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STRUCTURAL HEALTH MONITORING USING WAVELET TRANSFORM NUMERICALLY AND EXPERIMENTALLY

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ABSTRACT
Monitoring structure condition and detecting structural damage at the earliest possible stage have been a focus research recently. In this paper, a procedure for damage detection using wavelet analysis is presented. Damage identification in beams includes two steps. The first step is to detect the damage location, and the second step is to calculate the damage severity. A numerical simulation by Matlab is created to calculate the beam mode shapes. The damage location could be obtained by analyzing the fundamental mode shape by wavelet analysis. Effects of noise on detecting the damage are analyzed. Experimental tests are done to validate this technique in detecting damage location and damage severity. Two wavelet transforms (continuous, discrete) are used to analyze both the calculated and measured fundamental mode shape. Detailed parametric studies were conducted by changing damage location, intensity and wavelet families. These studies showed that the proposed technique could accurately predict the damage location and its severity.

Keywords: Beams – detection the damage location - damage severity - wavelet analysis

1. INTRODUCTION

Structural health monitoring (SHM) has been the subject of intense research development in the last two decades. In recent years, an increasing range of applications using SHM methodologies and technology, for infrastructure protection and resiliency, has been developed both exciting structures and for new constructions. The demand for the future intelligent and resilient structures requires enhanced system performance, structural safety and integrity with low maintenance cost. To meet these challenges, SHM has emerged as a reliable, efficient, and economical approach to monitor system performance.

Classical models based on the vibration analysis such as the changes in the natural frequencies and mode shapes of a structure can only detect large damages. This paper considers methods that can detect small damages. One of such methods is the Wavelet Transform (WT).

Wavelet analysis (WA) became an efficient tool in the problems of structural damage identification due to its high sensitivity to discontinuities in the signals caused by damages. Therefore, researchers are seeking for improving the sensitivity of wavelet-based-methods for high precision estimation of the presence of the damage and its position.

Identification of damage location using the continuous wavelet transform on mode shapes is studied by Douka et al.[1] and Bajabaet al [2]. They could detect the damage location without noise accurately. Silva et al. [3] estimated the crack location of the beam using the discrete wavelet transform on static deflection and first mode shape. Khatam et al. [4] detected the damage by applying wavelet transform on the acceleration response of harmonic dynamic loading applied to the beam. Ravanfar [5] detected the damage location by approach based on combination of the wavelet packet transform and entropy analysis.

Sivasubramanian et al. [6,7] detected the damage using the discrete wavelet transform and the continuous wavelet transform on the static deflection response. Rucka and Wilde [8] applied continuous wavelet transforms on static deflection lines and vibration mode shapes to detect the damage in beam, plate and shell. They found wavelet damage identification method might be suitable for detecting relatively small damage. Rucka [9] detected the damage for cantilever beam using continuous wavelet transform on the first eight modes and operational deflection shapes.

Masoumi, et al. [10] identified the damage location using the continuous and the stationary wavelet transforms on the uniform load surface. Janeliukstis et al. [11] detected the damage in polymer composite beams based on spatial continuous wavelet transform. They found that damage identification was demonstrated using statistical hypothesis approach to obtain damage localization with a confidence of 90 %. .Katunin [12] detected numerically the damage using fractional B-spline wavelets of a small crack in composite. Katunin [13-15] detected the damage in beams using high order of B-spline wavelets. He identified the crack in composite elements with non-linear geometry using spatial wavelet transform. He identified the multiple cracks in composite beams using discrete wavelet transform.

Douka et al.[17-19] estimated the crack location in double-cracked beams and plate from analyzing the mode shape using the continuous wavelet analysis. Rucka [20-21] detect numerically single damage in beam, plate and shell structures damage and multiple damages using the continuous wavelet transforms on mode shapes while the experimental tests couldn't estimate the damage location.

Janeliukstis et al. [22] estimated the damage based on 2D wavelet transform method on the mode shape of a square plate. Jaiswal et al. [23] detect the damage on beams by applying wavelet transform on the mode shape and mode shape curvature. Pakrashi et al.[24]detect the open crack using wavelet bases applied to mode shapes. Lima et al. [25] detect the damage in structures with nonlinear materials subjected to a real earthquake ground motion. They applied wavelet transform on the restoring force. Li et al. [26] detect the earthquake damage using wavelet transform applied to the acceleration response records. Zabel [27] estimated numerically the damage using wavelet analysis on the mode shape.

Estimation of damage intensity is the second step in damage identification process. Douka et al. [17-19] estimated the damage intensity basing on two approaches: the first is based on analyzing the magnitudes of detail wavelet coefficients at the location of damage, and the second is based on analyzing the energy of the signal (mode shape). Ravanfar et al. [5] estimated the severity of the damage by comparing the stiffness parameters before and after the occurrence of damage.

From the previous review, it is shown that the researchers try to apply wavelet transforms (continuous wavelet transform, discrete wavelet transform, spatial continuous wavelet transform, stationary wavelet transform) on different signal such as mode shapes, mode shape curvature, uniform load surface (ULS), static deflection, acceleration response due to impact or harmonic excitation force in order to estimate the damage location accurately and easily. So, the wavelet analysis method for damage identification is still under investigation.

The objectives of this research are summarized as follow:

1. Apply the discrete wavelet transform and continuous wavelet transform to mode shape difference instead of directly to mode shape to provide better results.
2. Examine the validation of this new methodology by experimental tests. As the experimental tests have noise due to measuring equipment.

3. Study some aspects on the identification of damage location using wavelet transform i.e. the various effects of noise and how to reduce its effect, changing the excitation location.
4. Estimate the damage severity using a proposed equation.

