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Interaction of laser pulse with a quantum electron-hole semiconductor plasma

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ABSTRACT

A laser-driven plasma is investigated theoretically in a quantum electron-hole semiconductor plasma by a short electromagnetic pulse. The laser beam is single, short, and high-intensity. Using the quantum hydrodynamic (QHD) physical model consists of continuity and momentum equations for electrons and holes. These equations are closed by Poisson's equation. The momentum equations include the wave vector of the electromagnetic field, pressure, and Bohm potential. It introduces the effect of two formulas about the pressure in the form of the last differential equations. An electromagnetic field is represented in the circularly polarized Gaussian profile. Laplace transformation and convolution theorem are used to obtain the final evolution equation of our theoretical model.

Key Words:

Quantum plasma ; Semiconductor plasma ; Laplace transformation ; Laser driven plasma ; Laser wakefeild acceleration

1. INTRODUCTION

Electromagnetic wave propagation through the plasma is a general and interesting topic with a variety of applications beginning from communications to laser-driven particle accelerators. In 1962, Shimoda used an optical maser of high power 10 kW/cm ^{2} to accelerate a beam of electrons by 10^9 eV/m [1]. In 1978, Shukla et al. introduced the nonlinear propagation of an intense circularly polarized laser field in collision-less electrons and ions plasma by using the wave equation and the governing relativistic hydrodynamic system [2]. They studied the effect of the relativistic mass, relativistic pondermotive force, and also nonlinear plasma dynamics. When the intensity of the laser light is relatively weak, the pondermotive force non-linearity can cause wave filamentation. In 1979, Tajima and Dawson [3] used an intense electromagnetic pulse to create weak plasma oscillations by a glass laser with characteristic, wavelength $\lambda = 1\mu m$, Power $I = 10^{18}$ W/cm ^{2}, to excite the electron plasma with a density of $n_0 = 10^{17}$ cm $^{-3}$. A wave packet of electromagnetic radiation (photons) injected in an under-dense plasma excites an electrostatic wake behind the photons. The wake plasma wave (plasmon) is excited by the ponderomotive force created by the photons with the phase velocity. The mechanism for generating the wakes can be simply seen by the following approximate treatment. Consider the light wave propagating in the x direction with the electric field in the y direction. The light wave sets the electrons into transverse oscillations. If the intensity is not so large that the transverse motion does not become relativistic. The wake plasmon, which propagates with phase velocity close to c, can trap electrons. The trapped electrons which execute trapping oscillations can gain a large amount of energy when they accelerate forward since they largely gain in mass and only get out of phase with this wave after a long time. In 1992, Tsintsadz [4] generated nonlinear plasma waves with arbitrary phase velocities by an intense laser pulse considered in order to determine the dependence of the maximum amplitude of driven waves on the group velocity of the laser pulse (or the phase velocity of excited wakefields). The success of the laser wakefield acceleration (LWFA) scheme depends on the use of relativistically intense, short laser pulses. They proved that the wakefield amplitude increases as the phase velocity decreases and, conversely, the wakefield amplitude decreases as the phase velocity increases and the phase velocity may be decreased (increased) by decreasing (increasing) the laser pulse frequency ω. In 1995, Cairns et al. [5] introduced the generation of large-amplitude plasma waves using high-power lasers in either the wakefield or beat wave. In 2012, Joker and Eslami [6], produced a huge amplitude plasma wakefield by the propagation of an intense short laser pulse through under-dense plasma. They explained the four basic schemes in plasma accelerators, which all work on the same principle: plasma wakefield acceleration (PWFA), plasma beat-wave acceleration (PBWA), self-modulated laser wakefield acceleration (SMLWFA), laser wakefield acceleration (LWFA). In the "*standard*" laser wakefield accelerator concept, a single short ($< 1ps$), ultrahigh intensity ($> 10^{18}$ W/cm ^{2}) laser pulse is injected into an under-dense plasma. Assuming the laser pulse maintains its shape, the plasma waves are excited by the laser pondermotive force. Thus, electrons are separated from the ions, which do not move because of their higher mass. Behind the laser, we see a cavities region with nearly zero electron density. These electrons are then drawn back by the stationary ions making a sphere-like wake structure, which is the plasma wave. The space the charge created by these plasma waves makes a field called wakefield. This wakefield is needed for particle acceleration. Laser wakefield acceleration is described as rich in nonlinear, relativistic, and ultra-fast physics and it is part of the rapidly emerging field of high-energy-density science. The laser pulse was chosen to be linearly polarized with a transverse Gaussian profile. A driving laser beam is then launched from the left boundary and propagates in the pre-ionized plasma of He+. This simulation considers a box of plasma in which the background plasma density is initially uniform. The number of grids in the 2D simulation box is $(N_x, N_y) = (1600{,}500)$ and the size of the plasma region is $102.4\mu m$ in the x-direction and $32\mu m$ in the y-direction with MKS units. An important parameter for the wake excitation in the discussion of the laser-plasma interaction is the laser pulse duration. The amplitude of the wakefields is smaller than the maximum values when the laser pulses are longer or shorter than the optimum pulse length, which is about $40fs$ equal to the plasma wavelength. Wakefield generation strongly depends on the laser intensity, pulse duration, beam spot size, and also temporal pulse shape. For a Gaussian pulse envelope, increasing the laser intensity enhances the excited plasma wakefield. A reverse behavior occurs when the wider spot size is used. It also showed the dependence of wakefield amplitude on the laser pulse duration.

