, k}$, $S_{n}^{m}[x]$ yields a number of known polynomials as its special cases. These include, among others, the Hermite polynomials, the Jacobi polynomials, the Laguerre polynomials, the Bessel's polynomials and several others (see for details, [2] and [3]).

On account of the general nature of our main integrals a large number of new and known integrals including an important result obtained by Garg and Mittal [4] and other integrals involving simpler Special functions and Polynomials, follow as its special cases. Next, we have obtained here four special cases of our main integral formulas (IF) that involves generalized Wright hypergeometric function, generalized Wright-Bessal function, the Hermite polynomials and the Laguerre polynomials,

[^0]which are also believed to be new.
The $\bar{H}$-function, due to Inayat-Hussain [5, 6] which is a generalization of familiar Fox H- function, contains, as special cases is given by Buschman and Srivastava [7]:
\[

$$
\begin{align*}
\bar{H}_{P, Q}^{M, N}[z] & =\bar{H}_{P, Q}^{M, N}\left[\left.z\right|_{\left(b_{j}, B_{j}\right)_{1, M},\left(b_{j}, B_{j} ; \beta_{j}\right)_{M+1, Q}} ^{\left(a_{j}, A_{j} ; \alpha_{j}\right)_{1, N},\left(a_{j}, A_{j}\right)_{N+1, P}}\right]  \tag{2}\\
& =\frac{1}{2 \pi i} \int_{-i \infty}^{+i \infty} z^{\xi} \bar{\varphi}(\xi) d \xi .(z \neq 0) \tag{3}
\end{align*}
$$
\]

where

$$
\begin{equation*}
\bar{\varphi}(\xi)=\frac{\prod_{j=1}^{M} \Gamma\left(b_{j}-B_{j} \xi\right) \prod_{j=1}^{N}\left\{\Gamma\left(1-a_{j}+A_{j} \xi\right)\right\}^{\alpha_{j}}}{\prod_{j=M+1}^{Q}\left\{\Gamma\left(1-b_{j}+B_{j} \xi\right)\right\}^{\beta_{j}} \prod_{j=N+1}^{P} \Gamma\left(a_{j}-A_{j} \xi\right)} \tag{4}
\end{equation*}
$$

It may be noted that the $\bar{H}$-function is even more general than familiar H -function.
A large number of special functions follow as its special cases. The details about this function and the nature of the contour can be seen in the paper referred above.

The following sufficient conditions for the absolute convergence of the defining integral for the $\bar{H}$-function given by the equation (3) has been given by Buschman and Srivastava [7].

$$
\begin{equation*}
\Omega=\sum_{j=1}^{M} B_{j}+\sum_{j=1}^{N}\left|\alpha_{j}\right| A_{j}-\sum_{j=M+1}^{Q}\left|\beta_{j}\right| B_{j}-\sum_{j=N+1}^{P} A_{j}>0 \tag{5}
\end{equation*}
$$

and

$$
\begin{equation*}
|\arg (z)|<\frac{1}{2} \pi \Omega \tag{6}
\end{equation*}
$$

where $\Omega$ is given by (5).
The behavior of the $\bar{H}$-function for small value of $|z|$ follows easily from a result given by Rathie[8].

$$
\begin{equation*}
\bar{H}_{P, Q}^{M, N}[z]=0\left(|z|^{\alpha}\right) \tag{7}
\end{equation*}
$$

and

$$
\begin{equation*}
\alpha=\min _{1 \leq j \leq M} \operatorname{Re}\left(b_{j} / B_{j}\right),|z| \rightarrow 0 \tag{8}
\end{equation*}
$$