2. WAVELET THEORY

There are two fundamental varieties of the wavelet transform [16]: CWT and DWT. Their theories are summarized in the following:

2.1 The Continuous Wavelet Transform

The CWT is the oldest wavelet transform based on the integration of square-integrable functions $f(x) \in L^2(R)$ in the Hilbert space. This transform convolves the signal with a wavelet function $\psi(a,b)$, where $a \in Z, a > 0$ and $b \in R$ denote scaling and translation parameters, respectively. The transform could be presented by the following equation:

$$Wf(a,b,s) = S^{-2} \int_{-\infty}^{\infty} f(x)\psi\left(\frac{x-b}{a}\right)dx \qquad (1)$$

where *Wf(a,s)* is a set of wavelet coefficients. It should be noted, that CWT has no such strong restrictions to the type of wavelet function, i.e. these functions could be almost arbitrary. The only condition, which should be satisfied is the admissibility condition:

$$\int_{-\infty}^{\infty} \psi(a,b)dx = 0 \qquad (2)$$

In most cases the wavelets characterizes by the vanishing moments:

$$\int x^{\alpha} \psi(x)dx = 0 \qquad (3)$$

however, there are the wavelets used in CWT, which have no vanishing moments, e.g. some types of wavelets bounded on interval[28]

2.2 The Discrete Wavelet Transform

The discrete wavelet transform (DWT) was developed in the late 80'sbasedon multi-resolution signal representation proposed by Mallat [28]. This representation forms the descending sequence of closed functional spaces $V_j \subset L^2(R)$:

$$....V_2 \subset V_1 \subset V_0 \subset V_{-1} \subset V_{-2}... \qquad (4)$$

with properties

$$\bigcup_{j\in Z} V_j = L^2(R) \; \bigcap_{j\in Z} V_j = \{0\} \qquad (5)$$

The orthogonal supplement in a space V_j for any j in a space $V_{j-1}$is a space with orthonormal base ψjk ,j, k∈Z. Thus, there is a limitation for the wavelet and scaling function. They should be orthonormal or (semi-, bi-) orthogonal. During DWT the signal $f(x)$ is decomposed to aset of approximation coefficients (A_j) and aset of detail coefficients (D_j) in each level of decomposition. The signal could be presented in the following form:

$$f(x) = \sum_{n=-\infty}^{\infty} f_n^0 \varphi(x-n) \qquad (6)$$

where $\varphi(x-n)$ denotes translation procedure by the scaling function.

The DWT-based decomposition could be presented by the set of filters. In this case, the resulting sets of approximation and detail coefficients could be presented as:

$$f_n^{(j)}(x) = \sum_l \tilde{h}_{2n-l} f_l^{(j-1)} \tag{7}$$

$$d_n^{(j)}(x) = \sum_l \tilde{g}_{2n-l} fd_l^{(j-1)} \tag{8}$$

where $\tilde{h}$ and $\tilde{g}$ are the impulse responses of the low-pass and high-pass filters, respectively. Due to the down sampling operation during DWT, the resulting sets of coefficients have a half length with respect to the original signal in the case of single-level decomposition. Following to the dyadic rule, the resulting length of sets of obtained coefficients reduces twice with each next level of decomposition.

3. PROCEDURE

Damage detection location in beams is obtained using the wavelet transform of the fundamental mode shape. To perform this technique some procedures are applied as follows:

1. The fundamental mode shape (φ_{d1}) of the beam is measured, in the current state.
2. The healthy fundamental mode shape (φ_1) is obtained from reference state or using finite element program that simulate the healthy beam.
3. The fundamental mode shape difference (D_m) is calculated using the following equation

$$D_m = \left| j_{d1} - j_1 \right| \tag{9}$$

In this research, three Matlab programs are prepared to make the following tasks:

1. Simulate the damaged beam to obtain the modal data of huge trails of beams to ensure the validity of the proposed technique.
2. Simulate the healthy beam to obtain the modal data when no reference data is found.
   Analyze the fundamental mode shape difference using wavelet transform to detect the location of damage in the beam.

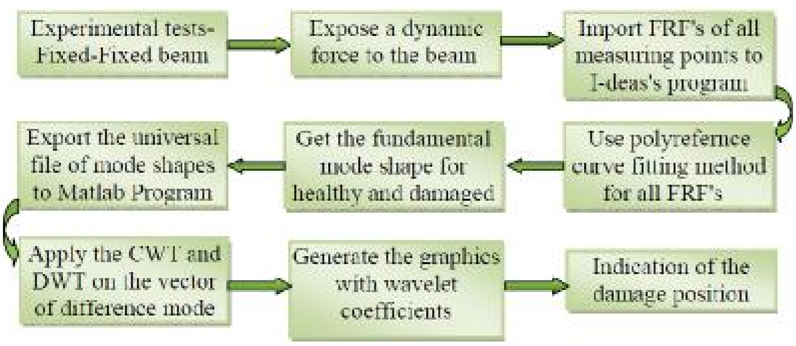


Fig.(1) General procedure adopted for the location of damage using wavelet analysis on the mode shape difference function

4. Beam FE simulation

4.1 Damaged Beam Modeling

The studied clamped-clamped beam is of length L, and has uniform rectangular cross section B x H with damage located at L_d .The beam is divided into n elements. Each element has four degrees of freedom (translation δ_i, $\delta_{i+1}$and rotation θ_i, θ_{i+1}) as shown in Fig. (2). The rotation degrees of freedom are neglected as they are very small with respect to translation degree of freedom.