In 2014, Hooker et al. [7] studied simulation for Laser-driven plasma accelerators that can generate accelerating gradients three orders of magnitude larger than radio-frequency accelerators and have achieved beam energies above 1 GeV in centimeter-long stages. It investigates the prospects for multi-pulse laser wakefield acceleration (MP-LWFA), in which the wake is excited by a train of low-energy laser pulses rather than by a single high-energy pulse. In a laser wakefield accelerator (LWFA) a single laser pulse, with a peak intensity of order 10^{18} W cm $^{-}2$, propagates through a plasma and excites a density wave via the ponderomotive force, which acts to expel plasma electrons from the region of the laser pulse. In MP-LWFA a train of low-energy laser pulses, rather than a single high-energy pulse, is used to excite the plasma wave. The plasma wakefields excited by the pulses will add coherently; so that the amplitude of the wakefield increases with each additional pulse, if the pulses are spaced by the plasma period $T_{pe} = 2\pi/\omega_{pe}$, where $\omega_{pe} = (n_e e^2/m^2\varepsilon_0)^{1/2}$ and n_e is the mean electron density. Particles can then be accelerated within a single period of the plasma wakefield trailing the driving train; usually, this would be the "bucket" immediately after the last pulse in the train. They showed that an MP-LWFA driven by a near-term laser system of this type could drive wakefields with an accelerating field of 4.7 GV/m, with a dephasing-limited energy gain of 0.75 GeV, and that with freq $f_{rep} = 10$ kHz these could drive the compact coherent and incoherent x-ray sources with average brightness's exceeding those available from large-scale, nonsuperconducting, RF accelerators. In 2017, Cowly et al. [8] investigated that, if the pulses are spaced by the plasma wavelength $\lambda_{p0} = 2\pi c/\omega_{p0}$, then the wakefields driven by the pulses in the train add coherently, causing the plasma wave amplitude to grow towards the back of the train. It presented the first demonstration of wakefield excitation by a laser pulse structure that is long compared to the plasma period, and for which there is sufficient control of the temporal profile to overcome relativistic saturation. We also take an important first step towards achieving energy recovery by showing that a suitably delayed laser pulse can dampen the plasma wave driven by a leading pulse. Since laser systems generating directly the pulse trains required for MP-LWFA are still under development, this first demonstration employed a Ti: sapphire laser the Gemini (Astra TA2) laser at the Rutherford Appleton Laboratory reconfigured to generate trains of laser pulses. In its standard, an arrangement this laser delivers to target approximately $600mJ, 40fs$ laser pulses with a center wavelength $\lambda_0 = 800nm$ at $f_{(rep)} = 5$ Hz. Also, it could be considered to be the first experimental demonstration of MP-LWFA or of beat-wave excitation with chirped laser pulses.