The following series representation for the $\bar{H}$-function given by Saxena et al. [9] will be required later on:

$$
\begin{equation*}
\bar{H}_{P, Q}^{M, N}\left[\left.z\right|_{\substack{\left(b_{j}, B_{j}\right)_{1, M},\left(b_{j}, B_{j} ; B_{j}\right)_{M+1, Q}}} ^{\substack{\left.a_{j}, A_{j} ; \alpha_{j}\right)_{1, N},\left(a_{j}, A_{j}\right)_{N+1, P} \\ \hline}}\right]=\sum_{T=0}^{\infty} \sum_{H=1}^{M} \bar{F}(\varsigma) z^{\varsigma} \tag{9}
\end{equation*}
$$

where

$$
\begin{equation*}
\bar{F}(\varsigma)=\frac{\prod_{j=1 j \neq H}^{M} \Gamma\left(b_{j}-B_{j} \varsigma\right) \prod_{j=1}^{N}\left\{\Gamma\left(1-a_{j}+A_{j} \varsigma\right)\right\}^{\alpha_{j}}}{\prod_{j=M+1}^{Q}\left\{\Gamma\left(1-b_{j}+B_{j} \varsigma\right)\right\}^{\beta_{j}} \prod_{j=N+1}^{P} \Gamma\left(a_{j}-A_{j} \varsigma\right)} \frac{(-1)^{T}}{T!B_{H}} \tag{10}
\end{equation*}
$$

$$
\begin{gather*}
\varsigma=\frac{b_{H}+T}{B_{H}}  \tag{11}\\
\mu_{1}=\sum_{j=1}^{M}\left|B_{j}\right|-\sum_{j=1}^{N}\left|a_{j} A_{j}\right|+\sum_{j=M+1}^{Q}\left|b_{j} B_{j}\right|-\sum_{j=N+1}^{P}\left|A_{j}\right|>0, \tag{12}
\end{gather*}
$$

and

$$
0<|z|<\infty
$$

We give here definitions of two functions which are particular cases of the $\bar{H}$ function but not of Fox H-function.
(i) The ${ }_{P} \bar{\psi}_{Q}(\mathrm{z})$-function will be defined and represented [10] as follows:

$$
{ }_{P} \bar{\psi}_{Q}\left[\begin{array}{l}
\left(a_{j}, A_{j} ; \alpha_{j}\right)_{1, P}  \tag{13}\\
\left(b_{j}, B_{j} ; \beta_{j}\right)_{1, Q}
\end{array} ; z\right]=\bar{H}_{P, Q+1}^{1, P}\left[-z \left\lvert\, \begin{array}{l}
\left(1-a_{j}, A_{j} ; \alpha_{j}\right)_{1, P} \\
(0,1),\left(1-b_{j}, B_{j} ; \beta_{j}\right)_{1, Q}
\end{array}\right.\right]
$$

(ii) The $\bar{J}_{\lambda}^{\nu, \mu}(z)$-function will be defined and represented [10] as follows:

$$
\begin{equation*}
\bar{J}_{\lambda}^{\nu, \mu}=\bar{H}_{0,2}^{1,0}\left[-\left.z\right|_{(0,1),(-\lambda, \nu ; \mu)} ^{-}\right] \tag{14}
\end{equation*}
$$

## 2. Main Integrals

Let $\Psi(z)$ denote the logarithmic derivative of gamma function $\Gamma(z)$ i.e. $\Psi(z)=\frac{\Gamma^{\prime}(z)}{\Gamma(z)}$.

### 2.1. Integral Formula 1.

$$
\begin{align*}
& \int_{0}^{\infty} F(X) \log x d x \\
& =2 \alpha^{-\nu}\left(\frac{\alpha}{2}\right)^{u} \Gamma(2 u) \sum_{k=0}^{[V / U]} \frac{(-V)_{U k}}{k!} A_{V, k} e_{1}^{k}(\alpha)^{-m k} \sum_{T=0}^{\infty} \sum_{H=1}^{M} \overline{F^{\prime}}(\varsigma) \\
& \left\{\log \left(\frac{\alpha}{2}\right)+2 \Psi(2 u)-\Psi(\nu-u+m k+n \varsigma)-\Psi(1+u+\nu+m k+n \varsigma)\right\} e_{2}^{\varsigma}(\alpha)^{-n \varsigma} \tag{15}
\end{align*}
$$