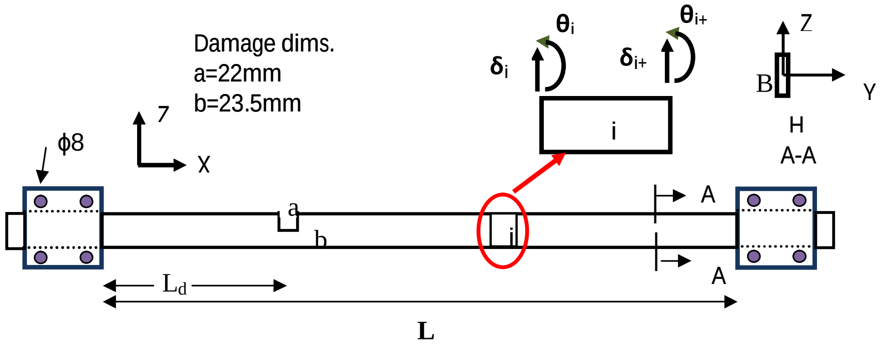


Fig. (2) Top view of a damaged steel beam

The first and second finite element programs started by constructing the undamaged stiffness matrix (K), damaged stiffness matrix (K_d) based on the assumed damage and the mass matrix (M). The damage is assumed to be reduction in one or more element's stiffness. Mass matrix is assumed unaffected by the damage. These matrices are constructed from the following equations [29]:

$$K_e = \frac{EI}{L^3}\begin{bmatrix} 12 & 6l & -12 & 6l \\ 6l & 4l^2 & -6l & 2l^2 \\ -12 & -6l & 12 & -6l \\ 6l & 2l^2 & -6l & 4l^2 \end{bmatrix} \qquad (10)$$

$$K_d = (1-a)*K_e \qquad (11)$$

$$M_e = \frac{rAL}{420}\begin{bmatrix} 156 & 22l & 54 & -13l \\ 22l & 4l^2 & 13l & -3l^2 \\ 54 & 13l & 156 & -22l \\ -13l & -3l^2 & -22l & 4l^2 \end{bmatrix} \qquad (12)$$

where: K_e, K_d, M_e, E, I, ρ, A, l and α are the stiffness matrix of undamaged element, the stiffness matrix of damaged element, element consistent mass matrix, Young's Modulus, second moment of inertia, mass density, cross section area, element length and damage ratio respectively. The element's stiffness and mass matrices are assembled to get the overall stiffness and mass matrices of the whole beam structure.

4.2SimulationResults

Four clamped-clamped steel beams of equal lengths of 1100 mm and equal cross-sections of 47x4 mm^2, are simulated. The beams have E= 210 GPa, ν =0.3 and ρ= 7800kg/m^3. The beams have single rectangular defect, a= 22mm, b =23.5mm and height of = 4mm that located at 0.37 L, 0.5L and 0.68L from left support.

Using the above described procedure, the beam is divided into 50 elements each having a length of 22mm.As a single damage, the damage is assumed in one element. The fundamental mode shape of beams undamaged and damaged at 0.37Larecalculated and displayed in Fig. (4).

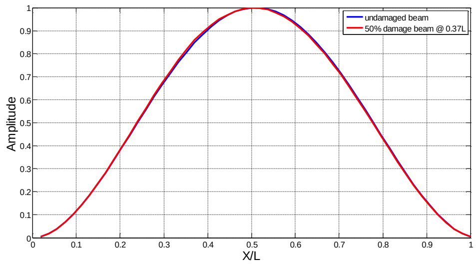
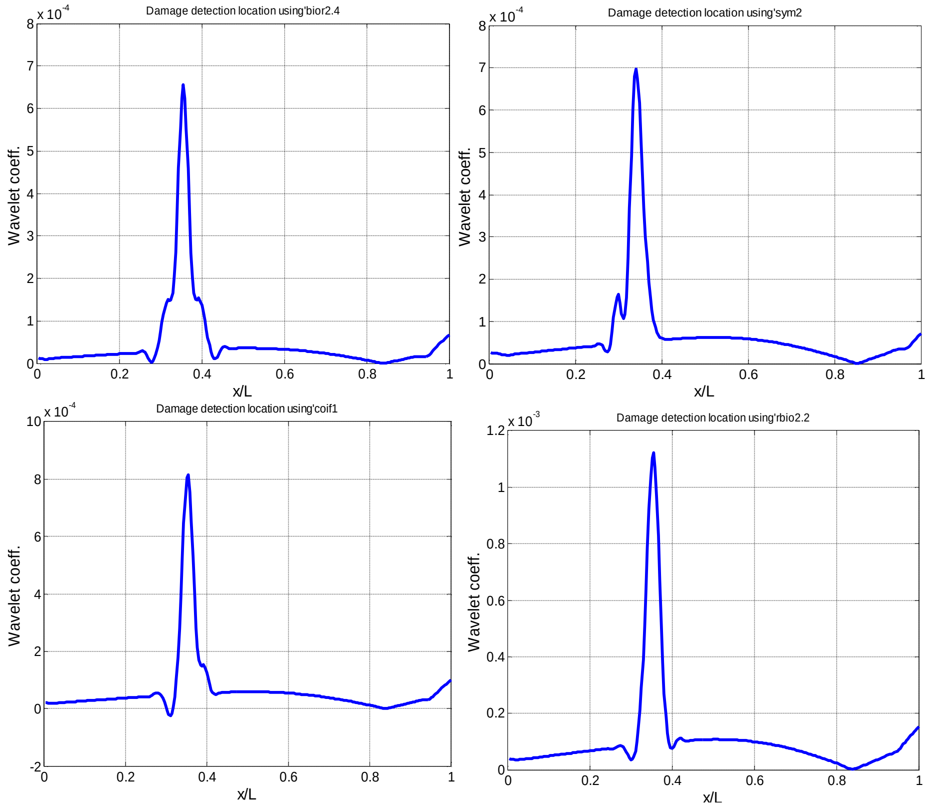
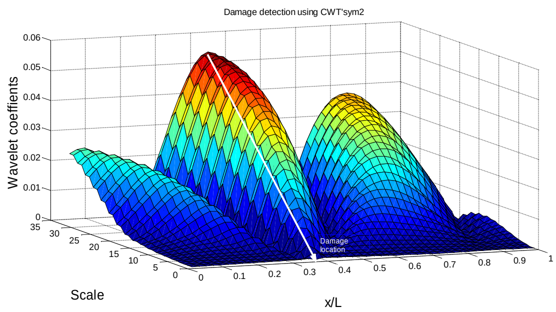