In semiconductors, the dynamics of free carriers is characterized by several parameters: the conductivity (frequency dependent), the plasma frequency ω_p, and the carrier damping rate Γ (the inverse of the carrier collision time). Γ and ω_p typically have THz frequencies. Ultra-fast THz Time-domain spectroscopy has been used to measure the complex conductivity of doped Silicon from low frequencies to frequencies higher than the plasma frequency and the carrier damping rate. The absorption spectrum of a bulk semiconductor subjected to a strong electrical field develops an exponential tail below the band gap and oscillations above it, which is known as the Franz-Keldysh effect. It can modify the band structure of the semiconductor at equilibrium. This effect has been observed by applying a strong THz field to a GaAs sample and by probing it with femtosecond white light. In this experiment, the single-cycle THz pulses, with energies of 2 μJ and field strengths exceeding 100 kV/cm, were generated by optical rectification in $LiNbO_3$, and the probe beam was obtained by focusing a femtosecond laser pulse onto a thick sapphire plate. The THz pulse energies reported here are consistent with those measured in LWFA-driven THz sources. In another experiment, THz pulses were used to observe stimulated THz emission from internal transitions of excitons in a Cu_2O semiconductor. Broadband THz pulses, also produced by optical rectification, monitored the electromagnetic response of the sample after photoexcitation [9].

The motivation of this work is to examine the theoretical treatment of the laser pulse propagates in a quantum electron-hole semiconductor plasma. The resulting differential equation simulated in two branches using two pressure quantities. For this purpose, we used the quantum hydrodynamic (QHD) system along with electrons and holes quantum plasma. These equations are reduced to the plasma wakefield equation in the presence of the electromagnetic field. Furthermore, the quantum pressure is examined by two approaches. The Laplace transformation plays an important role to get the solution of the non-homogeneous differential equation, which is reduced to a wakefield amplitude after the laser pulse propagates.

2. Mathematical model

Let us consider an electromagnetic wave propagating in an unmagnetized, collision-less, and quantum semiconductor plasma consisting of electrons (e) and holes (h). The governing equations consist of continuity and momentum equations in of electrons and holes closed by Poisson's equation represented in the quantum hydrodynamic (QHD) model [10, 11, 12] as

$$\frac{\partial n_e}{\partial t} + \nabla.(n_{eo} v_e) = 0, \tag{1}$$

$$\frac{\partial v_e}{\partial t} = \frac{e}{m_e}\nabla\phi - \frac{e^2}{2m_e^2c^2}\nabla|A|^2 + \frac{1}{m_e n_{eo}}\nabla P_e + \frac{\hbar^2}{4m_e^2}\nabla(\nabla^2 n_e), \tag{2}$$

$$\frac{\partial n_h}{\partial t} + \nabla.(n_{ho} v_h) = 0, \tag{3}$$

$$\frac{\partial v_h}{\partial t} = \frac{-e}{m_h}\nabla\phi - \frac{e^2}{2m_h^2c^2}\nabla|A|^2 + \frac{1}{m_h n_{ho}}\nabla P_h + \frac{\hbar^2}{4m_h^2}\nabla(\nabla^2 n_h), \tag{4}$$

the Poisson's equation is

$$\nabla^2\phi = 4\pi e(n_e - n_h). \tag{5}$$

where, $n_{e/h}$ is the (electron/hole) density, $n_{eo/ho}$ is the (electron/hole) initial density, $v_{e/h}$ is the (electron/hole) velocity, ϕ is the wakefield potential, $m_{e/h}$ is the (electron/hole) mass, $P_{e/h}$ is the (electron/hole) thermal pressure, e is the magnitude of the electron charge, $\hbar$ is the Plank constant divided by 2π, c is the light speed, A is the electromagnetic vector potential. The fourth term on the right-hand side of (2) and (4) is the quantum Bohm potential.