### 2.2. Integral Formula 2.

$$
\begin{align*}
& \int_{0}^{\infty} F(X) \log \left[\frac{1}{\left(x+\alpha+\left(x^{2}+2 \alpha x\right)^{1 / 2}\right)}\right] d x \\
& =2 \alpha^{-\nu}\left(\frac{\alpha}{2}\right)^{u} \Gamma(2 u) \sum_{k=0}^{[V / U]} \frac{(-V)_{U k}}{k!} A_{V, k} e_{1}^{k}(\alpha)^{-m k} \\
& \quad \sum_{T=0}^{\infty} \sum_{H=1}^{M} \bar{F}^{\prime}(\varsigma)\{-\log (\alpha)+\Psi(1+\nu+m k+n \varsigma)+\Psi(\nu-u+m k+n \varsigma) \\
& \quad-\Psi(1+u+\nu+m k+n \varsigma)-\Psi(\nu+m k+n \varsigma)\} e_{2}^{\varsigma}(\alpha)^{-n \varsigma} \tag{16}
\end{align*}
$$

### 2.3. Integral Formula 3.

$$
\begin{align*}
& \int_{0}^{\infty} F(X) \log \left[\frac{x}{\left(x+\alpha+\left(x^{2}+2 \alpha x\right)^{1 / 2}\right)}\right] d x \\
& =2 \alpha^{-\nu}\left(\frac{\alpha}{2}\right)^{u} \Gamma(2 u) \sum_{k=0}^{[V / U]} \frac{(-V)_{U k}}{k!} A_{V, k} e_{1}^{k}(\alpha)^{-m k} \sum_{T=0}^{\infty} \sum_{H=1}^{M} \overline{F^{\prime}}(\varsigma) \\
& \left\{\log \left(\frac{1}{2}\right)+2 \Psi(2 u)+\Psi(1+\nu+m k+n \varsigma)-2 \Psi(1+u+\nu+m k+n \varsigma)-\Psi(\nu+m k+n \varsigma)\right\} e_{2}^{\varsigma}(\alpha)^{-n \varsigma} \tag{17}
\end{align*}
$$

### 2.4. Integral Formula 4.

$$
\begin{align*}
& \int_{0}^{\infty} F(X) \log \left[x\left(x+\alpha+\left(x^{2}+2 \alpha x\right)^{1 / 2}\right)\right] d x \\
& =2 \alpha^{-\nu}\left(\frac{\alpha}{2}\right)^{u} \Gamma(2 u) \sum_{k=0}^{[V / U]} \frac{(-V)_{U k}}{k!} A_{V, k} e_{1}^{k}(\alpha)^{-m k} \sum_{T=0}^{\infty} \sum_{H=1}^{M} \bar{F}^{\prime}(\varsigma) \\
& \quad\left\{\log \left(\frac{\alpha^{2}}{2}\right)+2 \Psi(2 u)-\Psi(1+\nu+m k+n \varsigma)-2 \Psi(\nu-u+m k+n \varsigma)+\Psi(\nu+m k+n \varsigma)\right\} e_{2}^{\varsigma}(\alpha)^{-n \varsigma} \tag{18}
\end{align*}
$$

where

$$
\begin{gathered}
F(X)=x^{u-1}\left(x+\alpha+\left(x^{2}+2 \alpha x\right)^{1 / 2}\right)^{-v} S_{V}^{U}\left[e_{1}\left(x+\alpha+\left(x^{2}+2 \alpha x\right)^{1 / 2}\right)^{-m}\right] \\
\bar{H}_{P, Q}^{M, N}\left[e_{2}\left(x+\alpha+\left(x^{2}+2 \alpha x\right)^{1 / 2}\right)^{-n}\right]
\end{gathered}
$$

and

$$
\begin{equation*}
\overline{F^{\prime}}(\varsigma)=\bar{F}(\varsigma) \frac{\Gamma(\nu-u+m k+n \varsigma) \Gamma(1+\nu+m k+n \varsigma)}{\Gamma(\nu+m k+n \varsigma) \Gamma(1+\nu+u+m k+n \varsigma)} \tag{19}
\end{equation*}
$$

where $\bar{F}(\varsigma)$ and $\varsigma$ are given by (10) and (11) respectively.
The conditions of validity of above Integral Formulas are
(i) $n>0, \operatorname{Re}(u, \nu, m)>0$.
(ii) $|\arg z|<\frac{1}{2} \pi \Omega, \Omega>0$ where $\Omega$ is given by (5).
(iii) $\operatorname{Re}(u)-\operatorname{Re}(v)-n \min \operatorname{Re}\left(\frac{b_{j}}{B_{j}}\right)<0$