Fig.(3) The fundamental vibration mode of undamaged and damaged beam

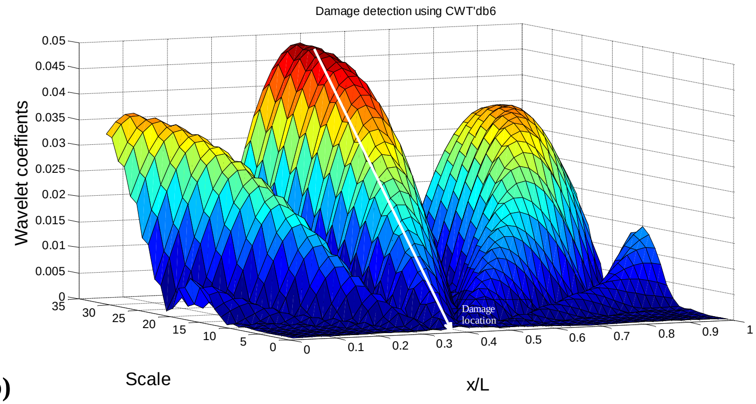
Fig. (3) show that the damaged fundamental mode shape is coincide with the healthy mode shape and cannot indicate the damage location. So, the mode difference of mode shape is analyzed by discrete wavelet transform and continuous wavelet transform to determine the location of the damage. For the discrete wavelet DWT, four wavelet families are applied. Fig. (4) indicate damage location of 50% damage at 0.37*L*with Symlets "sym2", Coiflets "coif 1", BiorSplines "bior2.4" and Reverse Bior "rbio2.2". For the continuous wavelet transform, two wavelet families are applied Fig.(5) show damage location of 50% damage at 0.37L with Symlets "sym2" and Daubechies "db6". Fig.(6) and Fig.(7) show the estimated damage location at 0.5L and 0.68L respectively.



Fig(4) DWC of 50% damage at 0.37L from left support using "bior2.4", "sym2", "coif1", "rbio2.2"



(a)



(b)

Fig.(5) CWT of 50% damage at 0.37L from left support using, (a) "sym2" and (b) "db6"

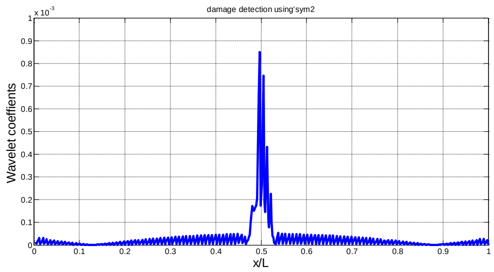


Fig.(6) DWC of 50% damage at 0.5L from left support using "sym2"

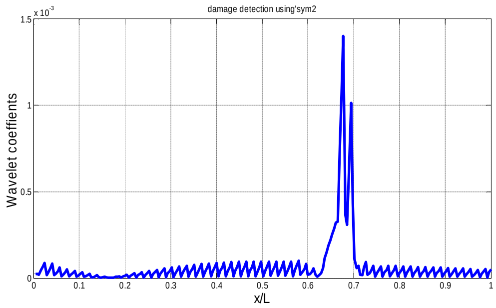


Fig.(7) DWC of 50% damage at 0.68L from left support using "sym2"

4.3 Effect of wavelet family

In order to investigate the wavelet family's effect on the identification of damage location, nine locations of single damage are analyzed by four families ("db2", "sym4", "bior 2.4" and "coif 5"). The errors in identification the damage location are shown in Fig.(8) and it is estimated from the following equation;

$$Error = \left| \frac{L_e - L_a}{L} \right| \qquad (13)$$

where; L_e is the estimated damage location,

L_a is the assumed damage location and L is the beam length

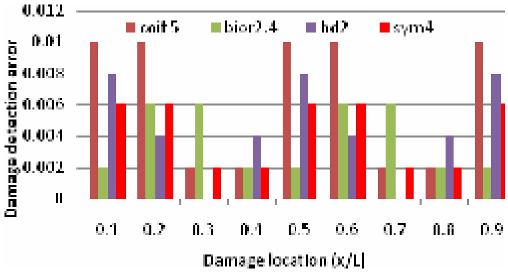


Fig.(8) Error in identification of the damage location using four wavelet families (''Coif 5'', ''bior2.4'', ''db2'', ''sym4'')

The overall estimation errors for the detection of damage location of all wavelet families are between 0 - 0.01. The errors increase near the boundary. As the analyzed signal (mode shape) have a finite length, the wavelet transform produces the high magnitude coefficients at the boundary of a signal due to sudden drop to zero- valued magnitude of mode shape in these regions. So, wavelet family has an effect on detecting the damage location.

4.4 Effect of noise on wavelet analysis

In order to investigate the method's performance with respect to noise containment of the measured data, mode shape is superimposed with simulated white noise 5% (random values with zero mean and variance equals 1) for the beam has damage at 0.375L and its severity 50% assumed in [22].

To reduce the effect of noise on damage identification, a smooth function is applied to the noisy mode shape. Fig.(9) shows how smoothing the mode shape reduces its noise.

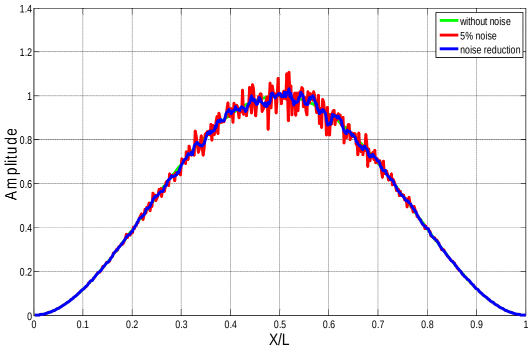


Fig.(9) Fundamental mode shape smoothing of damaged beam (at 0.37*L*)

Wavelet coefficients calculated due both the noisy and smoothed mode shapes are shown in Fig.(10). For the noisy mode shape, the max wavelet coefficients appear in several locations. This complicates identification of the damage. However, max wavelet coefficient appears on one location (at the true damage location at 0.37L). This indicated that the wavelet analysis could successfully detect damages due to experimentally measured mode shapes with modest signal-to-noise-ratios.