While semiconductors are nonlinear media, this model is suitable for small amplitude perturbation cases, but if the amplitude increases that requires the study of the non-linear properties.

For simplicity, let $n_h = \alpha n_e$, then the Poisson's Eq. takes the new form

$$n_e = \frac{\nabla^2\phi}{4\pi e(1-\alpha)} \quad , \quad n_h = \frac{\alpha\nabla^2\phi}{4\pi e(1-\alpha)}. \tag{6}$$

Subtracting Eq. (3) from (1) we get

$$\frac{\partial}{\partial t}(n_e - n_h) = -\nabla.(n_{eo} v_e - n_{ho} v_h), \tag{7}$$

using Poisson's Eq. and after integration by the position

$$\frac{\partial}{\partial t}\left(\frac{\nabla\phi}{4\pi e}\right) = -(n_{eo}v_e - n_{ho}v_h). \tag{8}$$

Multiply Eq.(2) by (n_{eo}) and Eq. (4) by (n_{ho}), then subtracting them we get

$$\frac{\partial}{\partial t}(n_{eo}v_e - n_{ho}v_h) = e\nabla\phi\left(\frac{n_{eo}}{m_e}+\frac{n_{ho}}{m_h}\right) - \nabla\left(\frac{P_e}{m_e}-\frac{P_h}{m_h}\right) - \frac{e^2\nabla|A|^2}{2c^2}\left(\frac{n_{eo}}{m_e^2}-\frac{n_{ho}}{m_h^2}\right) + \frac{\hbar^2}{4}\nabla\left[\nabla^2\left(\frac{n_{eo}n_e}{m_e^2}-\frac{n_{ho}n_h}{m_h^2}\right)\right] blue. \tag{9}$$

Use Eq. (8) in Eq. (9) and rearrangement it has a form

$$\nabla\left\{\frac{\partial^2}{\partial t^2}\left(\frac{\phi}{4\pi e}\right) + e\phi\left(\frac{n_{eo}}{m_e}+\frac{n_{ho}}{m_h}\right) - \left(\frac{P_e}{m_e}-\frac{P_h}{m_h}\right) - \frac{e^2|A|^2}{2c^2}\left(\frac{n_{eo}}{m_e^2}-\frac{n_{ho}}{m_h^2}\right)\right\} + \nabla\left\{\frac{\hbar^2}{4}\nabla^2\left(\frac{n_{eo}n_e}{m_e^2}-\frac{n_{ho}n_h}{m_h^2}\right)\right\} = 0 blue. \tag{10}$$

After integrating the last equation, we get

$$\frac{\partial^2\phi}{\partial t^2} + 4\pi e^2\phi\left(\frac{n_{eo}}{m_e}+\frac{n_{ho}}{m_h}\right) - 4\pi e\left(\frac{P_e}{m_e}-\frac{P_h}{m_h}\right) - \frac{4\pi e^3|A|^2}{2c^2}\left(\frac{n_{eo}}{m_e^2}-\frac{n_{ho}}{m_h^2}\right) + \frac{4\pi e\hbar^2}{4}\nabla^2\left(\frac{n_{eo}n_e}{m_e^2}-\frac{n_{ho}n_h}{m_h^2}\right) = 0. \tag{11}$$

To solve the last differential equation, it must be a function of the potential ϕ, which expresses the amplitude of the output wakefield after the laser pulse propagates in the e-h plasma.