To establish the IF 1. to IF 4., we required the following interesting integral:

$$
\begin{align*}
& I=\int_{0}^{\infty} x^{u-1}\left(x+\alpha+\left(x^{2}+2 \alpha x\right)^{1 / 2}\right)^{-v} S_{V}^{U}\left[e_{1}\left(x+\alpha+\left(x^{2}+2 \alpha x\right)^{1 / 2}\right)^{-m}\right] \\
& \bar{H}_{P, Q}^{M, N}\left[e_{2}\left(x+\alpha+\left(x^{2}+2 \alpha x\right)^{1 / 2}\right)^{-n}\right] d x \\
& =2 \alpha^{-\nu}\left(\frac{\alpha}{2}\right)^{u} \Gamma(2 u) \sum_{k=0}^{[V / U]} \frac{(-V)_{U k}}{k!} A_{V, k} e_{1}^{k}(\alpha)^{-m k} \\
& \bar{H}_{P+2, Q+2}^{M, N+2}\left[e_{2} \alpha^{-n} \left\lvert\, \begin{array}{c}
(1+u-\nu-m k, n ; 1),(-\nu-m k, n ; 1),\left(a_{j}, \alpha_{j} ; A_{j}\right)_{1, N},\left(a_{j}, \alpha_{j}\right)_{N+1, P} \\
\left(b_{j}, \beta_{j}\right)_{1, M},\left(b_{j}, \beta_{j} ; B_{j}\right)_{M+1, Q},(1-\nu-m k, n ; 1),(-u-\nu-m k, n ; 1)
\end{array}\right.\right] \\
& =  \tag{21}\\
& =2 \alpha^{-\nu}\left(\frac{\alpha}{2}\right)^{u} \Gamma(2 u) \sum_{k=0}^{[V / U]} \frac{(-V)_{U k}}{k!} A_{V, k} e_{1}^{k}(\alpha)^{-m k} \sum_{T=0}^{\infty} \sum_{H=1}^{M} \overline{F^{\prime}}(\varsigma) e_{2}^{\varsigma}(\alpha)^{-n \varsigma}
\end{align*}
$$

where $\overline{F^{\prime}}(\varsigma)$ is given by (19).
To evaluate I, we first express $S_{V}^{U}[$.$] in its series form with the help of (1) and change$ the order of integration and summation, put the value of $\bar{H}_{P, Q}^{M, N}[z]$ in terms of Mellin-Barnes contour integral with the help of (2), change the order of integration (which is permissible under the conditions stated with IF 1.) and integrate the x-integral with the help of Oberhettinger [11]:

$$
\begin{align*}
& \int_{0}^{\infty} x^{a-1}\left[x+c+\left(x^{2}+2 c x\right)^{1 / 2}\right]^{-b} d x  \tag{23}\\
& =2 b c^{-b}\left(\frac{1}{2} c\right)^{a} \frac{\Gamma(2 a) \Gamma(b-a)}{\Gamma(1+b+a)}, \quad 0<\operatorname{Re}(a)<b
\end{align*}
$$

Finally, interpreting the $\xi$ - contour integral in terms of the $\bar{H}$-function, we arrive at the right hand side of (22).

Method of Proof: To prove First Integral Formula (IF), by taking the partial derivative of both sides of (22) with respect to u. IF 2 is similarly established by taking the partial derivative of both sides of (22) with respect to v. To establish IF $3 \& 4$, we use the IF $1 \& 2$, first adding the IF $1 \& 2$, than we get the IF 3 . IF 4 is similarly established by subtracting IF 2 . from IF 1.