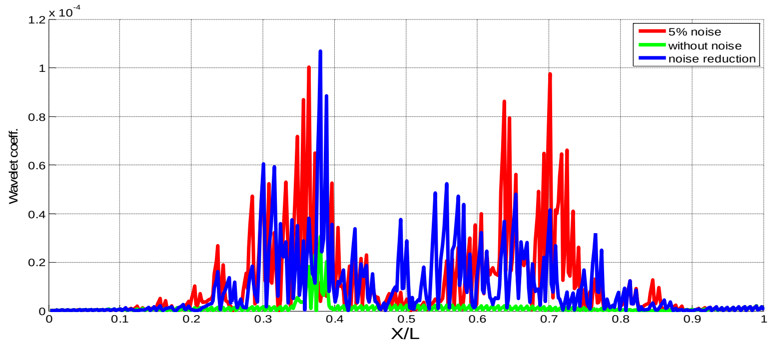


Fig.(10) Wavelet coefficients of fundamental mode shape with and without noise and with noise reduction

5. DAMAGE SEVERITY

The second step after damage location identification is the determination of the damage severity. In this research, the determination of damage severity is based on the analysis on magnitudes of detail wavelet coefficients in the location of damage. Many factors affect the damage severity such as damage location, wavelet transform, wavelet family, number of vanishing moments, wavelet coefficients, finite element division and location of the excitation force. These factors make the determination of damage severity very difficult. Therefore, numerous studies are done to examine the influence of damage location and wavelet coefficients on the intensity from damage location 0.01L to 0.99L. For this study, the determination of damage locations and severity are done using Symlts wavelet family "sym2". The excitation location was assumed at the beam middle. Fig.(11) represents the relation between absolute difference in detail wavelet coefficient at the damage location for different cases of single damage from 0.01L to 0.99L and intensity ratio. For any damage location, the AMDWC increase if damage intensity increase due to damage effect. The general procedure for damage severity presented in Fig.(12).

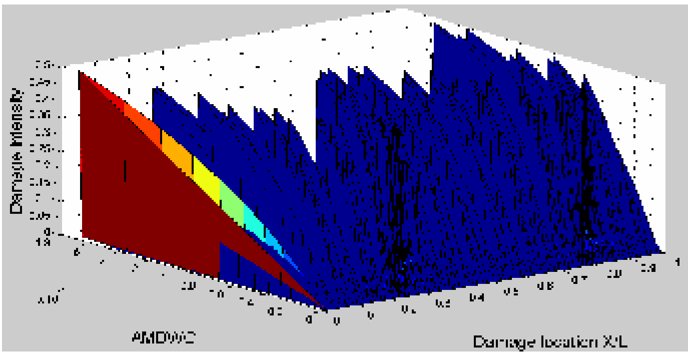


Fig.(11) Damage intensity of all damage location

To simplify the calculation of damage intensity, a proposed mathematical equation is driven as follows.

$$Damage.Intensty = 0.2255 + 0.03047\,L_d + 75.74C + 0.06188\,L_d^{\,2} - 0.923L_dC$$
$$-2862C^2 - 0.1108X_d^3 + 6.987L_d^2C - 192.7L_dC^2 + 3.346C^3$$

(14)

where X_d is the damage location and C is calculated from this equation

$$C = AMDWC / f \tag{15}$$

Where *AMDWC* is the absolute maximum of discrete wavelet coefficient at the damage location and *f* is a factor depends on the number of beam divisions (n) and calculated as follows

$$f = 3.335e^{-7} n^4 - 0.0001832\, n^3 + 0.0308\, n^2 - 2.114\, n + 54.27 \tag{16}$$

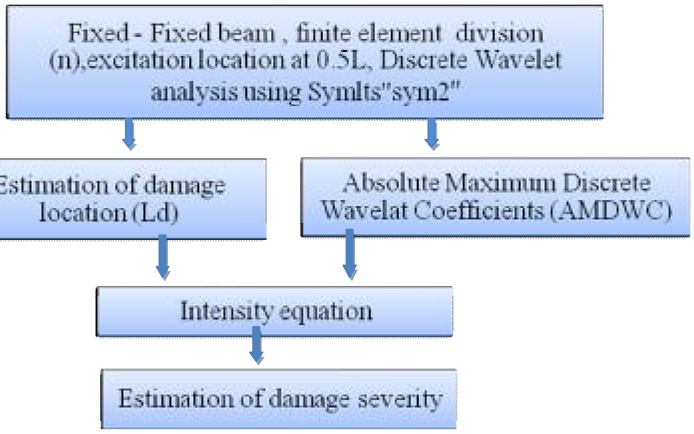


Fig.(12) General procedure adopted for the damage severity

For numerical damage simulation, the proposed equation is applied to damage cases (L_d from 0.1L to 0.9L each 0.1L and $\alpha_{assumed}$ from 5% to 50% each 5%). The error percentage in estimating the damage severity not exceed 10% except at 0.22L and 0.78L as they represent the static contra curvature points as indicated in Fig. (13).