By using the electron and hole plasma frequencies [12, 13]

$$\omega_{pe}^2 = \frac{4\pi e^2 n_{eo}}{m_e}, \qquad \omega_{ph}^2 = \frac{4\pi e^2 n_{ho}}{m_h}.$$

Then using relations in (6), Eq. (11) can be written in the formulation

$$\frac{\partial^2\phi}{\partial t^2} + \left(\omega_{pe}^2+\omega_{ph}^2\right)\phi - 4\pi e\left(\frac{P_e}{m_e}-\frac{P_h}{m_h}\right) + \frac{n_{eo}\hbar^2(1-\alpha M^2)}{4m_e^2(1-\alpha)}\nabla^4\phi = \frac{e}{2c^2}\left(\frac{\omega_{pe}^2}{m_e}-\frac{\omega_{ph}^2}{m_h}\right)|A|^2, \tag{12}$$

where $M = m_e/m_h$ is the mass ratio.

This manuscript introduces two approaches to quantum pressure.

3. Case I

If the pressure formula takes the Fermi velocity approach as [11]

$$P_e = \frac{v_F^2}{n_{eo}}n_e, \qquad P_h = \frac{v_F^2}{n_{ho}}n_h, \tag{13}$$

where $v_F = (2\pi\hbar/\sqrt{3}m_e)(3n_{eo}/8\pi)^{1/3}$ is the Fermi velocity. Using relations in (6), the differential equation (12) takes the form

$$\frac{\partial^2\phi}{\partial t^2} + \left(\omega_{pe}^2+\omega_{ph}^2\right)\phi - \frac{v_F^2(1-\alpha M)}{(1-\alpha)m_e n_{eo}}\nabla^2\phi + \frac{n_{eo}\hbar^2(1-\alpha M^2)}{4m_e^2(1-\alpha)}\nabla^4\phi = \frac{e}{2c^2}\left(\frac{\omega_{pe}^2}{m_e}-\frac{\omega_{ph}^2}{m_h}\right)|A|^2 blue. \tag{14}$$

Now, we will consider the electromagnetic pulse to excite the wakefield with the group velocity v_g. For simplicity; equation (14) must be converted from partial differential equations to ordinary differential equations. Assuming that $\xi = x - v_g t$ and substituting in Eq. (14)

$$\frac{n_{eo}\hbar^2(1-\alpha M^2)}{4m_e^2(1-\alpha)}\frac{\partial^4\phi}{\partial\xi^4} + \left(v_g^2 - \frac{v_F^2(1-\alpha M)}{(1-\alpha)m_e n_{eo}}\right)\frac{\partial^2\phi}{\partial\xi^2} + \left(\omega_{pe}^2 + \omega_{ph}^2\right)\phi = \frac{e}{2c^2}\left(\frac{\omega_{pe}^2}{m_e} - \frac{\omega_{ph}^2}{m_h}\right)|A|^2, \qquad (15)$$

for simplicity, the coefficient of Eq.(15) can be included in the form as follow

$$A_1\frac{d^4\phi(\xi)}{d\xi^4} + B_1\frac{d^2\phi(\xi)}{d\xi^2} + C_1\phi(\xi) = \eta|A(\xi)|^2, \quad \phi = \phi(\xi), \quad |A|^2 = |A(\xi)|^2, \qquad (16)$$

for A_1, B_1, C_1 and η are defined in **Appendix**.

When the laser field is represented as a Gaussian profile [11] $|A(\xi)|^2 = A_0^2 e^{[-\xi^2/L^2]}$, where the laser pulse length L is in the order of the plasma period $L \sim \lambda_p$, A_0 is the strength laser parameter defined as the peak amplitude of normalized vector potential of the laser field. For linear laser plasma interaction $A_0 \ll 1$, and for non linear interaction $A_0 \geq 1$ [14].