## 3. Special Cases of the Main Integral Formulas

The integral formulas, First, Second, Third and Fourth established here are unified in nature and act as key formulas. Thus the general class of polynomial involved in these integrals reduces to a large spectrum of polynomials listed by Srivastava and Singh [2] and we can further obtain various unified integral formulas involving a number of simpler polynomials. Again, the $\bar{H}$ - function occurring in the main results can be suitably specialized to a remarkably wide variety of useful functions which are expressible in terms of generalized Wright's Hyper-geometric function and generalized Wright's Bessal function. For example: A large number of integrals involving simpler functions of one variable can be easily obtained as their special cases. We however gave here only some special cases by way of illustration:
(i) If we take $M=1, N=P, Q=Q+1$ in (15) then the $\bar{H}$-function occurring therein breaks up into the ${ }_{P} \bar{\psi}_{Q}().[10]$ and the IF 1.(15) takes the following form after a little simplification which is also believed to be new:

$$
\begin{align*}
& \int_{0}^{\infty} x^{u-1}\left(x+\alpha+\left(x^{2}+2 \alpha x\right)^{1 / 2}\right)^{-v} S_{V}^{U}\left[e_{1}\left(x+\alpha+\left(x^{2}+2 \alpha x\right)^{1 / 2}\right)^{-m}\right] \\
& P \bar{\psi}_{Q}\left[-e_{2}\left(x+\alpha+\left(x^{2}+2 \alpha x\right)^{1 / 2}\right)^{-n}\right] \log x d x \\
& =2 \alpha^{-\nu}\left(\frac{\alpha}{2}\right)^{u} \Gamma(2 u) \sum_{k=0}^{[V / U]} \frac{(-V)_{U k}}{k!} A_{V, k} e_{1}^{k}(\alpha)^{-m k} \sum_{T=0}^{\infty} \overline{F^{\prime}}(T) \\
& \left\{\log \left(\frac{\alpha}{2}\right)+2 \Psi(2 u)-\Psi(\nu-u+m k+n T)-\Psi(1+u+\nu+m k+n T)\right\} e_{2}^{T}(\alpha)^{-n T} \tag{24}
\end{align*}
$$

where

$$
\bar{F}^{\prime}(T)=\bar{f}(T) \frac{\Gamma(\nu-u+m k+n T) \Gamma(1+\nu+m k+n T)}{\Gamma(\nu+m k+n T) \Gamma(1+\nu+u+m k+n T)}
$$

and

$$
\bar{f}(T)=\frac{\prod_{j=1}^{P}\left\{\Gamma\left(a_{j}+A_{j} T\right)\right\}^{\alpha_{j}}}{\prod_{j=1}^{Q}\left\{\Gamma\left(b_{j}+B_{j} T\right)\right\}^{\beta_{j}}}
$$

The conditions of validity of (24) easily follow from those given in (15).
(ii) If we take $M=1, N=P=0, Q=2$ in (15) than the $\bar{H}$-function occurring therein break up into the $\bar{J}_{\lambda}^{\mu, \eta}().[10]$ and the IF 1 takes the following form after a
little simplification which is also believed to be new:

$$
\begin{align*}
& \int_{0}^{\infty} x^{u-1}\left(x+\alpha+\left(x^{2}+2 \alpha x\right)^{1 / 2}\right)^{-v} S_{V}^{U}\left[e_{1}\left(x+\alpha+\left(x^{2}+2 \alpha x\right)^{1 / 2}\right)^{-m}\right] \\
& \bar{J}_{\lambda}^{\mu, \eta}\left[-e_{2}\left(x+\alpha+\left(x^{2}+2 \alpha x\right)^{1 / 2}\right)^{-n}\right] \log x d x \\
& =2 \alpha^{-\nu}\left(\frac{\alpha}{2}\right)^{u} \Gamma(2 u) \sum_{k=0}^{[V / U]} \frac{(-V)_{U k}}{k!} A_{V, k} e_{1}^{k}(\alpha)^{-m k} \\
& \sum_{T=0}^{\infty} \bar{F}^{\prime \prime}(T)\left\{\log \left(\frac{\alpha}{2}\right)+2 \Psi(2 u)-\Psi(\nu-u+m k+n T)-\Psi(1+u+\nu+m k+n T)\right\} e_{2}^{T}(\alpha)^{-n T} \tag{25}
\end{align*}
$$

where

$$
\bar{F}^{\prime \prime}(T)=\bar{f}^{\prime}(T) \frac{\Gamma(\nu-u+m k+n T) \Gamma(1+\nu+m k+n T)}{\Gamma(\nu+m k+n T) \Gamma(1+\nu+u+m k+n T)}
$$