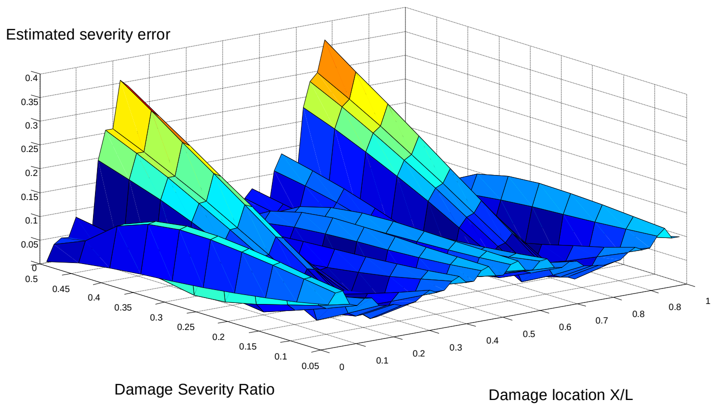


Fig.(13) Damage severity error for all damage location

6. EXPERIMENTAL VERIFICATION

6.1 Beam specimen set-up

To validate the analytical results of the proposed procedure, four steel beams (one undamaged and three damaged) have been tested. Each beam has a length of 1100 mm, rectangular cross-section of 47x4 mm and is clamped at both ends. Material properties of the beams are Young's modulus $E = 210$ GPa, Poisson ratio $\nu = 0.3$ and mass density $\rho = 7800 \text{kg/m}^3$. The damaged beams have single rectangular defects with a= 22mm, b = 23.5mm through the thickness, at L_d equal 0.5L, 0.37L, 0.73L respectively from left support.

Testing was performed in the structural dynamics laboratory at Aerospace Engineering Department, Cairo University. All beams are subjected to the same excitation force (multi-sine signal [B&K Guided tours]) at point ***f***. The excitation force is applied by B&K type 4809 permanent magnet exciter and B&K type 2816 Power Amplifier Unit. Acceleration responses are measured using B&K accelerometers. Two tests were performed, the first test used accelerometers (A1-A6) and the second test used accelerometers (A7-A12) as shown in Fig.(14). The accelerometers locations from left support are indicated at Table (1). The data were collected by B&K Multichannel Data Acquisition Unit type 2816 as shown in Fig.(15). Test specimens (D0, D1, D2 and D3) are shown in Figs.(16,17,18 and 19). Collected data are then delivered to a P.C. for further post processing. Natural frequency, modal damping and mode shape of the fundamental mode were extracted from the experimentally measured FRF's at the measured points.

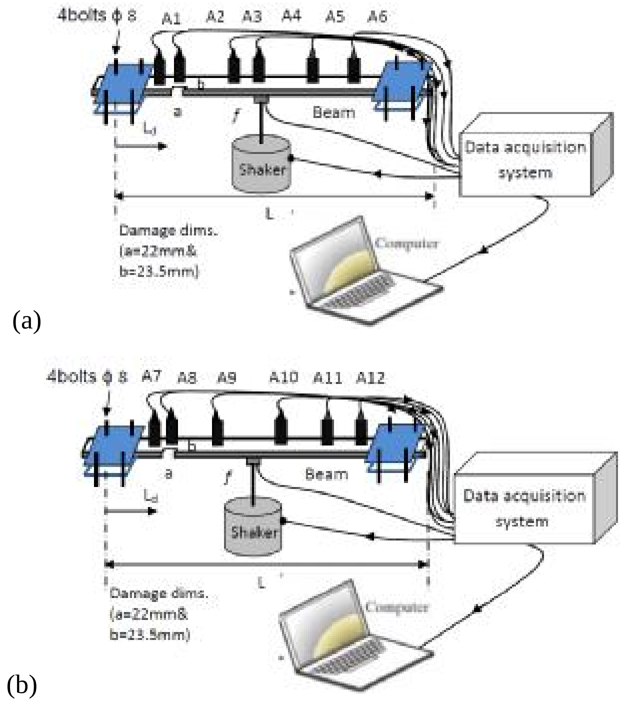


Fig.(14) experiment set up of (a) test1 and (b)test2

Table (1) accelerometers and their locations from left support

Accelerometer no.	L_d/L	Accelerometer no.	L_d/L
A1	**0.23**	**A7**	**0.29**
A2	**0.36**	**A8**	**0.36**
A3	**0.45**	**A9**	**0.42**
A4	**0.50**	**A10**	**0.55**
A5	**0.58**	**A11**	**0.64**
A6	**0.71**	**A12**	**0.77**

Table (2) Damage scenario and location

Beam no.	Damage Cases	Damage location(L_d/L)
I	Undamaged (D0)	-
II	Single (D1)	0.5
III	Single (D2)	0.37
IV	Single (D3)	0.68

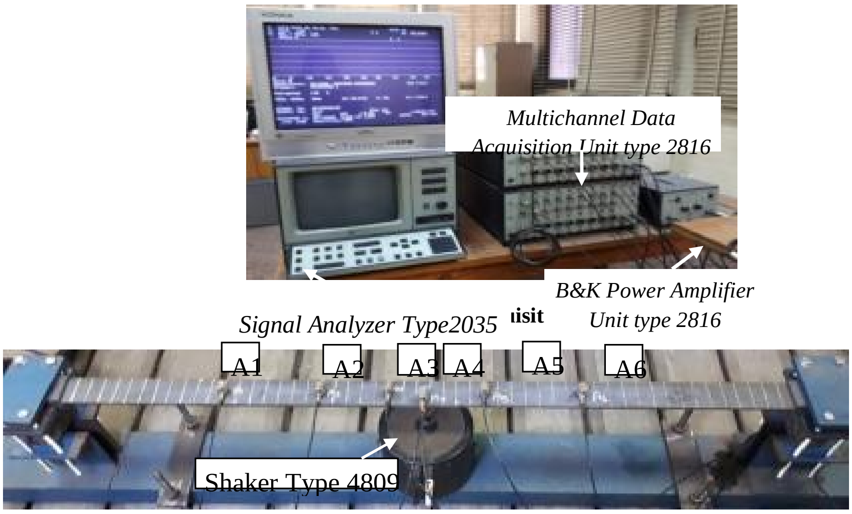


Fig.(16) Specimen D0 (undamaged)