The last differential equation can be solved by using Laplace transformation [15, 16]. By using Laplace effect $\mathcal{L}$ on Eq.(15) from the left direction

$$\mathcal{L}\{A_1\frac{d^4\phi(\xi)}{d\xi^4} + B_1\frac{d^2\phi(\xi)}{d\xi^2} + C_1\phi(\xi)\} = \mathcal{L}\eta|A(\xi)|^2, \qquad (17)$$

after the effect of the Laplace transform, when the initial values $\phi(0) = 0$, $\phi^{`}(0) = 0$, Eq.(16) takes the form

$$\{A_1S^4 + B_1S^2 + C_1\}\mathcal{L}\phi(\xi) = \eta\mathcal{L}|A(\xi)|^2, \qquad (18)$$

and

$$\Phi(S) = \mathcal{L}\phi(\xi) = \frac{\eta}{\{A_1S^4+B_1S^2+C_1\}}\mathcal{L}|A(\xi)|^2. \qquad (19)$$

For getting the wakefield potential, we'll use the inverse of Laplace transform on Eq. (18)

$$\mathcal{L}^{-1}\Phi(S) = \mathcal{L}^{-1}\mathcal{L}\phi(\xi) = \mathcal{L}^{-1}\{\frac{\eta}{\{A_1S^4+B_1S^2+C_1\}}\mathcal{L}|A(\xi)|^2\}, \qquad (20)$$

for simplicity

$$\phi(\xi) = \frac{\eta}{A_1}\mathcal{L}^{-1}\{\frac{1}{\{S^4+b_1S^2+c_1\}}\mathcal{L}|A(\xi)|^2\}, \qquad (21)$$

at $b_1 = B_1/A_1$ and $c_1 = C_1/A_1$. We can put the last equation in the form of the Convolution theorem as [16]

$$\phi(\xi) = \frac{\eta}{A_1}\mathcal{L}^{-1}\{G(s).F(s)\}, \qquad (22)$$

the Convolution theorem has the solution form as

$$\mathcal{L}^{-1}\{G(s).F(s)\} = \int_0^{\xi} g(\xi - u).f(u)du, \qquad (23)$$

where $g(\xi) = \mathcal{L}^{-1}G(s)$, and $f(u) = \mathcal{L}^{-1}F(s) = \mathcal{L}^{-1}\mathcal{L}|A(u)|^2 = |A(u)|^2$.

Using the partial fraction and the Laplace functions table, and a lot of analysis substitutes in Eq.(20), the wakefield potential has the solution as

$$\phi(\xi) = \frac{\eta A_0^2}{A_1(k_1^2-k_2^2)}\int_0^{\xi} \{\frac{sin[k_2(\xi-u)]}{k_2} - \frac{sin[k_1(\xi-u)]}{k_1}\}e^{[-u^2/L^2]}du, \quad (24)$$

at k_1^2 and k_2^2 are the solutions of Eq. $S^4 + b_1S^2 + c_1$.

4. Case II

If the pressure formula takes the Fermi temperature approach as [17]

$$P_e = \lambda_e n_e^3, \qquad P_h = \lambda_h n_h^3, \quad (25)$$

at

$$\lambda_e = \frac{k_B T_F}{n_{eo}^2}, \quad \lambda_h = \frac{k_B T_F}{n_{ho}^2}, \quad T_F = \frac{\hbar^2(3\pi^2 n_{eo})^{2/3}}{2m_e},$$

where T_F is the Fermi temperature and k_B is the Boltzmann constant.

The differential Eq. (12) takes the form

$$\frac{\partial^2\phi}{\partial t^2} + \left(\omega_{pe}^2 + \omega_{ph}^2\right)\phi - \frac{4\pi e\lambda_e\left(1-\lambda M\alpha^3\right)}{m_e(4\pi e(1-\alpha))^3}\nabla^6\phi + \frac{n_{eo}\hbar^2\left(1-\alpha M^2\right)}{4m_e^2(1-\alpha)}\nabla^4\phi = \frac{e}{2c^2}\left(\frac{\omega_{pe}^2}{m_e} - \frac{\omega_{ph}^2}{m_h}\right)|A|^2, \quad (26)$$

where $\lambda = \lambda_h/\lambda_e$ and $M = m_e/m_h$.