and

$$
\bar{f}^{\prime}(T)=\frac{1}{\{\Gamma(1+\lambda+\mu T)\}^{n}}
$$

The conditions of validity of (25) easily follow from those given in (15).
(iii) By applying our result given in (15) to the case of Hermite polynomial [2] and [3] and by setting $S_{V}^{2}(x) \rightarrow x^{V / 2} H_{V}\left[\frac{1}{\sqrt[2]{x}}\right]$ in which case $U=2, A_{V, k}=(-1)^{K}$, we have the following interesting consequences of the main result:

$$
\begin{align*}
& \int_{0}^{\infty} x^{u-1}\left(x+\alpha+\left(x^{2}+2 \alpha x\right)^{1 / 2}\right)^{-v} e_{1}^{V / 2}\left\{\left(x+\alpha+\left(x^{2}+2 \alpha x\right)^{1 / 2}\right)^{-m}\right\}^{V / 2} \\
& . H_{V}\left[\frac{1}{\left.2 \sqrt{e_{1}^{V / 2}\left\{\left(x+\alpha+\left(x^{2}+2 \alpha x\right)^{1 / 2}\right)^{-m}\right.}\right\}}\right] \bar{H}_{P, Q}^{M, N}\left[e_{2}\left(x+\alpha+\left(x^{2}+2 \alpha x\right)^{1 / 2}\right)^{-n}\right] \log x d x \\
& =2 \alpha^{-\nu}\left(\frac{\alpha}{2}\right)^{u} \Gamma(2 u) \sum_{k=0}^{[V / 2]} \frac{(-V)_{2 k}}{k!}(-1)^{k} e_{1}^{k}(\alpha)^{-m k} \sum_{T=0}^{\infty} \sum_{H=1}^{M} \bar{F}^{\prime}(\varsigma) \\
& \cdot\left\{\log \left(\frac{\alpha}{2}\right)+2 \Psi(2 u)-\Psi(\nu-u+m k+n \varsigma)-\Psi(1+u+\nu+m k+n \varsigma)\right\} e_{2}^{\varsigma}(\alpha)^{-n \varsigma} \tag{26}
\end{align*}
$$

valid under the same conditions as obtainable from (15).
(iv) For the Laguerre polynomials ([2] and [3]) $S_{V}^{1}(x) \rightarrow L_{V}^{\left(\alpha^{\prime}\right)}(x)$ in which case $U=1, A_{V, k}=\binom{V+\alpha^{\prime}}{V} \frac{1}{\left(\alpha^{\prime}+1\right)_{k}}$ the result (15) reduce to the following formula:

$$
\begin{gather*}
\int_{0}^{\infty} x^{u-1}\left(x+\alpha+\left(x^{2}+2 \alpha x\right)^{1 / 2}\right)^{-v} L_{V}^{\left(\alpha^{\prime}\right)}\left[e_{1}\left(x+\alpha+\left(x^{2}+2 \alpha x\right)^{1 / 2}\right)^{-m}\right] \\
\bar{H}_{P, Q}^{M, N}\left[e_{2}\left(x+\alpha+\left(x^{2}+2 \alpha x\right)^{1 / 2}\right)^{-n}\right] \log x d x \\
=2 \alpha^{-\nu}\left(\frac{\alpha}{2}\right)^{u} \Gamma(2 u) \sum_{k=0}^{[V]} \frac{(-V)_{k}}{k!}\binom{V+\alpha^{\prime}}{V} \frac{1}{\left(\alpha^{\prime}+1\right)_{k}} e_{1}^{k}(\alpha)^{-m k} \sum_{T=0}^{\infty} \sum_{H=1}^{M} \overline{F^{\prime}}(\varsigma) \\
\left\{\log \left(\frac{\alpha}{2}\right)+2 \Psi(2 u)-\Psi(\nu-u+m k+n \varsigma)-\Psi(1+u+\nu+m k+n \varsigma)\right\} e_{2}^{\varsigma}(\alpha)^{-n \varsigma} \tag{27}
\end{gather*}
$$

valid under the same conditions as required for (15).
If we take $\alpha_{j}=\beta_{j}=1$ in (21), then the $\bar{H}$-function reduces to the Fox $H$-function [12] and we arrive at the known result given by Garg and Mittal [4].

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