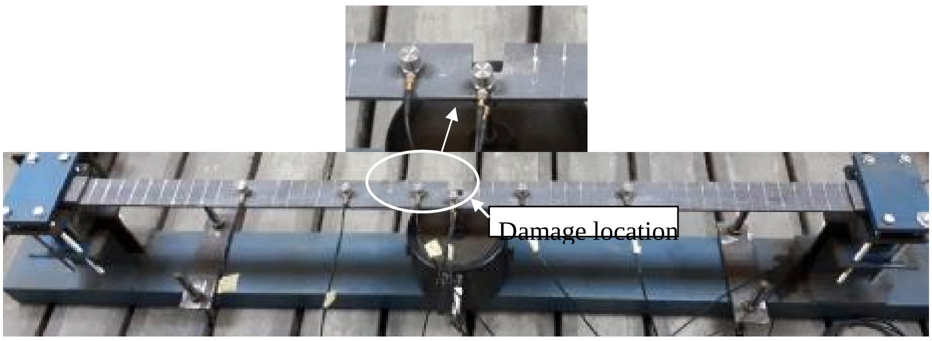


Fig.(17) Specimen D1;damaged at 0.5*L*

Fig.(18) Specimen D2;damaged at 0.37*L*

Fig.(19) Specimen D3;damaged at 0.68*L*

6.2 Experimental Results

6.2.1 Damage localization using wavelet transform on Mode shape difference

A plot of the calculated and measured fundamental mode shape of the undamaged beam is shown in Fig.(20). The measured mode shape is coincided on the calculated mode shape.

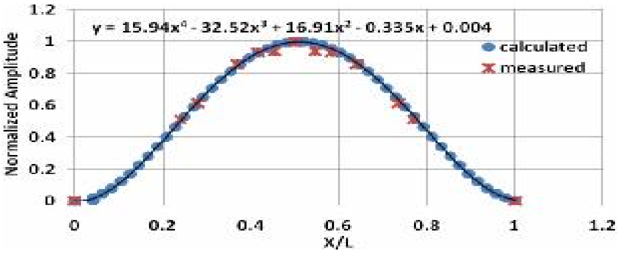


Fig.(20) The calculated and measured fundamental mode of the healthy beam

For damage case (D1), the fundamental mode shapes for undamaged and damaged beam are shown in Fig.(21). Which indicated that the damaged mode shape have lower values than healthy mode shape due to the introduced damage at 0.5L that make the beam more flexible than healthy case. Also, it is found that the mode shapes only don't indicate the damage location. So, the wavelet analysis is applied on the difference of mode shapes.

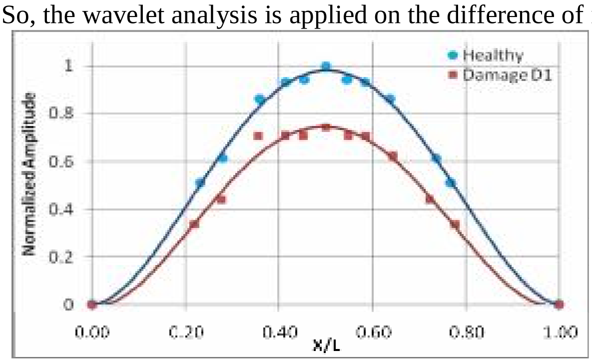


Fig.(21) Comparison between measured fundamental mode of the healthy and damage case D1

By using DWT on the mode shapes difference function of damage case D1 The damage location appears at the maximum wavelet coefficients as shown in Fig.(22). Also, using CWT presented MADWC appears at the estimated damage (0.5L) as shown in Fig.(23).

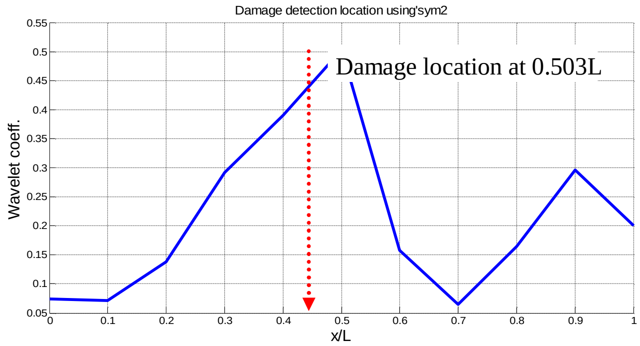


Fig.(22) DWC of the damaged beam D1

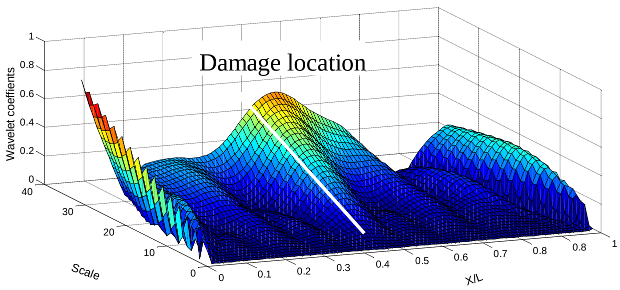


Fig.(23) CWC of the damaged beam D1 using ''sym2'

For damage casesD2 and D3 the estimated damage location are found at 0.375L and 0.68L as shown in Fig.(24) and Fig.(25) respectively. This reflects the ability of the discrete wavelet transform and the continuous wavelet transform in detecting the damage location.