Using $\xi = x - v_g t$ and substitution in Eq. (26)

$$\frac{\lambda_e\left(1-\alpha^3\lambda M\right)}{m_e(4\pi e)^2(1-\alpha)^3}\frac{\partial^6\phi}{\partial\xi^6} + \frac{n_{eo}\hbar^2\left(1-\alpha M^2\right)}{4m_e^2(1-\alpha)}\frac{\partial^4\phi}{\partial\xi^4} + \left(v_g^2\right)\frac{\partial^2\phi}{\partial\xi^2} + \left(\omega_{pe}^2 + \omega_{ph}^2\right)\phi = \frac{e}{2c^2}\left(\frac{\omega_{pe}^2}{m_e} - \frac{\omega_{ph}^2}{m_h}\right)|A|^2, \quad (27)$$

for simplicity, the coefficient of Eq.(27) can be included in the form as follow

$$A_2\frac{\partial^6\phi(\xi)}{\partial\xi^6} + B_2\frac{\partial^4\phi(\xi)}{\partial\xi^4} + C_2\frac{\partial^2\phi(\xi)}{\partial\xi^2} + D\phi(\xi) = \eta|A(\xi)|^2, \quad (28)$$

for A_2, B_2, C_2 and D are defined in **Appendix**.

Similarly, with The differential equation (16), it can be solved by using the Laplace transformation. By using Laplace effect $\mathcal{L}$ on Eq. (28) from the left direction

$$\mathcal{L}\{A_2\frac{\partial^6\phi(\xi)}{\partial\xi^6} + B_2\frac{\partial^4\phi(\xi)}{\partial\xi^4} + C_2\frac{\partial^2\phi(\xi)}{\partial\xi^2} + D\phi(\xi)\} = \mathcal{L}\eta|A(\xi)|^2, \quad (29)$$

after the effect of the Laplace transform, when the initial values $\phi(0) = 0$, $\grave{\phi}(0) = 0$, Eq.(29) takes the form

$$\{A_2S^6 + B_2S^4 + C_2S^2 + D\}\mathcal{L}\phi(\xi) = \eta\mathcal{L}|A(\xi)|^2, \quad (30)$$

and

$$\Phi(S) = \mathcal{L}\phi(\xi) = \frac{\eta}{\{A_2S^6+B_2S^4+C_2S^2+D\}}\mathcal{L}|A(\xi)|^2. \quad (31)$$

For getting the wakefield potential, we'll use the inverse of the Laplace transform on Eq.(31)

$$\mathcal{L}^{-1}\Phi(S) = \mathcal{L}^{-1}\mathcal{L}\phi(\xi) = \mathcal{L}^{-1}\{\frac{\eta}{\{A_2S^6+B_2S^4+C_2S^2+D\}}L|A(\xi)|^2\}, \quad (32)$$

for simplicity

$$\phi(\xi) = \frac{\eta}{A_2}\mathcal{L}^{-1}\{\frac{1}{\{S^6+b_2S^4+c_2S^2+d_2\}}\mathcal{L}|A(\xi)|^2\}, \tag{33}$$

at $b_2 = B_2/A_2$, $c_2 = C_2/A_2$ and $d_2 = D/A_2$.

We can put the last Eq. in the form of the Convolution theorem as

$$\phi(\xi) = \frac{\eta}{A_2}\mathcal{L}^{-1}\{G(s).F(s)\}, \tag{34}$$

the Convolution theorem has the solution form as

$$\mathcal{L}^{-1}\{G(s).F(s)\} = \int_0^{\xi} g(\xi-u).f(u)du, \tag{35}$$

where $g(\xi) = \mathcal{L}^{-1}G(s)$, and $f(u) = \mathcal{L}^{-1}F(s) = \mathcal{L}^{-1}\mathcal{L}|A(u)|^2 = |A(u)|^2$.

Using the partial fraction and the Laplace functions table, and a lot of analysis substitutes in eq.(33), the wakefield potential has the solution as

$$\phi(\xi) = \frac{\eta A_0^2}{A_2}\{I_1 + I_2 + I_3\}, \tag{36}$$

where

$$I_1 = \int_0^{\xi}\{\frac{\sin[k_1(\xi-u)]}{k_1(k_2^2-k_1^2)(k_3^2-k_1^2)}e^{\left[-\frac{u^2}{L^2}\right]}du, I_2 = \int_0^{\xi}\{\frac{\sin[k_2(\xi-u)]}{k_2(k_1^2-k_2^2)(k_3^2-k_2^2)}e^{\left[-\frac{u^2}{L^2}\right]}du, I_3 = \int_0^{\xi}\{\frac{sin[k_3(\xi-u)]}{k_3(k_2^2-k_3^2)(k_2^2-k_3^2)}e^{[-u^2/L^2]}du, \tag{37}$$

at k_1^2, k_2^2 and k_3^2 are the solutions of Eq. $S^6 + b_2S^4 + c_2S^2 + d_2$, when the laser field represented as a Gaussian profile $|A(u)|^2 = A_0^2e^{[-u^2/L^2]}$.

5. Summary

This study introduces the propagation of the laser pulse in the quantum semiconductor plasma. It examines the linear fluid system consisting of the continuity, and momentum equations for electrons and holes and closed by Poisson's equation. The effect of the quantum Bohm potential introduced in the system, also the pressure effect introduced in the two approaches. After using the pressure approaches, the resort differential equations in 4th order, and 6th order differential equations respectively are obtained. The differential equations are solved by using Laplace transformation and also by using the convolution theory. In case I, the pressure takes the Fermi approach, which also considers thermal pressure. The thermal pressure leads to get an ordinary differential equation in the 4th degree. In case II, the pressure takes an approach in a weakly relativistic degenerate regime with constant Fermi temperature, which leads to getting an ordinary differential equation in the 6th degree. The laser beam is introduced in the circularly polarized Gaussian profile [14, 18, 19]. The laser-driven plasma is called a laser wakefield accelerator (LWFA) when the used laser is a single beam. In the LWFA a single, short ($\leq$ 1 ps) and high intensity ($\geq 10^{17}$ w/cm ^{2}) laser pulse drive a plasma. The LWFA is driven efficiently when the laser pulse length is on the order of the plasma period $L \sim \lambda_p$, where $\lambda_p = 2\pi c/\omega_p = 2\pi/k$ or $\lambda_p(\mu m) = 3.3 \times 10^{10}\sqrt{n_{eo}(cm^{-3})}$ is the plasma wavelength. An important parameter in the discussion of laser-driven plasma is the laser strength parameter A_o, which is defined as the peak amplitude of the normalized vector potential of the laser field. The laser strength parameter related to the peak laser intensity I_0 is obtained.

6. Appendix

The coefficients of the differential equations in cases I and II are

$$A_1 = \frac{n_{eo}\hbar^2(1-\alpha M^2)}{4m_e^2(1-\alpha)}, \quad B_1 = \left(v_g^2 - \frac{v_F^2(1-\alpha M)}{(1-\alpha)m_e n_{eo}}\right), \quad C_1 = \left(\omega_{pe}^2 + \omega_{ph}^2\right).$$

$$A_2 = \frac{\lambda_e(1-\alpha^3\lambda M)}{m_e(4\pi e)^2(1-\alpha)^3}, \quad B_2 = \frac{n_{eo}\hbar^2(1-\alpha M^2)}{4m_e^2(1-\alpha)}, \quad C_2 = \left(v_g^2\right), \quad D = C_1.$$

$$\eta = \frac{e}{2c^2}\left(\frac{\omega_{pe}^2}{m_e} - \frac{\omega_{ph}^2}{m_h}\right).$$

7. References

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