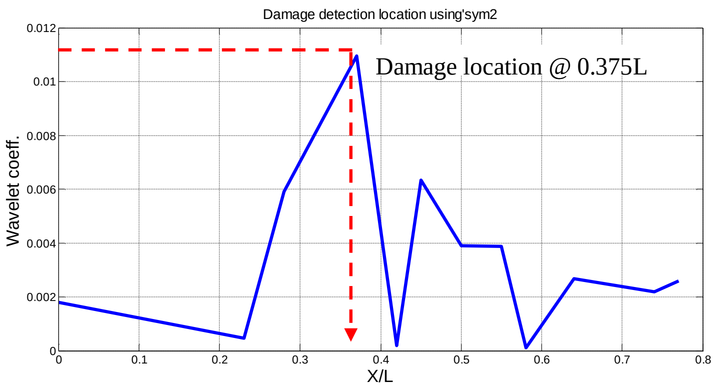


Fig.(24) DWC of the damage case D2

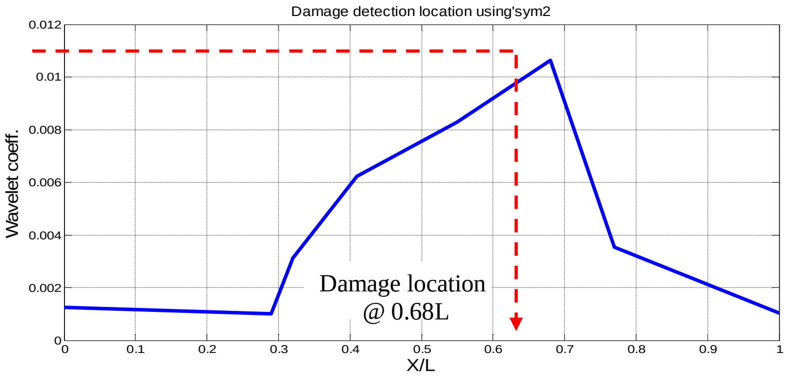


Fig.(25) DWC of the damage case D3

6.3 Effect of excitation location

To study of the effect of changing the excitation location on the proposed procedure, two locations of excitation at X_1= 0.5L and X_2= 0.58L are used. These two locations are applied to damage case D1. The normalized mode shapes are estimated from the measuring data points for two excitation locations. In Fig.(26) the mode shapes due to X_1 and X_2 seem to be nearly parallel. The values of normalized amplitude of mode shape at X_1 are higher than X_2, where X_1 is near the damage location. The maximum absolute discrete wavelet coefficients indicated in Fig.(27) for two mode shapes are indicated at the damage location. So, the excitation location doesn't affect the identification damage of location.

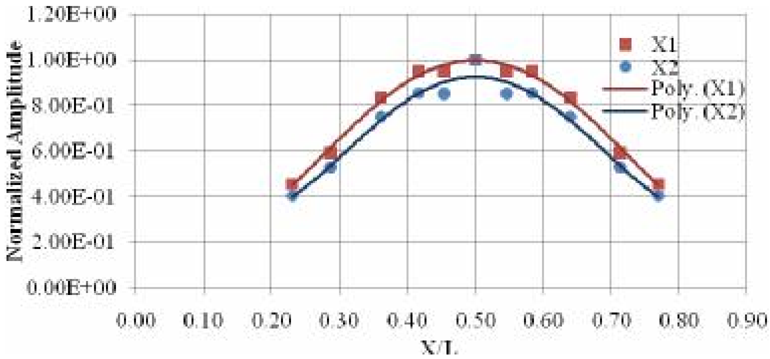
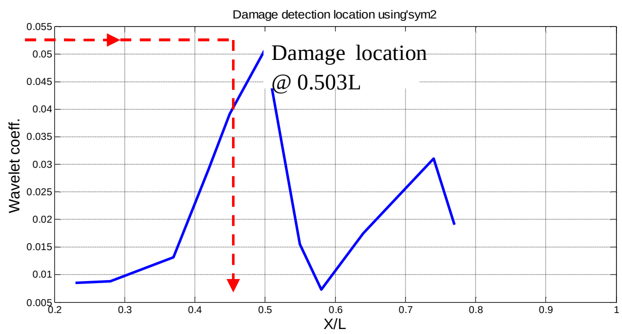
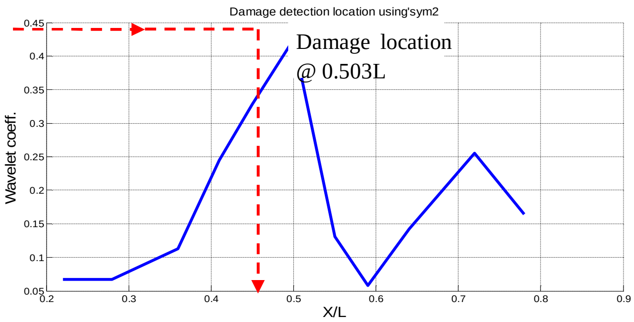


Fig.(26) Normalized mode shape of damaged case D1 For X_1 and X_2



(a)



(b)

Fig.(27) DWC of damaged case D1(a) Excitation location at X1 and (b) Excitation location at X2

6.4 Damage severity

By applying the proposed severity equation to the experimental damage results, the damage severity for three damages cases is shown in Table (3). The estimated errors for the damage severity are acceptable.

Table (3) The estimated severity for damage 50% for all cases

Damage case	Damage severity estimated	Error percentage %
D1	53	3
D2	41	9
D3	39	11

7- CONCLUSION

In this paper, the wavelet transform was applied to identification location and severity of damage in beam structures. This could be done by using the fundamental mode shape. The main conclusions drawn from this study are as follows:

محذوف:

1. Changing the type of wavelet function has an influence on the accuracy of damage identification.
2. Smoothing the mode shape reduces the noise effect.
3. Changing the excitation location has no effect on damage identification.
4. Less measuring data point needs wavelet transform family with low vanishing moment to get better results.
5. The proposed equation gives good results for the estimation of damage severity.
6. From numerical and experimental investigation, the proposed method was found able to accurately detect damage.

محذوف:

The proposed procedure can systematically extended for plates and other mechanical and civil engineering structures.

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